VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2012

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

									Letter
Figures									
Words									

SPECIALIST MATHEMATICS

Written examination 1

Friday 9 November 2012

Reading time: 9.00 am to 9.15 am (15 minutes)
Writing time: 9.15 am to 10.15 am (1 hour)

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are not permitted to bring into the examination room: notes of any kind, a calculator of any type, blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 9 pages with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.
Unless otherwise specified an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

Find an antiderivative of $\frac{6+x}{x^2+4}$.

2 marks

Question 2

Find all real solutions of the equation $2\cos(x) = \sqrt{3}\cot(x)$.

3 marks

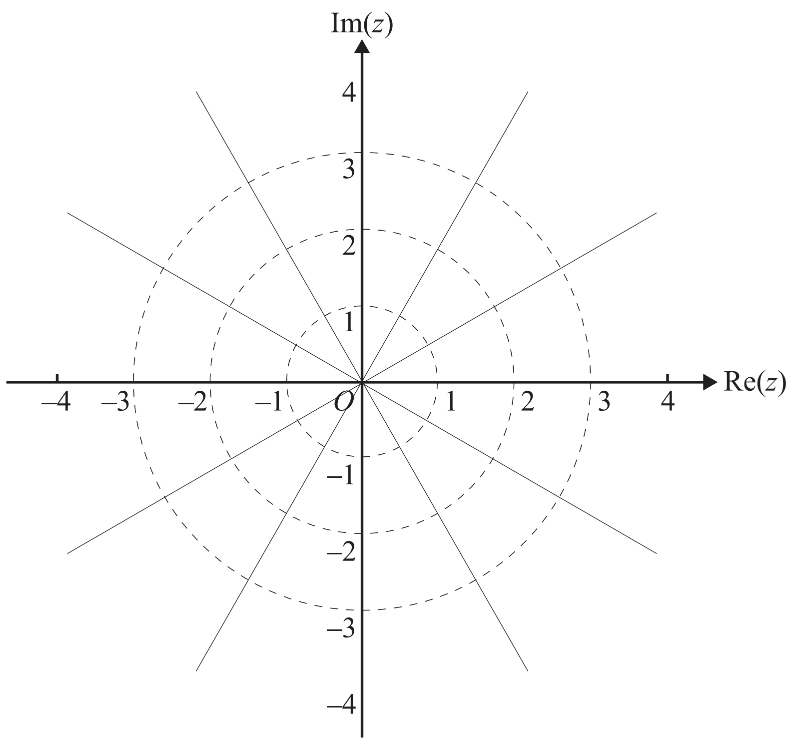
Question 3

Consider the equation $z^3 - z^2 - 2z - 12 = 0$, $z \in C$.

a. Given that $z = 2\text{cis}\left(\frac{2\pi}{3}\right)$ is a root of the equation, find the other two roots in the form $a + ib$, where $a, b \in R$.

3 marks

b. Plot all of the roots clearly on the Argand diagram below.

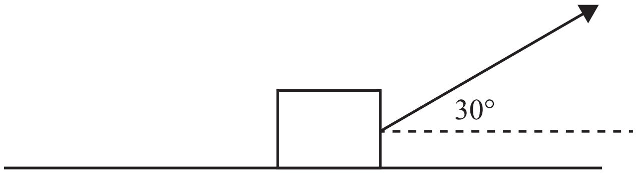


1 mark

Question 4

A crate of mass 50 kg sits on a rough floor. A tension force of T newtons is applied to the crate at 30° to the horizontal. The coefficient of friction is 0.2.

a. On the diagram below, show all forces acting on the crate and label them.



2 marks

b. Write down the maximum value of T that can be applied without the crate leaving the floor.

1 mark

c. Find the value of T required for the crate to be on the point of moving. Give your answer in the form $\frac{ag}{1+b\sqrt{c}}$, where a, b and c are integers, and g is the acceleration due to gravity.

3 marks

Question 5

Let $y = \arctan(2x)$.

Find the value of a given that $\dfrac{d^2y}{dx^2} = ax\left(\dfrac{dy}{dx}\right)^2$, where a is a real constant.

3 marks

Question 6

Find the gradient of the tangent to the curve $xy^2 + y + \left(\log_e(x-2)\right)^2 = 14$ at the point $(3, 2)$.

3 marks

Question 7

Consider the curve with equation $y = (x - 1)\sqrt{2 - x}$, $1 \le x \le 2$.

Calculate the area of the region enclosed by the curve and the x-axis.

3 marks

Question 8

The velocity, v m/s, of a body when it is x metres from a fixed point O is given by

$$v = \frac{2x}{\sqrt{1 + x^2}}.$$

Find an expression for the acceleration of the body in terms of x in simplest form.

3 marks

Question 9

The position of a particle at time t is given by

$$\underset{\sim}{r}(t) = \left(2\sqrt{t^2+2} - t^2\right)\underset{\sim}{i} + \left(2\sqrt{t^2+2} + 2t\right)\underset{\sim}{j},\ t \geq 0.$$

a. Find the velocity of the particle at time t.

1 mark

b. Find the speed of the particle at time $t = 1$ in the form $\frac{a\sqrt{b}}{c}$, where a, b and c are positive integers.

2 marks

c. Show that at time $t = 1$, $\frac{dy}{dx} = \frac{1+\sqrt{3}}{1-\sqrt{3}}$.

2 marks

d. Find the angle in terms of π, between the vector $-\sqrt{3}\ \underset{\sim}{i} + \underset{\sim}{j}$ and the vector $\underset{\sim}{r}(t)$ at time $t = 0$.

2 marks

Question 10

Consider the functions with rules $f(x) = \arcsin\left(\frac{x}{2}\right) + \frac{3}{\sqrt{25x^2 - 1}}$ and $g(x) = \arcsin(3x) - \frac{3}{\sqrt{25x^2 - 1}}$.

a. **i.** Find the maximal domain of $f_1(x) = \arcsin\left(\frac{x}{2}\right)$.

ii. Find the maximal domain of $f_2(x) = \frac{3}{\sqrt{25x^2 - 1}}$.

iii. Find the largest set of values of $x \in R$ for which $f(x)$ is defined.

1 + 1 + 1 = 3 marks

b. Given that $h(x) = f(x) + g(x)$ and that $\theta = h\left(\frac{1}{4}\right)$, evaluate $\sin(\theta)$.

Give your answer in the form $\frac{a\sqrt{b}}{c}$, $a, b, c \in Z$.

3 marks

END OF QUESTION AND ANSWER BOOK

SPECIALIST MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc\sin A$
sine rule:	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
cosine rule:	$c^2 = a^2 + b^2 - 2ab\cos C$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Circular (trigonometric) functions

$\cos^2(x) + \sin^2(x) = 1$

$1 + \tan^2(x) = \sec^2(x)$ $\cot^2(x) + 1 = \operatorname{cosec}^2(x)$

$\sin(x + y) = \sin(x)\cos(y) + \cos(x)\sin(y)$ $\sin(x - y) = \sin(x)\cos(y) - \cos(x)\sin(y)$

$\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$ $\cos(x - y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$ $\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$

$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$

$\sin(2x) = 2\sin(x)\cos(x)$ $\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + yi = r(\cos\theta + i\sin\theta) = r\,\mathrm{cis}\,\theta$

$|z| = \sqrt{x^2 + y^2} = r$ $\qquad -\pi < \mathrm{Arg}\, z \le \pi$

$z_1 z_2 = r_1 r_2\,\mathrm{cis}(\theta_1 + \theta_2)$ $\qquad \dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\mathrm{cis}(\theta_1 - \theta_2)$

$z^n = r^n\,\mathrm{cis}(n\theta)$ (de Moivre's theorem)

Calculus

$\dfrac{d}{dx}\left(x^n\right) = nx^{n-1}$ $\qquad \int x^n\,dx = \dfrac{1}{n+1}x^{n+1} + c, n \ne -1$

$\dfrac{d}{dx}\left(e^{ax}\right) = ae^{ax}$ $\qquad \int e^{ax}\,dx = \dfrac{1}{a}e^{ax} + c$

$\dfrac{d}{dx}\left(\log_e(x)\right) = \dfrac{1}{x}$ $\qquad \int \dfrac{1}{x}\,dx = \log_e|x| + c$

$\dfrac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$ $\qquad \int \sin(ax)\,dx = -\dfrac{1}{a}\cos(ax) + c$

$\dfrac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$ $\qquad \int \cos(ax)\,dx = \dfrac{1}{a}\sin(ax) + c$

$\dfrac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$ $\qquad \int \sec^2(ax)\,dx = \dfrac{1}{a}\tan(ax) + c$

$\dfrac{d}{dx}\left(\sin^{-1}(x)\right) = \dfrac{1}{\sqrt{1-x^2}}$ $\qquad \int \dfrac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\dfrac{x}{a}\right) + c, a > 0$

$\dfrac{d}{dx}\left(\cos^{-1}(x)\right) = \dfrac{-1}{\sqrt{1-x^2}}$ $\qquad \int \dfrac{-1}{\sqrt{a^2-x^2}}\,dx = \cos^{-1}\left(\dfrac{x}{a}\right) + c, a > 0$

$\dfrac{d}{dx}\left(\tan^{-1}(x)\right) = \dfrac{1}{1+x^2}$ $\qquad \int \dfrac{a}{a^2+x^2}\,dx = \tan^{-1}\left(\dfrac{x}{a}\right) + c$

product rule: $\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

quotient rule: $\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

chain rule: $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$

Euler's method: If $\dfrac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

acceleration: $a = \dfrac{d^2x}{dt^2} = \dfrac{dv}{dt} = v\dfrac{dv}{dx} = \dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right)$

constant (uniform) acceleration: $v = u + at$ $\qquad s = ut + \dfrac{1}{2}at^2$ $\qquad v^2 = u^2 + 2as$ $\qquad s = \dfrac{1}{2}(u + v)t$

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$

$\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos\theta = x_1 x_2 + y_1 y_2 + z_1 z_2$

$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$

Mechanics

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

friction: $F \le \mu N$

END OF FORMULA SHEET