2012 Assessment Report

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

2012 Mathematical Methods (CAS) GA 3: Examination 2

GENERAL COMMENTS

In 2012, 15 294 students sat for Mathematical Methods (CAS) examination 2. Students achieved scores across the range of available marks. Responses showed that the paper was accessible and that it provided an opportunity for students to demonstrate what they knew.

In Section 1, the majority of students answered Question 1 correctly. Less than 50 per cent of students obtained the correct answers for Questions 4, 8, 16, 17, 18, 19, 20 and 22. For Question 17, students were required to find the value of *m* for which the system of liner equations had no solution. Few students answered Question 20, on discrete random variables, correctly.

In Section 2, Question 2a., the graph-sketching was done very well. Question 2e., on transformations, was not well done. This question was distinctive in that it asked students to map the graph of *f*, where $f(x)=\frac{1}{2x-4}+3$, to the graph of *g*, where $g(x)=\frac{1}{x}$. Two different approaches are given in the more detailed comments below.

A variety of methods was employed to solve the probability question (Question 3bi.).

Students need to take care with exact answers. As clearly indicated in the instructions for Section 2, 'In all questions where a numerical answer is required, an exact value must be given unless otherwise specified'. Many students gave approximate answers as their final answers, especially for Question 5. Some students were rounding off too early; for example, in Questions 3d. and 4f. Others ignored the domain restrictions and often gave extra solutions. This occurred in Questions 1b., 1d., 4d. and 5bii.

In questions with a 'show that' instruction, students are required to show relevant working. Some students showed very little working in Questions 1a., 2c. and 3c. To successfully answer these questions, students needed to have sound algebraic understanding and skills.

SPECIFIC INFORMATION

This report provides sample answers or an indication of what the answers may have included. Unless otherwise stated, these are not intended to be exemplary or complete responses.

Section 1

The table below indicates the percentage of students who chose each option. The correct answer is indicated by shading.

Question	% A	% B	% C	% D	% E	% No answer	Comments
1	1	3	91	4	1	0	
2	4	10	5	11	70	0	
3	4	3	13	69	11	0	
4	45	21	14	14	5	1	$\frac{d}{dx}\left(g\left(e^{kx}\right)\right)=ke^{kx}g'\left(e^{kx}\right)=kg'\left(e^{kx}\right)e^{kx}$, using the chain rule.
5	3	66	23	4	4	0	
6	2	10	72	9	7	0	
7	5	11	13	67	3	0	

Question	% A	% B	% C	% D	% E	% No answer	Comments
8	49	14	18	14	4	1	The gradient of the graph of $y = f(x)$ is negative for $(-\infty,\ m) \cup (n,\ \infty)$. There is a local minimum at $x = m$ and a local maximum at $x = n$. Eighteen per cent of students chose option C, $(p,\ 0) \cup (q,\ \infty)$. This is when the graph of $y = f(x)$ is negative.
9	8	24	8	57	3	0	$y = \sqrt{b - x^2}$, $\frac{dy}{dx} = \frac{-2x}{2\sqrt{b - x^2}} = \frac{-1}{\sqrt{b-1}}$, when $x = 1$, $\frac{dy}{dx} = -\frac{1}{3}$ (gradient of the normal × gradient of the tangent = –1), $\frac{-1}{\sqrt{b-1}} = -\frac{1}{3}$, $\sqrt{b-1} = 3$, $b = 10$ Option B was obtained if the gradient of the tangent is 3, not the normal, $\frac{dy}{dx} = 3$, $\frac{-1}{\sqrt{b-1}} = 3$, $b = \frac{10}{9}$.
10	6	20	60	9	4	1	
11	18	5	7	8	62	1	
12	72	5	12	9	3	0	
13	11	52	20	12	4	1	$\Pr(B' \mid A) = \frac{\Pr(B' \cap A)}{\Pr(A)}$ $= \frac{\Pr(B' \cap A)}{\Pr(B' \cap A) + \Pr(A \cap B)} = \frac{\left(\frac{3}{7}\right)}{\left(\frac{3}{7} + \frac{2}{5}\right)} = \frac{15}{29}$ Option C was $\Pr(A \cap B) = \frac{14}{35}$.
14	7	10	11	64	7	1	
15	4	81	4	10	1	0	
16	13	17	27	34	8	1	The local maximum will be touching the x-axis when $c = 3$, giving two distinct solutions. So if $c < 3$ there will be one solution. The local minimum will be above the x-axis when $c > 8$. Hence $c < 3$ or $c > 8$.

Question	% A	% B	% C	% D	% E	% No answer	Comments
17	14	43	25	10	7	0	For the corresponding lines to be parallel require $\frac{m}{1}=\frac{3}{m+2}$ so m = –3 or 1. For lines to be distinct require $\frac{1}{m}\neq\frac{m}{1}$ so $m\neq 1$, hence m = –3. **OR** $\frac{m}{1}=\frac{3}{m+2}$, $\begin{vmatrix} m & 3 \\ 1 & m+2 \end{vmatrix}=0$, $m=-3$ or m = 1. When m = 1, the lines coincide hence $x+3y=1$ and $x+3y=1$. There is an infinite number of solutions. When $m=-3$, $-3x+3y=1$ and $x-y=-3$, there is no solution as the lines are parallel and distinct. Hence, $m=-3$. Twenty-five per cent of students chose option C, $m\in\{1,\ -3\}$.
18	30	11	32	16	10	1	By choosing a value of a between 1 and e, the x-intercept for the tangent is positive, which means option A is false. **OR** $y=\frac{1}{a}(x-a)+\log_e(a)$, $a>0$ When x = 0, $0=\frac{1}{a}(x-a)+\log_e(a)$ and $x=a-a\log_e(a)=b$. If b < 0, $a\log_e(a)>a$ $\log_e(a)>1$, $a>e$.
19	13	45	19	12	10	1	As $f(x)$ = cos(x) is positive in the first and fourth quadrants and negative in the second quadrant, then $f(\pi-\theta)=-f(\theta)$ and $f(\pi-\theta)=-f(-\theta)$. Alternatively, the given conditions imply the function is even; cos(x) is the only even function provided in the options.
20	19	18	27	16	19	1	$\Pr(X>1)=1-\left(\Pr(X=0)+\Pr(X=1)\right)$ $=1-(1-p)^0p-(1-p)p=1-p-p+p^2$ $=1-2p+p^2=(1-p)^2$

Question	% A	% B	% C	% D	% E	% No answer	Comments
21	63	7	17	8	4	1	
22	11	32	25	16	15	1	The graph of $y = f(x-2)$ has a local maximum at (a + 2, b) and a local minimum at (c + 2, d). The graph of $y = \lvert f(x-2) \rvert$ has local maxima at (a + 2, b) and (c + 2, $-d$). Hence, the graph of $y = -\lvert f(x-2) \rvert$ has local minima at (a + 2, $-b$) and (c + 2, d). Twenty-five per cent of students chose option C, local maxima at (a + 2, b) and (c + 2, $-d$), which is the case for the graph of $y = \lvert f(x-2) \rvert$.

Section 2

Question 1a.

Marks	0	1	2	Average
%	30	7	62	**1.3**

The total surface area = $2\left(xh + \frac{5x^2}{2} + \frac{5xh}{2}\right) = 6480$, so $2xh + 5x^2 + 5xh = 6480$, $7xh = 6480 - 5x^2$, hence

$$h = \frac{6480 - 5x^2}{7x}.$$

Some students incorrectly used volume instead of the total surface area. Others used $2xh + 4 \times \frac{5x^2}{2} = 6480$.

In this question, the relevant formulation was required.

Question 1b.

Marks	0	1	2	Average
%	34	28	38	**1.0**

Solve $V(x) > 0$ when $x > 0$, $0 < x < 36$

Some students correctly solved $V(x) = 0$ but left their answer as $x = -36$, $x = 0$ or $x = 36$, without considering the inequality. Others had incorrect notation such as [0, 36] or {0, 36}. Some solved *the total surface area* = 0 instead of $V(x) = 0$.

Question 1c.

Marks	0	1	2	3	Average
%	18	5	15	61	**2.2**

$$\frac{dV}{dx} = -\frac{75}{14}x^2 + \frac{16\,200}{7}$$

This question was quite well done. Some students attempted to find the derivative using a written method and made algebraic errors. Others had the correct answer with incorrect working. Some common incorrect answers were $\frac{dV}{dx} = \frac{-75x^2 - 16\ 200}{14}$ and $\frac{dV}{dx} = -75x^2 - 16\ 200$. Some students gave approximations, such as

$\frac{dV}{dx} = -5.357x^2 + 2314.29$, when exact answers were required. Students were expected to give the answer in the required form, $\frac{dV}{dx} = ax^2 + b$.

Question 1d.

Marks	0	1	2	Average
%	31	24	45	1.1

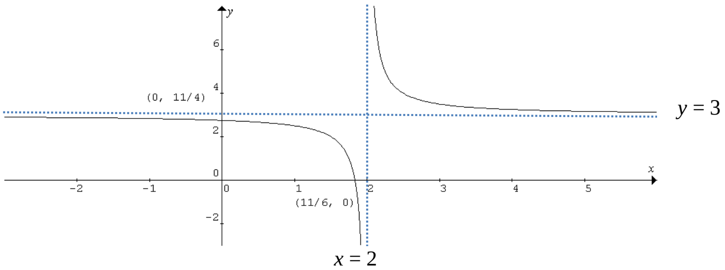
$\frac{dV}{dx} = 0$, $x = 12\sqrt{3}$, $h = \frac{120\sqrt{3}}{7}$

Some students included $x = -12\sqrt{3}$ as part of their answer. Others did not find the value for h. Some gave an approximate answer for x and h, such as $x = 20.78$, $h = 29.69$. Exact values were required.

Question 2a.

Marks	0	1	2	3	Average
%	7	12	38	43	2.2

The graph of $f(x) = \frac{1}{2x-4} + 3$ is shown below.



The graph was generally drawn well. Some students did not label the asymptotes with their equations or incorrectly labelled the vertical asymptote as $x = 4$. Others did not label the intercepts with their coordinates. Some did not give the exact answer for the x intercept, (1.83, 0) was a common response.

Question 2bi.

Marks	0	1	Average
%	10	90	0.9

$$f'(x) = \frac{-2}{(2x-4)^2} = \frac{-1}{2(x-2)^2} = \frac{-1}{(x-2)(2x-4)}$$

This question was very well done. Some students gave $f'(x) = \frac{-1}{(2x-4)^2}$ as the answer or $f'(x) = \frac{1}{(2x-4)^2}$.

Question 2bii.

Marks	0	1	Average
%	39	61	0.6

R^- or $(-\infty, 0)$

Some students had incorrect notations, such as $\{-\infty, 0\}$, $(0, -\infty)$ or $-R$. Others gave the domain of the derivative f' and some gave the range of f.

Question 2biii.

Marks	0	1	Average
%	46	54	**0.5**

$f'(x) < 0$ for $R \setminus \{2\}$

Therefore, $f'(x) \neq 0$; hence *f* has no stationary points.

Where the graph of f' was referred to, there was often discussion about asymptotes but no mention was made of the derivative never being zero. Some students gave other incomplete answers or answers that lacked detail, such as there was no solution or that the graph was continuous.

Question 2c.

Marks	0	1	2	Average
%	52	38	10	**0.6**

$y - q = \frac{-2}{(2p-4)^2}(x-p)$ or equivalent, substitute $q = \frac{1}{2p-4} + 3$, $y - \frac{1}{2p-4} - 3 = \frac{-2}{(2p-4)^2}(x-p)$,

$y - 3 = \frac{-2}{(2p-4)^2}(x-p) + \frac{1}{2p-4}$, $(2p-4)^2(y-3) = -2x + 2p + 2p - 4 = -2x + 4p - 4$

Many students were able to set up the first equation but then made algebraic errors. A common mistake was $y - \frac{1}{2p-4} + 3 = \frac{-2}{(2p-4)^2}(x-p)$.

As this was a 'show that' question, students needed to indicate how the given relation, or equivalent, was obtained.

Question 2d.

Marks	0	1	2	3	4	Average
%	74	6	4	2	14	**0.8**

Substitute $\left(-1, \frac{7}{2}\right)$ into $(2p-4)^2(y-3) = -2x + 4p - 4$, solve for *p*, *p* = 5 or *p* = 1, substitute *p* into *f*,

$\left(1, \frac{5}{2}\right)$, $\left(5, \frac{19}{6}\right)$

Many students did not attempt this question. Others did not realise that they could have used the result from part c. to find the coordinates. Some thought that $\left(-1, \frac{7}{2}\right)$ was a point on the hyperbola.

Question 2e.

Marks	0	1	2	Average
%	72	21	8	**0.4**

$T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix}$, dilate the graph of *f*, where $f(x) = \frac{1}{2x-4} + 3$, by a factor of 2 from the *y*-axis,

$y = \frac{1}{2\left(\frac{x}{2}\right) - 4} + 3 = \frac{1}{x-4} + 3$, then translate this graph 4 units to the left and 3 units down to obtain the graph of *g*,

where $g(x)=\frac{1}{x}$, $a=2,\ c=-4,\ d=-3$ or $y-3=\frac{1}{2x-4}$ image $y'=\frac{1}{x'}$, $y'=y-3\ \ x'=2x-4$,
$y'=y+d\ \ x'=ax+c,\ d=-3, a=2, c=-4$.

This question was done poorly. Some students gave the transformations that map the graph of *g* to *f*. Others produced simultaneous equations and did not give any answers.

Question 3ai.

Marks	0	1	Average
%	30	70	0.7

$\frac{1}{64}=0.015625$

This question was well answered. Some students rounded off their answer. An exact value was required.

Question 3aii.

Marks	0	1	2	Average
%	27	24	48	1.2

Binomial $n=20$, $p=\frac{1}{4}$, $\Pr(X\geq 10)=0.0139$ correct to four decimal places.

Some students gave only the answer. For questions worth more than one mark, working must be shown. Identifying the distribution with the correct parameters is sufficient working. Some students found $\Pr(X=10)$ or $\Pr(X<10)$, instead of the required probability.

Question 3aiii.

Marks	0	1	Average
%	45	55	0.6

$n\times\frac{1}{4}\times\frac{3}{4}=\frac{75}{16},\ n=25$

This question was answered quite well. Some students used the standard deviation, which was not necessary. Others did not know the formula for the variance.

Question 3bi.

Marks	0	1	2	3	Average
%	40	21	7	32	1.3

Two cases CCCC, ICCC, $\frac{1}{3}\times\left(\frac{3}{4}\right)^3+\frac{2}{3}\times\frac{1}{3}\times\left(\frac{3}{4}\right)^2$, $\frac{17}{64}=0.265625$

Alternatively, a transition matrix, $\begin{matrix} & C_i & C_i' \\ C_{i+1} & & \\ C'_{i+1} & & \end{matrix}\begin{bmatrix}\frac{3}{4} & \frac{1}{3}\\ \frac{1}{4} & \frac{2}{3}\end{bmatrix}^2=\begin{bmatrix}\frac{31}{48} & \frac{17}{36}\\ \frac{17}{48} & \frac{19}{36}\end{bmatrix}$, $\frac{17}{36}\times\frac{3}{4}\times\frac{3}{4}=\frac{17}{64}$, or a tree diagram could be used.

Various methods were used to calculate the answer. Some students misinterpreted the question and found the probability of answering Question 3 correctly, then Question 4 and then Question 5. Other students did not read or interpret the statement '…Katrina answers Question 1 incorrectly'. Some students rounded off their answer. An exact value was required.

Question 3bii.

Marks	0	1	2	Average
%	34	36	30	1.0

$$\begin{bmatrix} \frac{3}{4} & \frac{1}{3} \\ \frac{1}{4} & \frac{2}{3} \end{bmatrix}^{24} \times \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.57142.... \\ 0.42857.... \end{bmatrix}$$, 0.5714 correct to four decimal places.

Many students had the correct transition matrix. Often incorrect working was shown; for example, T^{25} and $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ instead of T^{24} and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

Question 3c.

Marks	0	1	2	Average
%	76	12	13	0.4

$25p^{24}(1-p)+p^{25}=6p^{25},\ 25p^{24}-30p^{25}=0,\ 5p^{24}(5-6p)=0,\ p>0,\ p=\frac{5}{6}$

Some students used $1-\Pr(X=23)$ or $\Pr(Y \ge 23) = 6\Pr(Y=25)$. Suitable working was required as this was a 'show that' question.

Question 3d.

Marks	0	1	2	3	4	Average
%	71	9	5	3	12	0.7

Y is binomial with $p=\frac{5}{6}$, $n = 25$, $\Pr(Z \ge \frac{25-a}{b}) = \Pr(Y \ge 22) = 0.381566\ldots$, $\Pr(Z \ge \frac{20-a}{b}) = \Pr(Y \ge 18) = 0.955268\ldots$

$\frac{20-a}{b} = -1.69823...$, $\frac{25-a}{b} = 0.30137...$, $a = 24.246$, $b = 2.500$ rounded off to three decimal places.

Many students did not realise that Y was binomial. Some students tried to solve $\frac{20-a}{b} = 0.3815...$ and $\frac{25-a}{b} = 0.9552...$, and some rounded off their answers too soon.

Question 4ai.

Marks	0	1	Average
%	47	53	0.5

$r=\frac{h}{5}$

Some students found h in terms of r instead.

Question 4aii.

Marks	0	1	Average
%	45	55	0.5

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{h}{5}\right)^2 h = \frac{\pi h^3}{75}$

Students who answered part ai. correctly tended to answer this question correctly.

Question 4b.

Marks	0	1	Average
%	15	85	0.8

$h(20) = 10 + \frac{1}{1600}(20^3 - 1200 \times 20) = 0$

This question was answered well.

Question 4ci.

Marks	0	1	Average
%	40	60	0.6

$h(5) = \frac{405}{64} = 6\frac{21}{64} = 6.328125$ m

Some students gave an approximate answer when an exact value was required.

Question 4cii.

Marks	0	1	2	3	Average
%	54	21	8	17	0.9

$\frac{dV}{dh} = \frac{\pi h^2}{25}$, $\frac{dh}{dt} = \frac{3}{1600}t^2 - \frac{3}{4}$, $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$, $\frac{dV}{dt} = \frac{\pi h^2}{25} \times \left(\frac{3}{1600}t^2 - \frac{3}{4}\right)$, $t = 5$ and $h(5) = \frac{405}{64}$, $\frac{dV}{dt} = -3.538\ldots$

$= -3.5$ correct to one decimal place

The rate at which the volume is decreasing is 3.5 m^3/min, correct to one decimal place.

Many students could not set up the related rates equation, $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$. Others had problems substituting the correct values into the equation.

Question 4d.

Marks	0	1	2	Average
%	31	16	53	1.2

$h(t) = 10 + \frac{1}{1600}(t^3 - 1200t) = 2$, $t = 12.2$ min, correct to one decimal place

Many students knew they had to solve $h(t) = 2$. Some gave extra solutions.

Question 4e.

Marks	0	1	2	Average
%	77	11	11	0.3

$V(2) = \frac{8\pi}{75}$, time = $\frac{8\pi}{75 \times 0.2}$, time = $\frac{8\pi}{15}$ min

Some students gave an approximate answer when an exact answer was required.

Question 4f.

Marks	0	1	Average
%	89	11	0.1

time = $(20 - 12.167...) + \frac{8\pi}{15} = 9.5$ minutes, correct to one decimal place.

Many students did not attempt this question. Of those who did, some set up the incorrect equation. Some students rounded off too early in their calculations.

Question 5ai.

Marks	0	1	Average
%	22	78	0.8

$$\int_{-2}^{0} f\left(x\right)dx = 1 - e^{-2} = 1 - \frac{1}{e^2}$$

This question was well answered. Some students gave an approximate answer when an exact value was required.

Question 5aii.

Marks	0	1	Average
%	55	45	0.4

Area = $1 - e^{-2} = 1 - \frac{1}{e^2}$

Some students realised that the area was the same as that in part ai.

Question 5aiii.

Marks	0	1	Average
%	64	36	0.4

Area $= e - e^{-2} = e - \frac{1}{e^2}$

Some students gave an approximate answer when an exact answer was required.

Question 5bi.

Marks	0	1	2	Average
%	40	26	34	0.9

Solve $g(x) = k(x)$ for x, $x = \frac{a \pm \sqrt{a^2 - 4}}{2}$

Many students had the correct working and knew to solve $g(x) = k(x)$ for x. Some students rejected the solution $x = \frac{a - \sqrt{a^2 - 4}}{2}$ but it was in the required domain.

Question 5bii.

Marks	0	1	Average
%	89	11	**0.1**

$a^2 - 4 > 0$, $a > 2$, as a is positive

Some students included extra solutions, ignoring the requirement that a had to be a positive real number. Some solved $a^2 - 4 \geq 0$ for x.

Question 5c.

Marks	0	1	2	Average
%	82	7	11	**0.3**

$\frac{1}{2}\left(\frac{a+\sqrt{a^2-4}}{2} + \frac{a-\sqrt{a^2-4}}{2}\right) = \sqrt{2}$, $a = 2\sqrt{2}$ OR $\frac{a}{2} = \sqrt{2}, a = 2\sqrt{2}$

Many students used $x_m = \frac{x_1 - x_2}{2}$ for the midpoint formula. Others used the distance formula.