VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2012

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

Letter

Figures

Words

SPECIALIST MATHEMATICS

Written examination 2

Monday 12 November 2012

Reading time: 3.00 pm to 3.15 pm (15 minutes)
Writing time: 3.15 pm to 5.15 pm (2 hours)

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 25 pages with a detachable sheet of miscellaneous formulas in the centrefold.
- Answer sheet for multiple-choice questions.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- All written responses must be in English.

At the end of the examination
- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

The graph with equation $y = \dfrac{1}{2x^2 - x - 6}$ has asymptotes given by

A. $x = -\dfrac{3}{2}$, $x = 2$ and $y = 1$

B. $x = -\dfrac{3}{2}$ and $x = 2$ only

C. $x = \dfrac{3}{2}$, $x = -2$ and $y = 0$

D. $x = -\dfrac{3}{2}$, $x = 2$ and $y = 0$

E. $x = \dfrac{3}{2}$ and $x = -2$ only

Question 2

A rectangle is drawn so that its sides lie on the lines with equations $x = -2$, $x = 4$, $y = -1$ and $y = 7$.
An ellipse is drawn inside the rectangle so that it just touches each side of the rectangle.
The equation of the ellipse could be

A. $\dfrac{x^2}{9} + \dfrac{y^2}{16} = 1$

B. $\dfrac{(x+1)^2}{9} + \dfrac{(y+3)^2}{16} = 1$

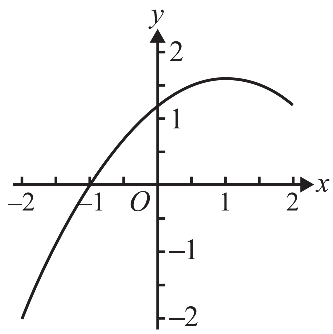
C. $\dfrac{(x-1)^2}{9} + \dfrac{(y-3)^2}{16} = 1$

D. $\dfrac{(x+1)^2}{36} + \dfrac{(y+3)^2}{64} = 1$

E. $\dfrac{(x-1)^2}{36} + \dfrac{(y-3)^2}{64} = 1$

Question 3

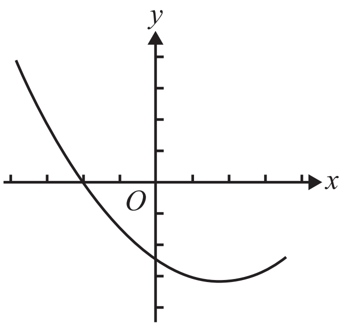
The graph of $y = f(x)$ is shown below.



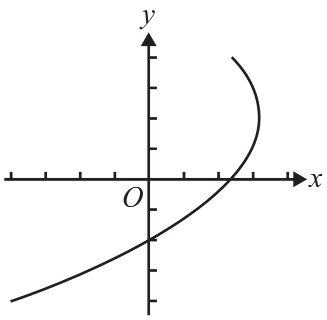
All of the axes below have the same scale as the axes in the diagram above.

The graph of $y = \dfrac{1}{f(x)}$ is best represented by

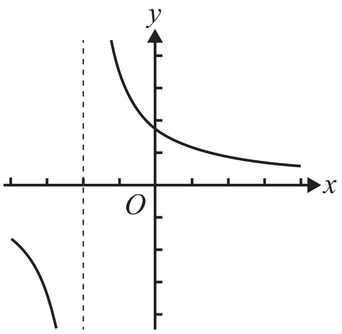
A.



B.



C.



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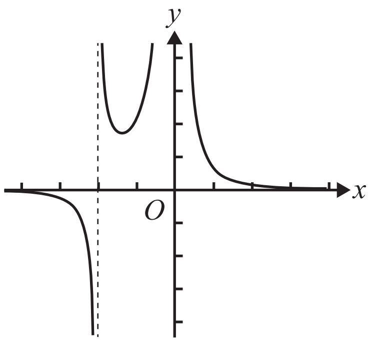
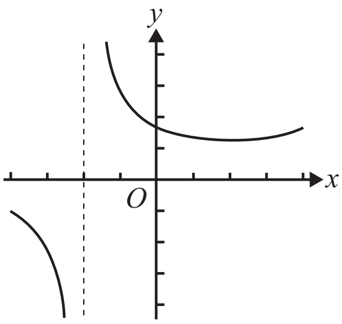


E.



Question 4

The domain and range of the function with rule $f(x) = \arccos(2x-1) + \dfrac{\pi}{2}$ are respectively

A. $[-2, 0]$ and $[0, \pi]$

B. $[-2, 0]$ and $\left[\dfrac{\pi}{2}, \dfrac{3\pi}{2}\right]$

C. $[0, 1]$ and $[0, \pi]$

D. $[0, 1]$ and $\left[\dfrac{\pi}{2}, \dfrac{3\pi}{2}\right]$

E. $[0, \pi]$ and $[0, 1]$

Question 5

If $z = \sqrt{2}\text{cis}\left(-\dfrac{4\pi}{5}\right)$ and $w = z^9$, then

A. $w = 16\sqrt{2}\text{cis}\left(\dfrac{36\pi}{5}\right)$

B. $w = 16\sqrt{2}\text{cis}\left(-\dfrac{\pi}{5}\right)$

C. $w = 16\sqrt{2}\text{cis}\left(\dfrac{4\pi}{5}\right)$

D. $w = 9\sqrt{2}\text{cis}\left(-\dfrac{\pi}{5}\right)$

E. $w = 9\sqrt{2}\text{cis}\left(\dfrac{4\pi}{5}\right)$

Question 6

For any complex number z, the location on an Argand diagram of the complex number $u = i^3\,\overline{z}$ can be found by

A. rotating z through $\dfrac{3\pi}{2}$ in an anticlockwise direction about the origin

B. reflecting z about the x-axis and then reflecting about the y-axis

C. reflecting z about the y-axis and then rotating anticlockwise through $\dfrac{\pi}{2}$ about the origin

D. reflecting z about the x-axis and then rotating anticlockwise through $\dfrac{\pi}{2}$ about the origin

E. rotating z through $\dfrac{3\pi}{2}$ in a clockwise direction about the origin

Question 7

The set of points in the complex plane defined by $|z + 2i| = |z|$ corresponds to

A. the point given by $z = -i$

B. the line $\text{Im}(z) = -1$

C. the line $\text{Im}(z) = -i$

D. the line $\text{Re}(z) = -1$

E. the circle with centre $-2i$ and radius 1

Question 8

If $z = a + bi$, where both a and b are non-zero real numbers and $z \in C$, which of the following does **not** represent a real number?

A. $z + \overline{z}$

B. $|z|$

C. $z\overline{z}$

D. $z^2 - 2abi$

E. $(z - \overline{z})(z + \overline{z})$

Question 9

Euler's formula is used to find y_2, where $\frac{dy}{dx} = \cos(x)$, $x_0 = 0$, $y_0 = 1$ and $h = 0.1$

The value of y_2 correct to four decimal places is

A. 1.1000 and this is an underestimate of $y(0.2)$

B. 1.1995 and this is an overestimate of $y(0.2)$

C. 1.1995 and this is an underestimate of $y(0.2)$

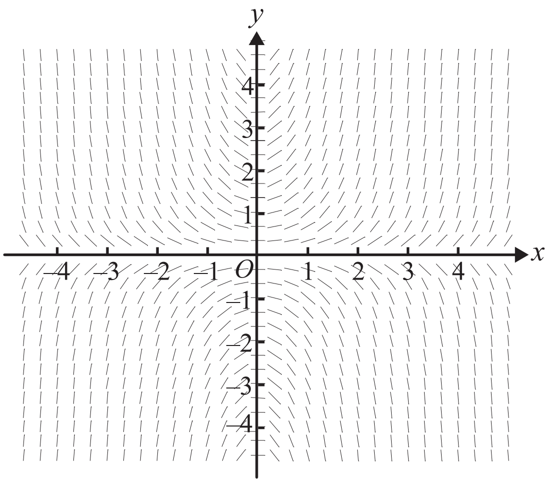
D. 1.2975 and this is an underestimate of $y(0.2)$

E. 1.2975 and this is an overestimate of $y(0.2)$

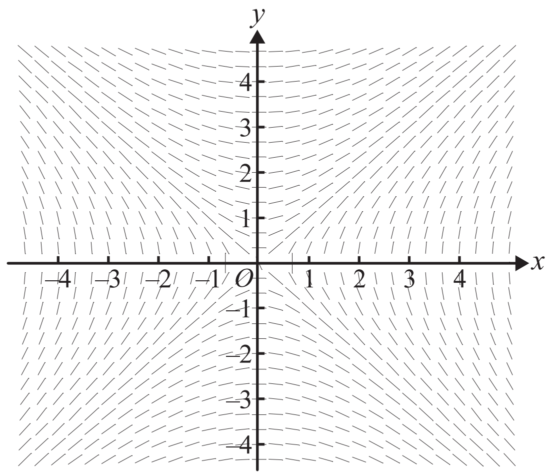
Question 10

The diagram that best represents the direction field of the differential equation $\frac{dy}{dx} = xy$ is

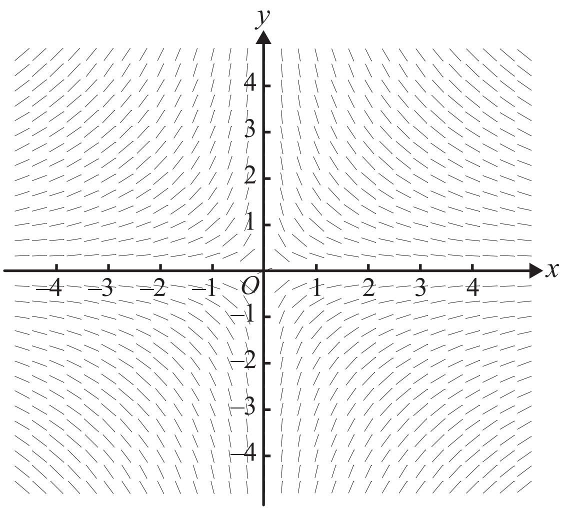
A.



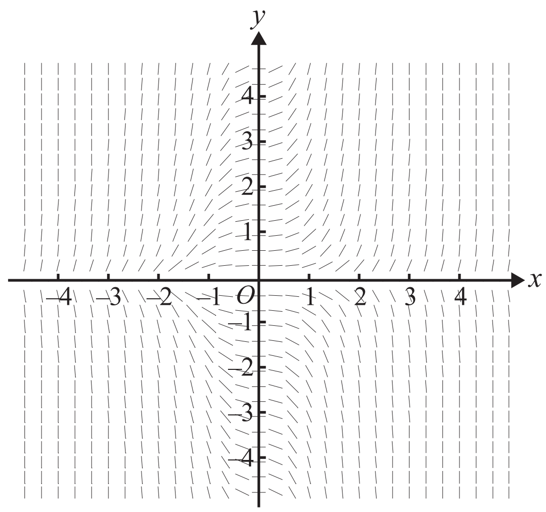
B.



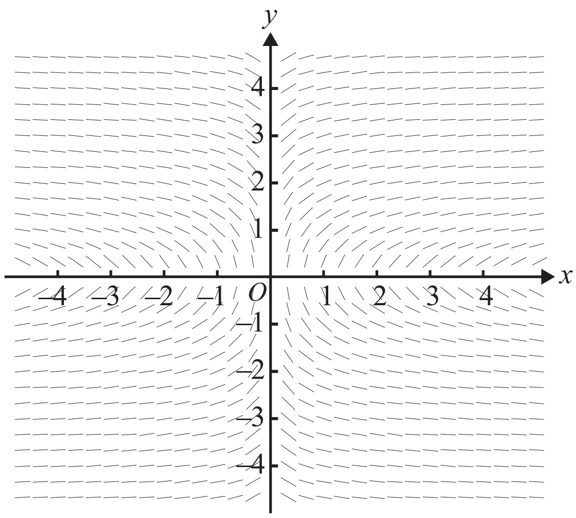
C.



D.



E.



Question 11

If $\dfrac{d^2y}{dx^2} = x^2 - x$ and $\dfrac{dy}{dx} = 0$ at $x = 0$, then the graph of y will have

A. a local minimum at $x = \dfrac{1}{2}$

B. a local maximum at $x = 0$ and a local minimum at $x = 1$

C. stationary points of inflection at $x = 0$ and $x = 1$, and a local minimum at $x = \dfrac{3}{2}$

D. a stationary point of inflection at $x = 0$, no other points of inflection and a local minimum at $x = \dfrac{3}{2}$

E. a stationary point of inflection at $x = 0$, a non-stationary point of inflection at $x = 1$ and a local minimum at $x = \dfrac{3}{2}$

Question 12

The volume of the solid of revolution formed by rotating the graph of $y = \sqrt{9-(x-1)^2}$ about the x-axis is given by

A. $4\pi(3)^2$

B. $\pi\int_{-3}^{3}\left(9-(x-1)^2\right)dx$

C. $\pi\int_{-2}^{4}\left(\sqrt{9-(x-1)^2}\right)dx$

D. $\pi\int_{-2}^{4}\left(9-(x-1)^2\right)^2dx$

E. $\pi\int_{-4}^{2}\left(9-(x-1)^2\right)dx$

Question 13

Using a suitable substitution, $\int_0^{\frac{\pi}{3}} \sin^3(x)\cos^4(x)dx$ can be expressed in terms of u as

A. $\int_0^{\frac{\pi}{3}} \left(u^6 - u^4\right)du$

B. $\int_1^{\frac{1}{2}} \left(u^6 - u^4\right)du$

C. $\int_{\frac{1}{2}}^{1} \left(u^6 - u^4\right)du$

D. $\int_0^{\frac{\sqrt{3}}{2}} \left(u^6 - u^4\right)du$

E. $\int_0^{\frac{\sqrt{3}}{2}} \left(u^4 - u^6\right)du$

Question 14

A particle is acted on by two forces, one of 6 newtons acting due south, the other of 4 newtons acting in the direction N60° W.

The magnitude of the resultant force, in newtons, acting on the particle is

A. 10

B. $2\sqrt{7}$

C. $2\sqrt{19}$

D. $\sqrt{52 - 24\sqrt{3}}$

E. $\sqrt{52 + 24\sqrt{3}}$

Question 15

The vectors $\underset{\sim}{a} = 2\underset{\sim}{i} + m\underset{\sim}{j} - 3\underset{\sim}{k}$ and $\underset{\sim}{b} = m^2\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}$ are perpendicular for

A. $m = -\frac{2}{3}$ and $m = 1$

B. $m = -\frac{3}{2}$ and $m = 1$

C. $m = \frac{2}{3}$ and $m = -1$

D. $m = \frac{3}{2}$ and $m = -1$

E. $m = 3$ and $m = -1$

Question 16

The distance between the points $P(-2, 4, 3)$ and $Q(1, -2, 1)$ is

A. 7

B. $\sqrt{21}$

C. $\sqrt{31}$

D. 11

E. 49

Question 17

If $\underset{\sim}{u} = 2\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{v} = 3\underset{\sim}{i} - 6\underset{\sim}{j} + 2\underset{\sim}{k}$, the vector resolute of $\underset{\sim}{v}$ in the direction of $\underset{\sim}{u}$ is

A. $\frac{20}{49}\left(3\underset{\sim}{i} - 6\underset{\sim}{j} + 2\underset{\sim}{k}\right)$

B. $\frac{20}{3}\left(2\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}\right)$

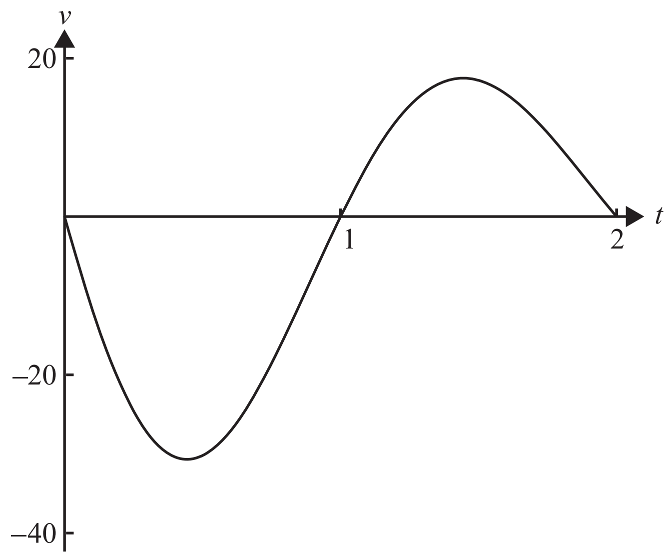
C. $\frac{20}{7}\left(3\underset{\sim}{i} - 6\underset{\sim}{j} + 2\underset{\sim}{k}\right)$

D. $\frac{20}{9}\left(2\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}\right)$

E. $\frac{1}{9}\left(-2\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}\right)$

Question 18

The velocity–time graph for the first 2 seconds of the motion of a particle that is moving in a straight line with respect to a fixed point is shown below.



The particle's velocity v is measured in cm/s. Initially the particle is x_0 cm from the fixed point.

The distance travelled by the particle in the first 2 seconds of its motion is given by

A. $\int_0^2 v\,dt$

B. $\int_0^2 v\,dt + x_0$

C. $\int_1^2 v\,dt - \int_0^1 v\,dt$

D. $\left|\int_0^2 v\,dt\right|$

E. $\int_1^2 v\,dt - \int_0^1 v\,dt + x_0$

Question 19

A body is moving in a straight line and, after t seconds, it is x metres from the origin and travelling at v ms^{-1}. Given that $v = x$, and that $t = 3$ where $x = -1$, the equation for x in terms of t is

A. $x = e^{t-3}$

B. $x = -e^{3-t}$

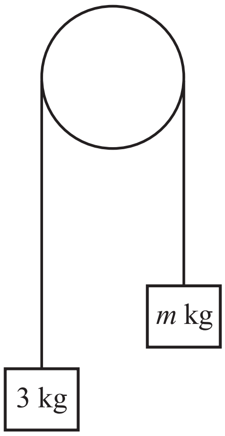
C. $x = \sqrt{2t-5}$

D. $x = -\sqrt{2t-5}$

E. $x = -e^{t-3}$

Question 20

Particles of mass 3 kg and m kg are attached to the ends of a light inextensible string that passes over a smooth pulley, as shown.



If the acceleration of the 3 kg mass is 4.9 m/s^2 upwards, then

A. $m = 4.5$

B. $m = 6.0$

C. $m = 9.0$

D. $m = 13.5$

E. $m = 18.0$

Question 21

A particle of mass 3 kg is acted on by a variable force, so that its velocity v m/s when the particle is x m from the origin is given by $v = x^2$.

The force acting on the particle when $x = 2$, in newtons, is

A. 4

B. 12

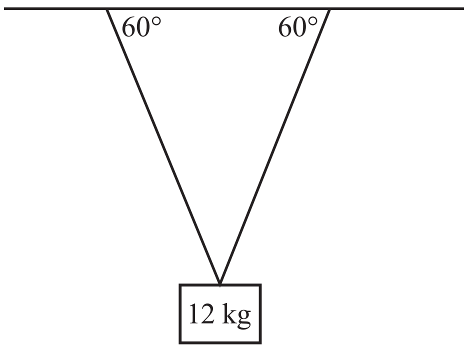
C. 16

D. 36

E. 48

Question 22

A 12 kg mass is suspended in equilibrium from a horizontal ceiling by two identical light strings. Each string makes an angle of 60° with the ceiling, as shown.



The magnitude, in newtons, of the tension in each string is equal to

A. $6g$

B. $12g$

C. $24g$

D. $4\sqrt{3}\,g$

E. $8\sqrt{3}\,g$

END OF SECTION 1

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.
Unless otherwise specified an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

A curve is defined by the parametric equations

$$x = \operatorname{cosec}(\theta) + 1$$
$$y = 2\cot(\theta)$$

a. Show that the curve can be expressed in the cartesian form

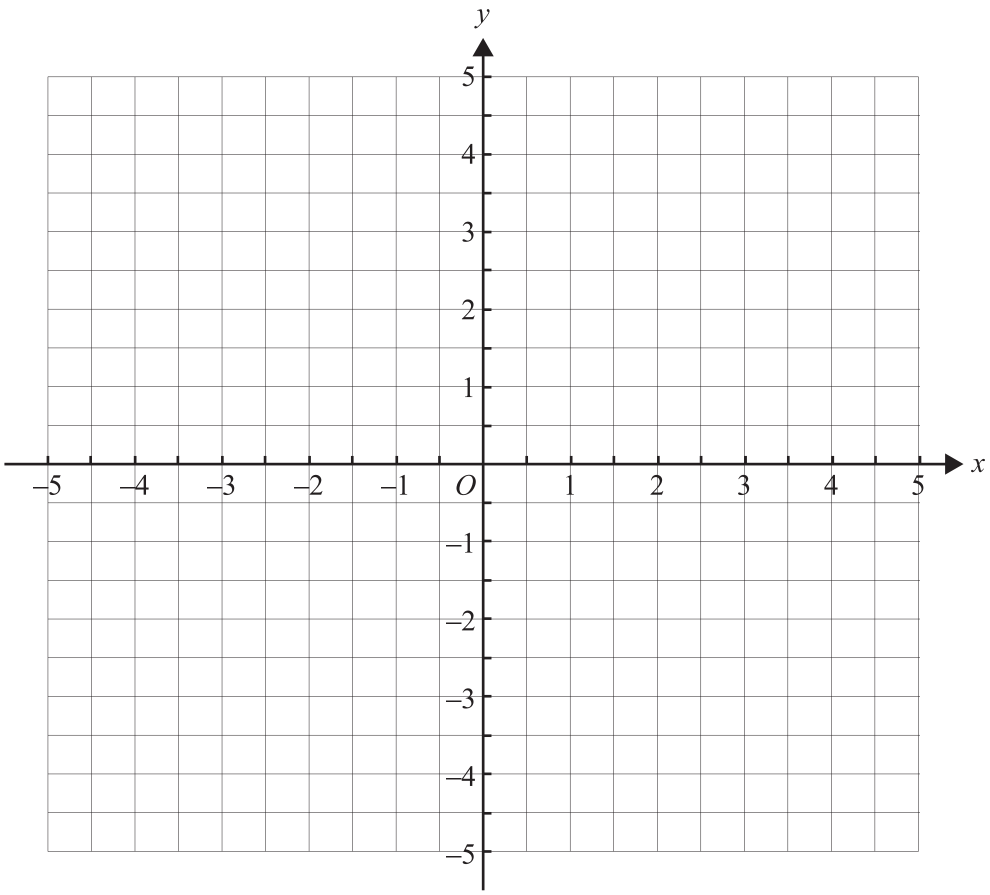
$$(x-1)^2 - \frac{y^2}{4} = 1$$

2 marks

b. Sketch the curve defined by the parametric equations

$$x = \operatorname{cosec}(\theta) + 1$$
$$y = 2\cot(\theta)$$

labelling any asymptotes with their equations.



4 marks

c. The region bounded by the curve and the line $x = 3$ is rotated about the x-axis.
Find the volume of the solid formed.

2 marks

d. Find the gradient of the curve where $\theta = \frac{7\pi}{6}$.

3 marks

Question 2

a. Given that $\cos\left(\frac{\pi}{12}\right) = \frac{\sqrt{\sqrt{3}+2}}{2}$, show that $\sin\left(\frac{\pi}{12}\right) = \frac{\sqrt{2-\sqrt{3}}}{2}$.

2 marks

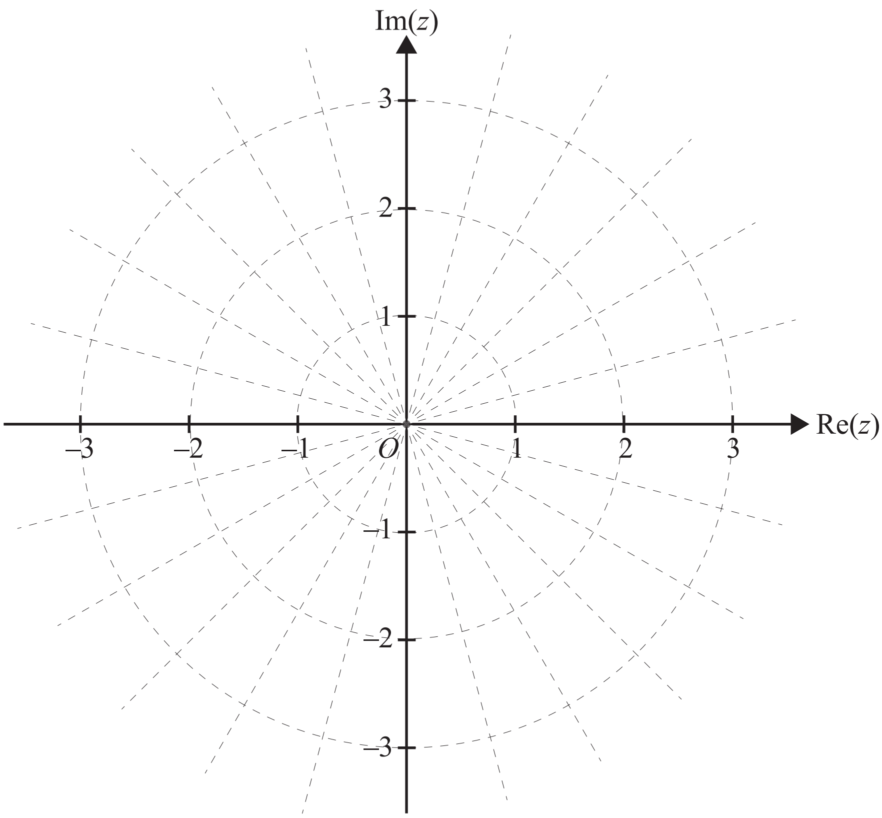
b. **i.** Express $z_1 = \frac{\sqrt{\sqrt{3}+2}}{2} + i\frac{\sqrt{2-\sqrt{3}}}{2}$ in polar form.

ii. Write down z_1^4 in polar form.

1 + 1 = 2 marks

c. On the Argand diagram below, shade the region defined by

$$\left\{z : \text{Arg}(z_1) \le \text{Arg}(z) \le \text{Arg}(z_1^4)\right\} \cap \{z : 1 \le |z| \le 2\},\ z \in C.$$



2 marks

d. Find the area of the shaded region in **part c.**

2 marks

e. **i.** Find the value(s) of n such that $\text{Re}\left(z_1{}^n\right) = 0$, where $z_1 = \frac{\sqrt{\sqrt{3}+2}}{2} + i\frac{\sqrt{2-\sqrt{3}}}{2}$.

ii. Find $z_1{}^n$ for the value(s) of n found in **part i.**

3 + 1 = 4 marks

Question 3

A car accelerates from rest. Its speed after T seconds is V ms^{-1}, where

$$V = 17\tan^{-1}\left(\frac{\pi T}{6}\right),\ T \geq 0$$

a. Write down the limiting speed of the car as $T \to \infty$.

1 mark

b. Calculate, correct to the nearest 0.1 ms^{-2}, the acceleration of the car when $T = 10$.

1 mark

c. Calculate, correct to the nearest second, the time it takes for the car to accelerate from rest to 25 ms^{-1}.

2 marks

After accelerating to 25 ms^{-1}, the car stays at this speed for 120 seconds and then begins to decelerate while braking. The speed of the car t seconds after the brakes are first applied is v ms^{-1} where

$$\frac{dv}{dt} = -\frac{1}{100}(145 - 2t),$$

until the car comes to rest.

d. **i.** Find v in terms of t.

ii. Find the time, in seconds, taken for the car to come to rest while braking.

2 + 2 = 4 marks

e. **i.** Write down the expressions for the distance travelled by the car during each of the three stages of its motion.

ii. Find the total distance travelled from when the car starts to accelerate to when it comes to rest. Give your answer in metres correct to the nearest metre.

2 + 1 = 3 marks

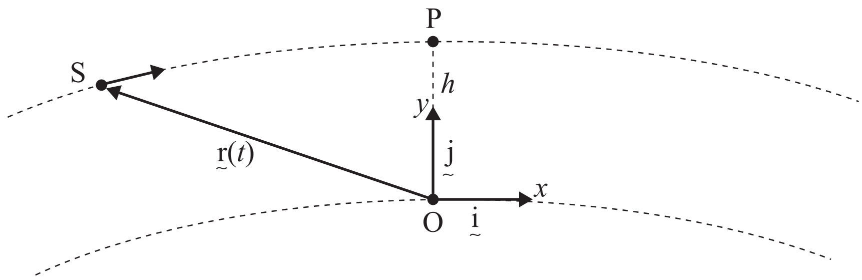
Question 4

The position vector of the *International Space Station* (S), when visible above the horizon from a radar tracking location (O) on the surface of Earth, is modelled by

$$\underset{\sim}{r}(t) = 6800\sin\left(\pi(1.3t - 0.1)\right)\underset{\sim}{i} + \left(6800\cos\left(\pi(1.3t - 0.1)\right) - 6400\right)\underset{\sim}{j},$$

for $t \in \left[0, 0.154\right]$,

where $\underset{\sim}{i}$ is a unit vector relative to O as shown and $\underset{\sim}{j}$ is a unit vector vertically up from point O. Time t is measured in hours and displacement components are measured in kilometres.



a. Find the height, h km, of the space station above the surface of Earth when it is at point P, directly above point O.

Give your answer correct to the nearest km.

1 mark

b. Find the acceleration of the space station, and show that its acceleration is perpendicular to its velocity.

3 marks

c. Find the speed of the space station in km/h.
Give your answer correct to the nearest integer.

2 marks

d. Find the equation of the path followed by the space station in cartesian form.

2 marks

e. Find the times when the space station is at a distance of 1000 km from the radar tracking location O.
Give your answers in hours, correct to two decimal places.

3 marks

Question 5

At her favourite fun park, Katherine's first activity is to slide down a 10 m long straight slide. She starts from rest at the top and accelerates at a constant rate, until she reaches the end of the slide with a velocity of 6 ms^{-1}.

a. How long, in seconds, does it take Katherine to travel down the slide?

1 mark

When at the top of the slide, which is 6 m above the ground, Katherine throws a chocolate vertically upwards. The chocolate travels up and then descends past the top of the slide to land on the ground below. Assume that the chocolate is subject only to gravitational acceleration and that air resistance is negligible.

b. If the initial speed of the chocolate is 10 m/s, how long, correct to the nearest tenth of a second, does it take the chocolate to reach the ground?

2 marks

c. Assume that it takes Katherine four seconds to run from the end of the slide to where the chocolate lands.

At what velocity would the chocolate need to be propelled upwards, if Katherine were to immediately slide down the slide and run to reach the chocolate just as it hits the ground?

Give your answer in ms^{-1}, correct to one decimal place.

2 marks

Katherine's next activity is to ride a mini speedboat. To stop at the correct boat dock, she needs to stop the engine and allow the boat to be slowed by air and water resistance.

At time t seconds after the engine has been stopped, the acceleration of the boat, a ms^{-2}, is related to its velocity, v ms^{-1}, by

$$a = -\frac{1}{10}\sqrt{196 - v^2}.$$

Katherine stops the engine when the speedboat is travelling at 7 m/s.

d. **i.** Find an expression for v in terms of t.

ii. Find the time it takes the speedboat to come to rest.

Give your answer in seconds in terms of π.

iii. Find the distance it takes the speedboat to come to rest, from when the engine is stopped. Give your answer in metres, correct to one decimal place.

3 + 2 + 3 = 8 marks

END OF QUESTION AND ANSWER BOOK

SPECIALIST MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc\sin A$
sine rule:	$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$
cosine rule:	$c^2 = a^2 + b^2 - 2ab\cos C$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ hyperbola: $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

Circular (trigonometric) functions

$\cos^2(x) + \sin^2(x) = 1$

$1 + \tan^2(x) = \sec^2(x)$ $\cot^2(x) + 1 = \operatorname{cosec}^2(x)$

$\sin(x + y) = \sin(x)\cos(y) + \cos(x)\sin(y)$ $\sin(x - y) = \sin(x)\cos(y) - \cos(x)\sin(y)$

$\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$ $\cos(x - y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$ $\tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$

$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$

$\sin(2x) = 2\sin(x)\cos(x)$ $\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + yi = r(\cos\theta + i\sin\theta) = r\operatorname{cis}\theta$

$|z| = \sqrt{x^2 + y^2} = r$ $\qquad$ $-\pi < \operatorname{Arg} z \le \pi$

$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$ $\qquad$ $\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\operatorname{cis}(\theta_1 - \theta_2)$

$z^n = r^n \operatorname{cis}(n\theta)$ (de Moivre's theorem)

Calculus

$\dfrac{d}{dx}\left(x^n\right) = nx^{n-1}$ $\qquad$ $\int x^n\,dx = \dfrac{1}{n+1}x^{n+1} + c, n \ne -1$

$\dfrac{d}{dx}\left(e^{ax}\right) = ae^{ax}$ $\qquad$ $\int e^{ax}\,dx = \dfrac{1}{a}e^{ax} + c$

$\dfrac{d}{dx}\left(\log_e(x)\right) = \dfrac{1}{x}$ $\qquad$ $\int \dfrac{1}{x}\,dx = \log_e|x| + c$

$\dfrac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$ $\qquad$ $\int \sin(ax)\,dx = -\dfrac{1}{a}\cos(ax) + c$

$\dfrac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$ $\qquad$ $\int \cos(ax)\,dx = \dfrac{1}{a}\sin(ax) + c$

$\dfrac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$ $\qquad$ $\int \sec^2(ax)\,dx = \dfrac{1}{a}\tan(ax) + c$

$\dfrac{d}{dx}\left(\sin^{-1}(x)\right) = \dfrac{1}{\sqrt{1-x^2}}$ $\qquad$ $\int \dfrac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\dfrac{x}{a}\right) + c, a > 0$

$\dfrac{d}{dx}\left(\cos^{-1}(x)\right) = \dfrac{-1}{\sqrt{1-x^2}}$ $\qquad$ $\int \dfrac{-1}{\sqrt{a^2-x^2}}\,dx = \cos^{-1}\left(\dfrac{x}{a}\right) + c, a > 0$

$\dfrac{d}{dx}\left(\tan^{-1}(x)\right) = \dfrac{1}{1+x^2}$ $\qquad$ $\int \dfrac{a}{a^2+x^2}\,dx = \tan^{-1}\left(\dfrac{x}{a}\right) + c$

product rule: $\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

quotient rule: $\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

chain rule: $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$

Euler's method: If $\dfrac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

acceleration: $a = \dfrac{d^2x}{dt^2} = \dfrac{dv}{dt} = v\dfrac{dv}{dx} = \dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right)$

constant (uniform) acceleration: $v = u + at$ $\qquad$ $s = ut + \dfrac{1}{2}at^2$ $\qquad$ $v^2 = u^2 + 2as$ $\qquad$ $s = \dfrac{1}{2}(u + v)t$

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$

$\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos\theta = x_1 x_2 + y_1 y_2 + z_1 z_2$

$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$

Mechanics

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

friction: $F \le \mu N$

END OF FORMULA SHEET