2012 Specialist Mathematics GA 3: Written examination 2

GENERAL COMMENTS

The number of students who sat for the 2012 Specialist Maths examination 2 was 3895. The examination comprised 22 multiple-choice questions (worth 22 marks) and five extended-answer questions (worth 58 marks). The paper seemed to be accessible to the full range of students, with most of them making substantial attempts at all questions in Section 2. Detailed statistical information that is related to the examination is published on the VCAA website.

In 2012, there were only two 'show that' questions: Question 1a. and Question 2a. It needs to be emphasised again that in this type of question, students need to show the steps of working that enable them to deduce the required result.

A number of students did not keep in mind the instruction for Section 2 that stated, 'In questions where more than one mark is available, appropriate working **must** be shown'. A number of students simply wrote down answers for Questions 3c., 3dii., 4d. and 5dii., without displaying relevant working.

A larger number of students did not keep in mind the other important instruction for Section 2 that stated, 'Unless otherwise specified an **exact** answer is required to a question'. Too often, a correct exact answer was obtained and then the student went on to give an approximation for the final answer. This occurred in Questions 3a., 5a. and, to a lesser extent, in Questions 1c. and 1d.

The examination revealed areas of strength and weakness in student performance.

Areas of strength included

- the use of CAS technology to perform differentiation and to evaluate definite integrals – Questions 1c., 3b., 3eii. and 5diii.
- the use of CAS technology to solve equations – Questions 3c., 3dii. and 4e., although all could be solved using a written method
- the ability to manage implicitly defined functions and rates of change – Question 1d.
- the ability to identify and sketch regions in the complex plane – Question 2c.
- the ability to work with appropriate forms of acceleration in a kinematics context – Question 5d.

Areas of weakness included

- untidy working, lack of logical development and lack of clarity about what a student intends to be their final answer (clearly flagging a final answer by underlining or circling would be helpful – Questions 2ei., 4b., 5b. and 5c.)
- work being done in very light pencil, making it very difficult to read
- graphs being done in pen rather than pencil – Question 1b.
- poor presentation of sketch graphs, including straight-line asymptotes being drawn freehand, and poor asymptotic behaviour where a curve moves away from an asymptote or touches it – Question 1b.
- omission of the constant of integration – Question 3di. And, to a lesser extent, Question 5diii.
- the use of constant acceleration formulas in variable acceleration situations – Question 3c. and, to a lesser extent, Questions 5di. and 5diii.

SPECIFIC INFORMATION

Section 1

The table below indicates the percentage of students who chose each option. The correct answer is indicated by shading.

Question	% A	% B	% C	% D	% E	% No Answer	Comments
1	1	11	2	85	1	0	
2	1	5	85	1	8	0	Centre is $(1,\ 3)$, x and y semi-axes lengths are 3 and 4 respectively, hence option C was the correct answer.
3	2	6	16	10	66	0	
4	1	5	5	88	1	0	
5	9	14	74	1	2	0	
6	11	19	37	27	7	1	$i^3\overline{z} = -r\sin(\theta) - ri\cos(\theta)$ Option C is $i\left(-r\cos(\theta)+ri\sin(\theta)\right)$ A diagrammatic solution is also possible.
7	6	58	15	6	15	0	In cartesian form, $y=-1$, hence option E was the correct answer.
8	5	3	5	16	71	0	
9	6	59	28	5	4	0	$y_1 = y_0 + 0.1\times\cos(0) = 1.1$ $y_2 = 1.1 + 0.1\times\cos(0.1) = 1.1995$ As $\sin(x)$ is 'concave down' around $x=0.2$, 1.1995 is an overestimate.
10	63	17	9	6	5	0	Differential equation requires zero gradients on each axis and negative gradients in second quadrant, hence option A was the correct answer.
11	2	6	16	25	51	0	Points of inflexion at x = 0 and x = 1, stationary points at x = 0 and $x=\frac{3}{2}$, checking values gives option E.
12	48	7	8	16	21	0	Volume of sphere centre $(1,\ 0)$ and radius 3 simplifies to give option A.
13	4	53	21	15	8	1	Letting $u=\cos x$ gives option B.
14	3	51	19	17	9	1	Adding vectors 'tip to tail' and applying the cosine rule, $\lvert \underset{\sim}{F}\rvert^2 = 4^2 + 6^2 - 2\times4\times6\times\cos(60^\circ)$, gives option B.
15	2	5	8	85	2	0	
16	77	11	7	3	1	1	$\lvert\overrightarrow{PQ}\rvert = \lvert -3\underset{\sim}{i} + 6\underset{\sim}{j} + 2\underset{\sim}{k}\rvert = 7$, hence option A was correct.
17	11	16	10	60	3	0	Require $(\underset{\sim}{v}.\hat{\underset{\sim}{u}})\hat{\underset{\sim}{u}}$, hence option D was correct.
18	2	5	60	8	25	0	
19	24	16	9	8	44	1	Integrating gives $\log_e(\lvert x\rvert) = t-3$, then $\pm x = e^{t-3}$, option E gives $x=-1$ when t = 3.
20	11	14	66	6	2	1	

21	5	27	12	5	50	1	$F = 3\times v\dfrac{dv}{dx} = 3\times x^2 \times 2x$ when $x = 2$, $F = 48$, hence option E was correct.
22	8	9	6	66	10	1	$2T\cos(30^\circ) = 12g$, and so option D was correct.

Section 2

Question 1a.

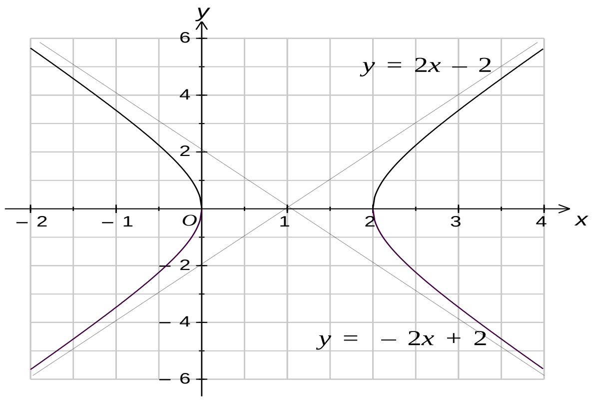
Marks	0	1	2	Average
%	16	11	73	1.6

$$x-1 = \operatorname{cosec}(\theta),\ \frac{y}{2} = \cot(\theta), \text{ and as } \cot^2(\theta)+1 = \operatorname{cosec}^2(\theta),\ \left(\frac{y}{2}\right)^2 + 1 = (x-1)^2,\ \Rightarrow (x-1)^2 - \frac{y^2}{4} = 1$$

This question was fairly well answered, although a number of convoluted methods were seen. With this type of question, the result to be shown should appear at the end of the working and not at the start. Students should work forward in a 'show that' question and not start with what they have to show.

Question 1b.

Marks	0	1	2	3	4	Average
%	14	7	9	28	42	2.8



Most students knew what to sketch for this question, but many did not show the correct asymptotic behaviour of each branch of the hyperbola. Other problems were asymptotes that were not ruled straight or not labelled. A number of students drew their graphs in pen rather than pencil, which made it very messy when corrections had to be made.

Question 1c.

Marks	0	1	2	Average
%	24	19	57	1.3

$$V = \int_2^3 \pi y^2\,dx = \pi\int_2^3 \left(4(x-1)^2 - 4\right)dx = \frac{16\pi}{3}$$

This question was generally well answered. However, some students omitted π and others had difficulty rearranging to get y^2 in terms of x.

A minority of students attempted to integrate with respect to y.

Question 1d.

Marks	0	1	2	3	Average
%	27	12	14	47	1.8

$\frac{dy}{dx}=\frac{dy}{d\theta}\times\frac{d\theta}{dx}$, $\frac{dy}{dx}=-2\operatorname{cosec}^2(\theta)\times\frac{1}{-\operatorname{cosec}(\theta)\cot(\theta)}=\frac{2}{\cos(\theta)}$, and where $\theta=\frac{7\pi}{6}$, $\frac{dy}{dx}=-\frac{4}{\sqrt{3}}$

Some students implicitly differentiated the cartesian equation of the hyperbola and substituted $x=-1$ and $y=2\sqrt{3}$ in most cases to get the correct answer.

Others solved explicitly for *y*, then differentiated to get $\frac{dy}{dx}=\pm\frac{2(x-1)}{\sqrt{x^2-2x}}$, but often gave two answers, or the answer with the wrong sign.

Question 2a.

Marks	0	1	2	Average
%	22	22	56	1.4

$\sin^2\left(\frac{\pi}{12}\right)=1-\left(\frac{\sqrt{\sqrt{3}+2}}{2}\right)^2$, $\sin^2\left(\frac{\pi}{12}\right)=1-\frac{\sqrt{3}+2}{4}=\frac{4-\sqrt{3}-2}{4}=\frac{2-\sqrt{3}}{4}$, $\sin\left(\frac{\pi}{12}\right)=\frac{\sqrt{2-\sqrt{3}}}{2}$, taking the positive root as $\frac{\pi}{12}$ is in the first quadrant.

This question was fairly well answered, although a number of students did not use $\cos\left(\frac{\pi}{12}\right)$. A number of students did not show enough arithmetic detail leading to the given result. Alternative methods involved using a right-angled triangle approach or the identity $\sin\left(\frac{\pi}{6}\right)=2\sin\left(\frac{\pi}{12}\right)\cos\left(\frac{\pi}{12}\right)$.

Question 2bi.

Marks	0	1	Average
%	19	81	0.8

$z_1=r\operatorname{cis}(\theta)=\operatorname{cis}\left(\frac{\pi}{12}\right)$

The question was well answered, with the most common error being $r\neq1$.

Question 2bii.

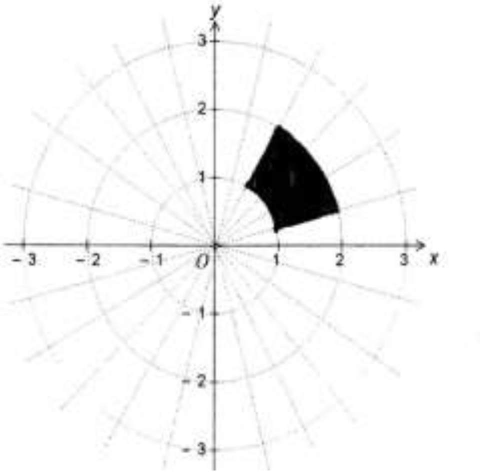
Marks	0	1	Average
%	15	85	0.9

$z_1^{\,4}=\operatorname{cis}\left(\frac{\pi}{3}\right)$

This question was well answered. Most students who obtained an answer for part 2bi. were able to apply De Moivre's theorem correctly.

Question 2c.

Marks	0	1	2	Average
%	23	17	60	1.4



This question was reasonably well answered. However, poor shading of the required area was often seen and some students omitted the corner points from the region.

Question 2d.

Marks	0	1	2	Average
%	42	14	44	1

$A = \pi\left(2^2 - 1^2\right) \times \frac{1}{8} = \frac{3\pi}{8}$

Most students realised that they needed the area of a portion of an annulus. However, the correct portion seemed to elude many. Some students calculated the area of a segment. A few attempted to use integration.

Question 2ei.

Marks	0	1	2	3	Average
%	43	26	6	25	1.1

$\cos\left(\frac{n\pi}{12}\right) = 0$, and so $n = 6 + 12k$ where $k \in Z$

The general solution for n eluded most students. Many gave some specific values for n and some moved from a correct general solution to specific solutions. Others gave the correct general solution, but failed to define k.

Question 2eii.

Marks	0	1	Average
%	64	36	0.4

$\pm i$

This question was not answered very well. A number of students obtained $\pm i$ independently of their efforts in part 2ei. Equivalent forms of the answer were also accepted. Some students gave only i as their answer.

Question 3a.

Marks	0	1	Average
%	42	58	0.6

$\frac{17\pi}{2}$

This question was reasonably well answered. The most common error was a decimal approximation being given. Some students tried to provide an interval of values. Some had an unclear understanding of 'limiting value'.

Question 3b.

Marks	0	1	Average
%	18	82	**0.8**

0.3

This question was quite well answered, with most students finding the numerical derivative directly on their calculators.

Question 3c.

Marks	0	1	2	Average
%	23	5	72	**1.5**

$25 = 17\tan^{-1}\left(\frac{\pi T}{6}\right),\ T = 18.995,\ T = 19$

This question was quite well answered, with most students using the 'solve' facility on their calculators. A minority of students did not treat this as a two-mark question and simply wrote down the answer. Some students misread the question and took the acceleration to be $25\,\text{ms}^{-2}$, and others attempted to use constant acceleration formulas.

Question 3di.

Marks	0	1	2	Average
%	20	22	58	**1.4**

$v = -\frac{1}{100}\left(145t - t^2\right) + 25$

This question was fairly well answered. However, a surprising number of students omitted the constant of integration.

Question 3dii.

Marks	0	1	2	Average
%	15	34	51	**1.4**

$0 = -\frac{1}{100}\left(145t - t^2\right) + 25,\ t = 20$

This question was well answered by those who managed to answer Question 3di. correctly. Most students knew to let $v = 0$.

Question 3ei.

Marks	0	1	2	Average
%	44	20	34	**0.9**

$\int_0^{19} 17\tan^{-1}\left(\frac{\pi T}{6}\right) dT,\ 25 \times 120,\ \int_0^{20} -\frac{1}{100}\left(145t - t^2\right) + 25\ dt$

Incorrect terminals in the integral for stage three were a common error. Some students set up an integral of the acceleration instead of velocity for the third stage.

Question 3eii.

Marks	0	1	Average
%	70	30	**0.3**

3637

Only a minority of students managed to answer this question correctly.

Question 4a.

Marks	0	1	Average
%	51	49	**0.5**

400

The most common incorrect answer was 67 m, obtained by finding the y-coordinate of the space station when t = 0.

Many students did not realise that when the sine term was zero in $\underset{\sim}{r}(t)$, the corresponding cosine term would be one.

Question 4b.

Marks	0	1	2	3	Average
%	28	13	25	34	**1.7**

$\dot{\underset{\sim}{r}}(t) = 6800 \times 1.3\pi \cos(\pi(1.3t - 0.1))\underset{\sim}{i} - 6800 \times 1.3\pi \sin(\pi(1.3t - 0.1))\underset{\sim}{j}$

$\ddot{\underset{\sim}{r}}(t) = -6800 \times 1.3^2 \pi^2 \sin(\pi(1.3t - 0.1))\underset{\sim}{i} - 6800 \times 1.3^2 \pi^2 \cos(\pi(1.3t - 0.1))\underset{\sim}{j}$

$\dot{\underset{\sim}{r}}(t).\ddot{\underset{\sim}{r}}(t) = -6800^2 \times 1.3^3 \pi^3 \cos(u)\sin(u) + 6800^2 \times 1.3^3 \pi^3 \sin(u)\cos(u)$, where $u = \pi(1.3t - 0.1)$

= 0, hence perpendicular.

Of those students who attempted this part, most seemed to know how to answer it. However, there were many errors, such as incorrect differentiation, $\underset{\sim}{i}$ and $\underset{\sim}{j}$ being dropped, and failure to spell out the terms of the scalar product between $\dot{\underset{\sim}{r}}(t)$ and $\ddot{\underset{\sim}{r}}(t)$, to demonstrate that the result was zero.

Question 4c.

Marks	0	1	2	Average
%	45	21	34	**0.9**

speed = $|\dot{\underset{\sim}{r}}(t)|$ = 27 772

Many students seemed to know what to do in this question, but could not bring it to a successful conclusion.

Poor rounding to the nearest integer and non-numerical answers (8840π) were often seen.

Question 4d.

Marks	0	1	2	Average
%	39	15	46	**1.7**

$x = 6800\sin(\pi(1.3t - 0.1)), y = 6800\cos(\pi(1.3t - 0.1)) - 6400$, $x^2 + (y + 6400)^2 = 6800^2$

This question was only moderately well done. A frequent error was $(y - 6400)^2$ in the cartesian equation to the path and a less frequent error was the omission of the 6400 altogether.

A number of students used the velocity vector and eliminated t instead of using the position vector.

Question 4e.

Marks	0	1	2	3	Average
%	51	16	4	29	**1.1**

$6800^2 \sin^2(u) + (6800\cos(u) - 6400)^2 = 1000^2$, where $u = \pi(1.3t - 0.1)$

expanding gives $6800^2 - 2 \times 6400 \times 6800\cos(u) + 6400^2 = 1000^2$, then $\cos(\pi(1.3t - 0.1)) = 0.9903$

which gives $t = 0.04$ and $t = 0.11$

Alternatively, as an approximate answer was required, this can be readily solved numerically using CAS.

This question was not very well answered. A significant number of students knew to work with $\left|\underset{\sim}{r}(t)\right| = 1000$, but only a smaller number managed to find both solutions for t.

Question 5a.

Marks	0	1	Average
%	27	73	0.8

$s = \dfrac{u+v}{2} \times t$, $10 = \dfrac{6}{2} \times t$, $t = \dfrac{10}{3}$

This question was quite well answered. However, too many students went on to forfeit this mark by giving their final answer as an approximate 3.3, rather than as an exact value.

Question 5b.

Marks	0	1	2	Average
%	44	7	49	1.1

Using $s = ut + \dfrac{1}{2}at^2$, $-6 = 10t - 4.9t^2$, $t = 2.5$

Consistency of signs in the constant acceleration formula was a challenge for many. Few students demonstrated any understanding of the concept of 'displacement' and made hard work of this question by splitting the motion up into three stages. A poor understanding of kinematics was demonstrated by one group of students who attempted to solve the problem by assuming that $v = 0$ when the chocolate hit the ground.

Question 5c.

Marks	0	1	2	Average
%	70	7	23	0.6

$s = ut + \dfrac{1}{2}at^2$, $-6 = u \times \left(\dfrac{22}{3}\right) - 4.9 \times \left(\dfrac{22}{3}\right)^2$, $u = 35.1$

A number of students took 4 as the time to use in the above equation and others used 4 added to their answer to part 5b. instead of using $\dfrac{22}{3}$. The problems detailed in part 5b. were very much repeated in this part. When the motion was split up into several stages, complicated equations (some correct) for the total time appeared, such as

$\dfrac{22}{3} = \dfrac{2u}{g} + \dfrac{1}{g}\left(\sqrt{u^2 + 2 \times g \times 6} - u\right)$.

Question 5di.

Marks	0	1	2	3	Average
%	19	9	20	52	2.1

$t = \displaystyle\int \frac{-10}{\sqrt{196 - v^2}}\,dv$, $-\dfrac{t}{10} = \sin^{-1}\left(\dfrac{v}{14}\right) + c$, $v = 14\sin\left(\dfrac{\pi}{6} - \dfrac{t}{10}\right)$. The form $v = 14\cos\left(\dfrac{\pi}{3} + \dfrac{t}{10}\right)$ was also accepted.

This question was generally quite well done, with the main errors being the incorrect evaluation of the constant of integration or its omission altogether.

Question 5dii.

Marks	0	1	2	Average
%	27	24	49	1.2

$0 = 14\sin\left(\dfrac{\pi}{6} - \dfrac{t}{10}\right)$, $t = \dfrac{5\pi}{3}$

This question was reasonably well done. Most students knew to substitute $v = 0$ in their result to part 5di.

Question 5diii.

Marks	0	1	2	3	Average
%	37	11	15	37	1.5

$$\frac{dx}{dt} = 14\sin\left(\frac{\pi}{6} - \frac{t}{10}\right),\ x = \int_0^{5\pi/3} 14\sin\left(\frac{\pi}{6} - \frac{t}{10}\right)dt,\ x = 18.8$$

This question was generally done well by those students who completed the paper up to this last stage. One error, though not common, was the use of constant acceleration formulas for this part.