2013 Further Mathematics GA 3: Written examination 2

GENERAL COMMENTS

The selection of modules by students in the Further Mathematics examination 2 in 2013 is shown in the table below.

MODULE	% 2013
1 – Number patterns	26
2 – Geometry and trigonometry	63
3 – Graphs and relations	41
4 – Business-related mathematics	30
5 – Networks and decision mathematics	43
6 – Matrices	72

Examination 2 is designed to present students with straightforward questions at the commencement of each module, where the mathematics required is reasonably evident. Questions then become more challenging towards the end of each module.

Students are reminded to read a question carefully and ensure they understand it before answering. The focus should then be on using the data provided and carefully applying and communicating the relevant mathematics. Students should check that their answer fits the context and requirements of the question.

Some students inadvertently transpose digits when transferring data from the question or their own working, resulting in an incorrect answer. Where this is the case, marks may still be available to students if they have shown their working out. Other students provide working that is very difficult to follow. This was sometimes seen in responses to a ‘show that’ question where a relevant calculation needed to be shown and the required result identified. In order to maximise marks, students must clearly communicate the appropriate mathematics.

Students may find that estimating an answer first may help them to decide if their final answer is reasonable. If they find that an answer is not reasonable, then an error has likely occurred and can be corrected.

The rounding of answers in dollars is a continuing issue and applies to the Core and all modules. Some students rounded to the nearest dollar, despite instructions to round to the nearest cent. While Australia does not use one-cent pieces in its physical currency, students must not round currency answers to the nearest five cents unless specifically instructed to do so. Banking, money exchange, supermarkets, petrol stations and other organisations deal with single cents, or even fractions of a cent, electronically on a daily basis.

SPECIFIC INFORMATION

The statistics in this report may be subject to rounding errors resulting in a total less than 100 per cent.

Core

Questions 1a. and 1b.

Marks	0	1	2	Average
%	5	25	70	**1.7**

1a.
31

1b.
61%

$$\frac{49+45}{153}\times 100 = 61.4379...$$

Questions 2a. and 2b.

Marks	0	1	2	3	Average
%	8	28	12	52	**2.1**

2a.
the mode = 78, the range = 9

2b.
$Q_1 - 1.5 \times IQR = 75 - 1.5 \times 3 = 70.5$. Therefore, 70 is an outlier because it is less than 70.5.

This question asked for an explanation of why 70 was an outlier for this group of countries. Many students calculated a value of 70.5 and then wrote that 'it is therefore an outlier'. Further explanation, including a direct comparison between 70 and 70.5, was expected.

Other common, incomplete or unacceptable answers included

- 70 is outside the bulk of the data – this does not compare 70 with $Q_1 - 1.5 \times IQR$
- it is outside 2SD from the mean – standard deviations do not define an outlier
- there is no other country close to it – this does not compare 70 with $Q_1 - 1.5 \times IQR$.

Question 3a.

Marks	0	1	Average
%	40	60	**0.6**

1.8

$$z = \frac{y-\bar{y}}{s_y} = \frac{91-85.6}{2.99} = 1.806...$$

Most students applied the correct rule to find the *z* score, but many used the wrong variable. The rule on the formula sheet was given as $z = \dfrac{x-\bar{x}}{s_x}$ and applied to the *z* score for the pay rate (independent variable, *x*).

However, the question asked for the *z* score for the development index (the dependent variable, *y*) for which $z = \dfrac{y - \bar{y}}{s_y}$ must be used.

Many students failed to use brackets to evaluate $\dfrac{91 - 85.6}{2.99}$, which must be entered as (91 – 85.6) ÷ 2.99. Without brackets, this calculation becomes $91 - \dfrac{85.6}{2.99} = 62.371...$, which was a common, incorrect answer.

Questions 3b. and 3c.

Marks	0	1	2	3	4	Average
%	34	8	17	9	32	**2**

3b.

For the equation $y = a + bx$, the value of

$$b = r\frac{S_y}{S_x} = 0.488 \times \frac{2.99}{5.37} = 0.2717... \approx 0.272$$

and $a = \bar{y} - b\bar{x} = 85.6 - 0.2717... \times 15.7 = 81.334 \approx 81.3$

3c.

– 2.2

predicted = 81.3 + 0.272 × 14.3 = 85.1896…

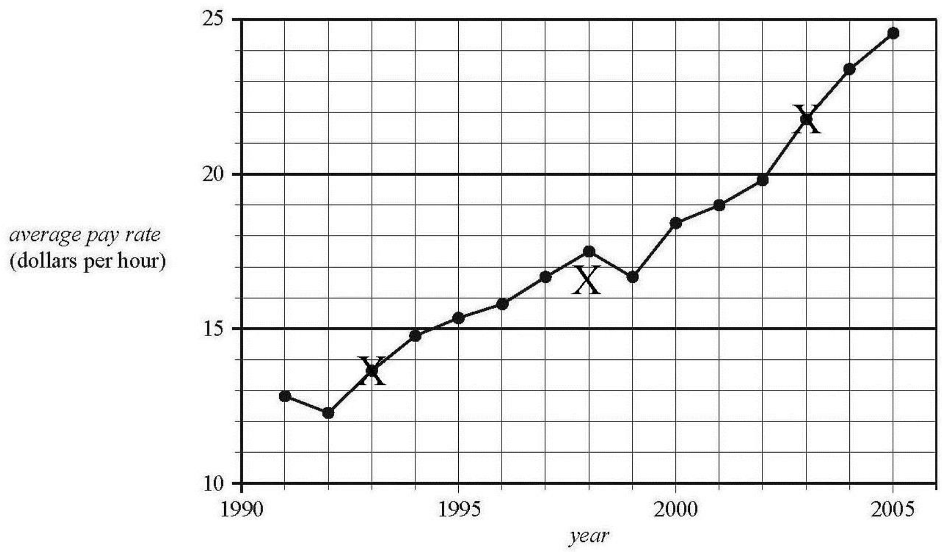
∴ residual = 83 – 85.1896…= – 2.1896…

Many students correctly found the predicted value but did not continue to find the residual value as required.

Questions 4ai. and 4aii.

Marks	0	1	2	3	Average
%	31	31	13	26	**1.4**

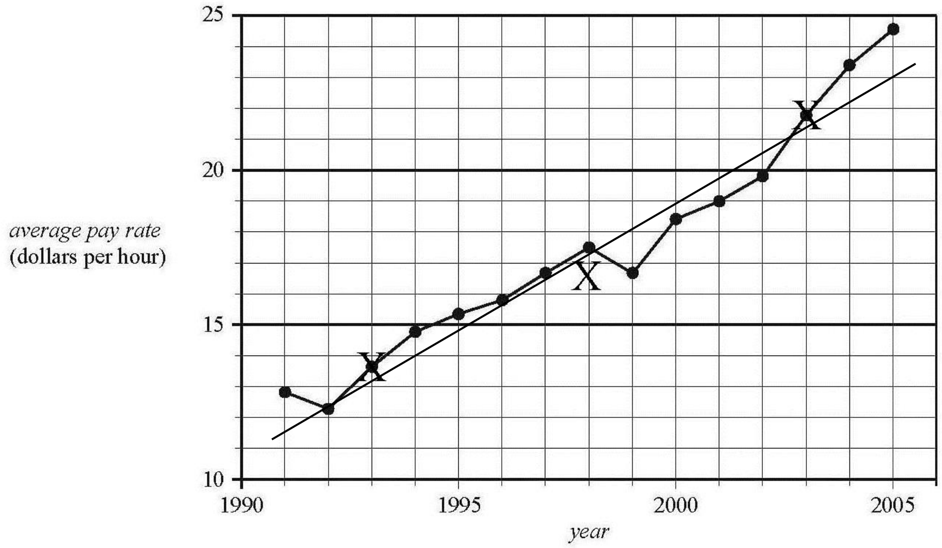
4ai.



Most students found the two correct outer points but few found the correct middle point. Commonly, the middle point was incorrectly located on the dot for 1999 and was in line with the two outer points at 1992 and 2003. A parallel movement of the line that connected the two outer points one third of the way towards the middle point could not then be demonstrated.

There are several concepts in the Core that are often confused. For this question, some students found the 13 points that would result from three-median smoothing instead of the three-median line as required.

4aii.



Questions 4b. and 4c.

Marks	**0**	**1**	**2**	**Average**
%	54	29	17	**0.7**

4b.
0.8

$$\frac{21.8-13.7}{2003-1993}=0.81$$

4c.
The average pay rate increases by \$0.80 per hour each year.

An interpretation of the slope of a line was required. Several students wrote 'strong, positive, linear relationship' but this was not appropriate. These students may have copied this from their bound book of notes.

The interpretation of the slope had to relate to the *average pay rate* and the *year*. This means that appropriate units must have been given, in this case in dollars per hour per year. An answer that simply indicated 'an increase of 0.8 each year' did not explain what the number 0.8 referred to and was not accepted.

Module 1: Number patterns

It appears that students' understanding of difference equations is improving, but students must read the requirements of each question carefully.

Question 1

1a. and 1b.

Marks	0	1	2	Average
%	5	25	70	**1.7**

1a.
3

1b.
21

2a.–3c.

Marks	0	1	2	3	Average
%	16	10	26	49	**2.1**

2a.
16.6

$8 \times 1.2^{(5-1)} = 16.58$

2b.
9th year

$t_8 = 28.7$
$t_9 = 34.4$
$t_{10} = 41.3$

2c.
208

$$S_{10} = \frac{8\left(1.2^{10} - 1\right)}{1.2 - 1} = 207.66...$$

Questions 3ai.–3b.

Marks	0	1	2	3	4	Average
%	22	15	23	23	18	**2**

3ai.
150

$P_3 = 250 = P_2 + 50 \rightarrow P_2 = 200$

$P_2 = 200 = P_1 + 50 \rightarrow P_1 = 150 = c$

3aii.
Arithmetic

3b.
$a = 100$, $b = 50$

The most common incorrect answer was $a = 150$

Questions 4a.–4c.

Marks	0	1	2	3	Average
%	38	11	19	32	**1.5**

4a.
20%

The most common incorrect answer was 80%

4b.
11 700

$A_1 = 18\ 000$
$A_2 = 14\ 500$
$A_3 = 11\ 700$

4c.
3600

Solve $18\ 000 = 0.8 \times 18\ 000 + k$

Questions 5a. and 5b.

Marks	0	1	2	3	Average
%	75	10	12	4	**0.5**

5a.
$L_{n+1} = 0.9\, L_n$, $L_{2012} = 20$

$$r = \frac{18}{20} = 0.9$$

This question asked students to write a difference equation for a sequence.

A rule has one part that models the growth or decay in a sequence and does not need a starting value.
A difference equation has two parts: a rule and a starting value. Together these define the particular sequence.

Some students only wrote the rule $L_{n+1} = 0.9\, L_n$ for this question. Inclusion of the initial value would have turned this rule into a difference equation as required.

The question defined L_n and gave the values of L_{2012} and L_{2013}. Of those students who wrote a difference equation, most incorrectly called the initial value L_1 rather than L_{2012}.

5b.
2016

n	Year	tonnes	$/*t*	income
1	2012	20	1500	30 000
2	2013	18	1300	23 400
3	2014	16.2	1100	17 820
4	2015	14.58	900	13 122
5	2016	13.12	700	9 185

Below $10 000 in 5th year

Conversion from Year 5 to 2016 was necessary for full marks.

Module 2: Geometry and trigonometry

Questions 1a.–1c.

Marks	0	1	2	3	4	Average
%	4	10	11	22	52	**3.1**

1a.
42 m

1bi.
77 m

Some students ignored the obtuse angle and inappropriately applied Pythagoras's theorem to this triangle.

1bii.
957 m^2

$$A = \frac{1}{2} \times 40 \times 52 \times \sin(113^\circ) = 957.325...$$

1c.
14°

$$\tan^{-1}\left(\frac{13}{52}\right) = 14.036...$$

Many students did not answer this question.

Questions 2a. and 2b.

Marks	0	1	2	3	Average
%	27	26	7	40	**1.6**

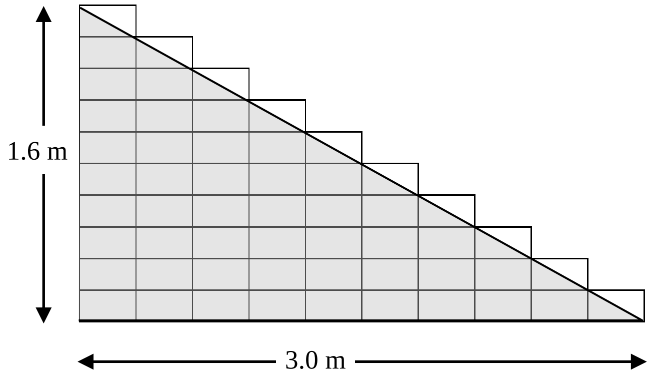
2a.
0.048 m^2

Converting units of area was an issue for some students. Many students correctly found the shaded area to be 480 cm^2 but then incorrectly converted this to 4.8 m^2.

2b.
6.6 m^3

Area of the cross-section = 55 × area of one of the rectangles

Many students inappropriately used $A = \frac{1}{2} \times 3.0 \times 1.6$. This would give the area shaded in the diagram below and is the equivalent of five rectangles fewer than the cross-section area required.



Questions 3a. and 3b.

Marks	0	1	2	Average
%	34	34	32	**1**

Question 3 was, in general, very poorly answered. Many students did not attempt this question.

3a.
$200 + \pi d = 400$

The given diameter of 63.66 m needed to be the end result of a calculation in this 'show that' question.

A common unacceptable answer included an appropriate equation involving the radius, *r*, but did not include an explanation of how this gave the diameter, *d*, as required. While this last step might appear to be unnecessary, it is essential that connections between variables are explained.

3b.
9549 m^2

$$A = 100 \times 63.66 + \pi\left(\frac{63.66}{2}\right)^2 = 9548.9...$$

Question 3c.

Marks	0	1	2	Average
%	58	15	26	**0.7**

3c.
340 m^3

Area within outer boundary = $100 \times 79.66 + \pi\left(\frac{79.66}{2}\right)^2 = 12\ 949.9..$

Volume = (12 950 – 9549) × 0.1 = 340.1

This question was very poorly answered. Common errors included using a diameter of 63.66 + 8 = 71.66 m for the outside track and using the inappropriate formula for the volume of a sphere.

Some students treated the running track as a prism, 0.1 m deep, 8 m wide and 400 m long. Many students did not attempt this question.

Questions 4 and 5

Marks	0	1	2	3	4	Average
%	68	11	12	4	5	**0.7**

Question 4
1.728

$$k^2 = \frac{72}{50} = \frac{36}{25} = 1.44$$

$$\therefore\ k = \frac{6}{5} = 1.2$$

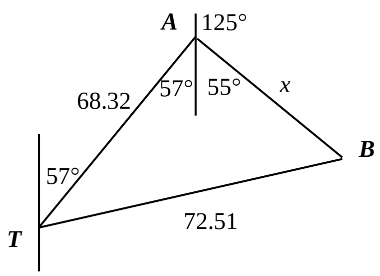
$$\therefore\ k^3 = 1.2^3 = \frac{216}{125} = 1.728$$

The ratio of dimensions of the larger discus to the smaller discus was required. The value of $k > 1$ since a larger volume is scaled up from the smaller volume.

A common error was to work with the reciprocal of the area ratio. This would give a value less than one for the linear and the volume ratios. If this is multiplied with the smaller discus dimensions, the result would be a smaller discus, not a larger one.

This question was very poorly answered. Many students did not attempt this question.

Question 5
9.7 m



One method to determine the length *AB* uses the sine rule to first determine $\angle ABT$. Then, since $\angle TAB = 112°$, $\angle ATB$ can be calculated and used in the cosine rule to find the length *AB*.

Many students drew a diagram that showed all of the bearings. However, many misread the question and incorrectly used 125° as the bearing of point *B* from *T* rather than of point *B* from *A*. Others incorrectly used 72.51 m as the length of *AB*.

Module 3: Graphs and relations

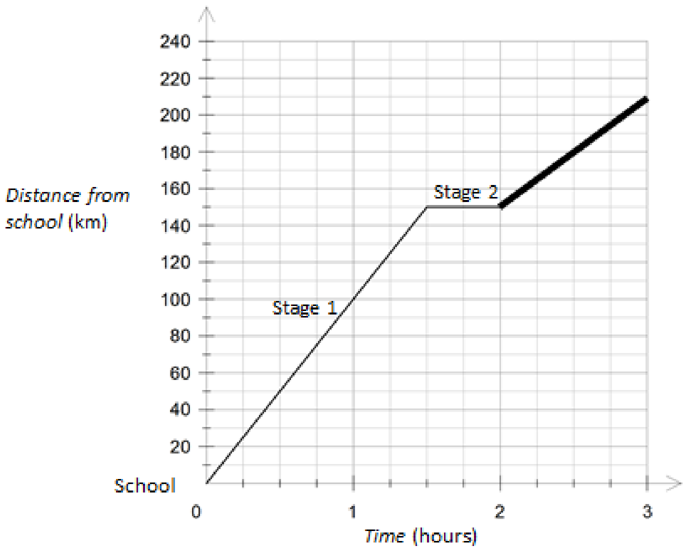
Questions 1a.–1c.

Marks	0	1	2	3	Average
%	3	9	21	68	**2.6**

1a.
100 km/h

1b.
30 minutes

1c.



Questions 1d. and 1e.

Marks	0	1	2	Average
%	34	39	26	0.9

1d.
70 km/h

$$\frac{210}{3} = 70$$

1e.
$k = 30$

$60 \times 2 + k = 150$
$\therefore\ k = 30$

A common incorrect answer was 150.

Questions 2a. and 2b.

Marks	0	1	2	Average
%	8	18	74	1.7

2a.
7 hours

$20s + 30(2) = 200$
$\therefore\ s = 7$

2b.
4 hours each

$20t + 30t = 200$
$\therefore\ t = 4$

Questions 3a. and 3b.

Marks	0	1	2	Average
%	34	30	36	1

3a.
$18n - 260$

Profit = $24n - (6n + 260)$

Many students forgot to include the bracket needed for the expression for cost.

3b.
43 students

$18n - 260 \geq 500$
$\therefore n \geq 42.2$

Students must take the context of a question into account before rounding numbers. Many students rounded 42.2 down to an answer of 42. The question asked for the minimum number of students who will need to participate in order to make a profit of at least $500, which will not quite be achieved with only 42 students.

Questions 4a. and 4b.

Marks	0	1	2	Average
%	32	43	25	1

4a.
$a = 6$, $b = 4$

Unacceptable answers included $a \leq 6$ and $b \leq 4$. The values of a and b are constants within an inequation and only equals signs can apply to the constants in this question.

4b.
$y \geq 2x$

At least two unpowered sites (y) were needed for every powered site (x). To write this as an inequation, it may have been useful to first explore the minimal relationship between x and y; that is, to look first at **exactly** two unpowered sites for every one powered site to build a table.

x	y
1	2
2	4
3	6

This leads to the equation $y = 2x$

However, **at least** two unpowered sites for every one powered site were needed. Then, the following table would also be suitable.

x	y
1	2
2	5
3	10

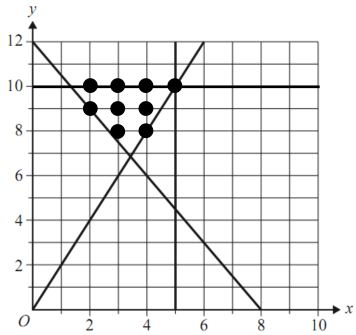
Here, the value of y is greater than, or equal to, twice the value of x. Hence, the inequation is $y \geq 2x$

A common incorrect answer was $2y \geq x$ and, if tested against the second table, would have been found to be inappropriate.

Questions 4c.–4eii.

Marks	0	1	2	3	4	Average
%	57	19	13	7	5	0.9

4c.



Only whole numbers apply to the number of powered and unpowered sites. Therefore, it was expected that nine points within the feasible region, as above, were identified or listed. However, in this question for this year, a correctly shaded region was accepted.

4d.
11

Either of the points (2, 9) or (3, 8) give the minimum number of powered and unpowered sites, 11.

4ei.
$390

The two points (2, 9) or (3, 8) can now be tested to find the lowest cost.
(3, 8) gives Cost = $60 \times 3 + 30 \times 8 = \$420$
(2, 9) gives Cost = $60 \times 2 + 30 \times 9 = \$390$

4eii.
$480

24 boys in site A + 24 girls in site B

This requires at least $x = 4$ powered sites and $y = 6$ unpowered sites. However, the point (4, 6) is not inside the correct feasible region. It does not satisfy the correct inequality 4 from Question 4b. as at least two unpowered sites (y) are needed for every powered camp site (x).

$y \geq 2x$ is needed

$\therefore \quad y \geq 2 \times 4$

$\therefore$ minimum cost at (4, 8) = $240 + $240 = $480

Module 4: Business-related mathematics

In this module, some students simply wrote answers without showing any working and missed out on method marks or consideration for rounding or consequential errors.

Answers must be rounded as instructed. This is often to the nearest cent and not to the nearest five or ten cents or the nearest dollar. When cents are included in a sum of money, two decimal places should be written. For example, $5245.40 rather than $5245.4

Questions 1a.–1c.

Marks	0	1	2	3	4	Average
%	1	7	27	39	27	2.8

1a.
$8000

1bi.
8000 – 6500 = 1500

1bii.
$500

8000 – 5 × 1500 = 500

1c.
12 000

$0.25 \times k = 3000$

Many students did not attempt this question.

Questions 2a.–2bii.

Marks	0	1	2	3	Average
%	40	22	20	18	1.2

2a.
$5245.35

$$5000 \times \left(1 + \frac{4.8}{100 \times 12}\right)^{12} = 5425.3510...$$

Optionally, a calculator TVM solver function could have been used in this question.

An incorrect answer of \$5245.40 was often seen.

2bi.
\$7698.86

N = 12
I % = 4.8
PV = –5000
PMT = –200
FV = **7698.8614…**
P/Y = 12
C/Y = 12

Students attempted to use various formulas that often resulted in calculation errors. The TVM function of technology provides an effective tool for solving annuity problems.

2bii.
\$298.86

$7698.86 - (5000 + 12 \times 200) = 298.86$

Many students did not allow for the \$200 that Hugo had added each month for a year.

Questions 3a.–3d.

Marks	0	1	2	3	4	5	6	Average
%	33	12	14	13	9	10	9	2.2

3a.
\$362.50

$I = 7500 \times 0.08 \times 2 = 1200$

$$\text{Payment} = \frac{7500+1200}{24} = 362.5$$

Students often confuse simple (flat) interest with compound interest calculations. Many students inappropriately treated this as a reducing balance problem. Students should understand that the term ‘flat interest rate’ refers to simple interest.

3b.
15.36%

$$\text{Effective rate} = \frac{2\times 24}{24+1}\times 8 = 15.36$$

3c.
Simple interest does not take into account reductions in the outstanding balance as repayments are made.

The \$50 is only 8% per annum of the loan balance in the first month. Each monthly repayment reduces the balance of the loan and the regular \$50 interest payment becomes a greater percentage of this remaining balance. Hugo will be paying increasingly more than 8% per annum as the loan balance reduces. The effective rate is approximately double the flat rate and represents an average actual rate paid in the life of the loan.

3d.
\$3328

Depreciation rate = $\dfrac{7500 - 6375}{7500} = 15\%$

Value after 5 years
= 7500×0.85^5 = \$3327.789…

Question 4

Marks	0	1	2	Average
%	67	14	19	0.5

\$11 029

Students needed to read this question carefully to identify the three steps needed for the solution. Writing and labelling such steps can be very helpful in organising thoughts, but very few students showed any TVM input or other working.

Stage 1– Find the quarterly payment that would repay the loan in 4 years (16 repayments)
N = 16
I % = 12
PV = 25 000
PMT = **–1990.2712…**
FV = 0
P/Y = 4
C/Y = 4

Stage 2 – Use the payment found from Stage 1 to find the principal remaining after 2 years (8 repayments)
N = 8
I % = 12
PV = 25 000
PMT = –1990.27 (from Stage 1 above)
FV = **–13 971.09…**
P/Y = 4
C/Y = 4

To complete the answer,
Stage 3 – Amount paid off the principal = (loan value) – (principal remaining after eight quarters)
= (25 000) – (13 971.09)
= 11 028.91

The convention used above for input values is that any money coming to Hugo (such as a loan to him) is treated as positive. Any money leaving Hugo (such as paying out on a loan or investing money elsewhere) is treated as negative.

In Stage 2 above, the value of PV is positive since it is the loan Hugo had received. The value of PMT is negative because it is money that leaves Hugo when he pays it.

The value of FV is found to be negative. Following the convention used above, this negative value must be interpreted as an amount that will 'leave' Hugo in the future. In other words, the principal remaining is negative and he will still owe \$13 971.09 after eight months.

An alternative sign convention for TVM input is merely the opposite of what has been used above. That is, any money coming to Hugo would be regarded as negative, and any money leaving Hugo would be regarded as positive. It does not matter which convention is used as long as interpretations are consistent with the chosen convention.

Module 5: Networks and decision mathematics

Questions 1a.–1d.

Marks	0	1	2	3	4	5	Average
%	1	1	3	7	18	70	**4.5**

1a.
3

1b.
1000 m

1ci.
P4

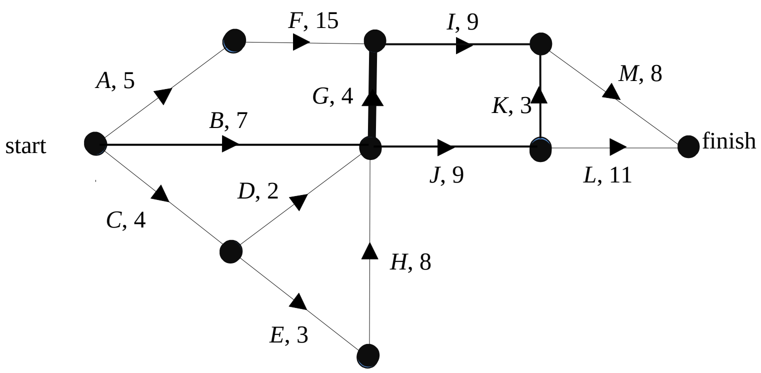
1cii.
Euler path

1d.
E – *P*5 – *P*4 – *P*6 – *P*3 – *P*2 – *P*1

Questions 2a. and 2b.

Marks	0	1	2	Average
%	10	26	64	**1.6**

2a.



2b.
7 hours

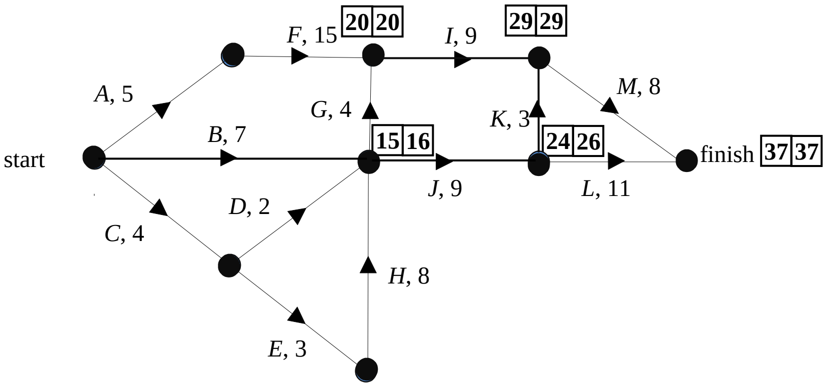
Activity *H* cannot start until activities *C* and *E* have both been completed, taking a minimum of 7 hours in total.

Questions 2ci.–2e.

Marks	0	1	2	3	4	Average
%	25	23	28	18	7	**1.6**

2ci.
A-F-I-M

2cii.
14



The critical path *AFIM* takes 37 hours.
The latest starting time of activity *D* depends upon the latest starting time of activities *G* and *J*, of which *G* is the most critical.
Activity *G* connects to the critical path as a prerequisite for activity *I* and must finish no later than 20 hours.
Activity *G* takes 4 hours to complete and its latest starting time = 20 – 4 = 16
Then, activity *D* must end no later than 16 hours.
Therefore, its latest starting time = 16 – 2 = 14

A common incorrect answer was 15 hours.

2d.
This will happen only if the crashed activity is on the (single) critical path *A-F-I-M* in this project.

This question was generally poorly answered.

2e.
36 hours

The critical path *A-F-I-M* is 37 hours long.
If activity *F* is crashed by 2 hours, the path *C-E-H-G-I-M* (36 hours long) will then become the new critical path.
Crashing activity *F* by more than one hour will not reduce the completion time of the project below 36 hours since *F* is not on the new critical path *C-E-H-G-I-M*.

Questions 3a.–3bii.

Marks	0	1	2	3	4	Average
%	20	20	35	21	3	1.7

3a.
37

The numbers on the edges of the directed network gave the maximum number of people who are permitted to walk along any one of the tracks.

The question required the maximum number of people permitted to walk from *A* to *D* each day. Many students did not see this question as a minimum cut/maximum flow problem.

Maximum flow = minimum cut of 37 through *CD* and *ED* or through *AB*, *FB* and *FE* or through *BC*, *EC* and *ED*.

3bi.
A-B-E-C-D

3bii.

Group	Maximum group size	Path taken from *A* to *D*
1	17	answered in **part b.i.**
2	11	*A-F-E-D*
3	7	*A-G-F-B-C-D*
4	2	*A-B-E-D*

Once 17 students in group 1 had plotted their route *ABECD*, there was a maximum of another 37 – 17 = 20 students who would be permitted to walk from *A* to *D*.

The table required the maximum group size for the remaining 20 students, from which the second-largest possible group was 11, leaving 9 students remaining to form another one group or more.

Often, group sizes were unreasonable since they exceeded 24, which was the number of students permitted initially on the most vacant track *C* to *D*.

Module 6: Matrices

Questions 1a.–1c.

Marks	0	1	2	3	Average
%	5	16	33	46	2.2

1a.
2

1b.
Two ponds connect directly by pipe to pond R.

1c.

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

A common incorrect answer had 0, 1, 1, 1, 0 in the first row, which was unchanged from the original matrix *W*.

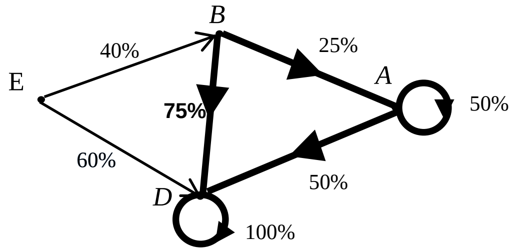
Questions 2ai.–2aii.

Marks	0	1	2	3	Average
%	19	19	31	31	1.7

2ai.
6000

60% × 10 000 = 6000

2aii.



The 100% cycle drawn at *D* (all that die this year will be dead next year) was a common omission.

Students should not add edges marked 0% that are in the opposite direction to the given edges. Similarly, loops with 0% at *E* and *B* should not be included.

Questions 2bi.–2d.

Marks	0	1	2	3	4	5	6	Average
%	21	14	16	20	19	4	6	2.4

2bi.

$$S_1 = \begin{bmatrix} 0 \\ 4000 \\ 650 \\ 7150 \end{bmatrix}$$

$$S_1 = T\,S_0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.4 & 0 & 0 & 0 \\ 0 & 0.25 & 0.5 & 0 \\ 0.6 & 0.75 & 0.5 & 1 \end{bmatrix}\begin{bmatrix} 10\,000 \\ 1000 \\ 800 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 4000 \\ 650 \\ 7150 \end{bmatrix}$$

2bii.
331.25 adult trout

$$S_4 = T^4\,S_0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.4 & 0 & 0 & 0 \\ 0 & 0.25 & 0.5 & 0 \\ 0.6 & 0.75 & 0.5 & 1 \end{bmatrix}^4 \times \begin{bmatrix} 10\,000 \\ 1000 \\ 800 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 331.25 \\ 11\,468.75 \end{bmatrix}$$

On average, the state matrix predicts that there will be 331.25 adult trout after four years. Since this average is a predicted value, a decimal number of adult trout is valid. An answer of 331 adult trout was common and also accepted.

2ci.
13 years

$$S_{12} = T^{12}\, S_0 = \begin{bmatrix} 0 \\ 0 \\ 1.2939... \\ 11\,790.706... \end{bmatrix} \quad S_{13} = T^{13}\, S_0 = \begin{bmatrix} 0 \\ 0 \\ 0.64697... \\ 11\,799.353... \end{bmatrix}$$

S_{13} is the first state matrix where the number of adult trout is less than one, as asked for in the question.

2cii.
1325 adult trout

$$S_1 = T\, S_0 = \begin{bmatrix} 0 \\ 4000 \\ 650 \\ 7150 \end{bmatrix} \quad S_2 = T^2\, S_0 = \begin{bmatrix} 0 \\ 0 \\ 1325 \\ 10\,475 \end{bmatrix} \quad S_3 = T^3\, S_0 = \begin{bmatrix} 0 \\ 0 \\ 662.5 \\ 11\,137.5 \end{bmatrix} \quad S_4 = T^4\, S_0 = \begin{bmatrix} 0 \\ 0 \\ 331.25 \\ 11468.75 \end{bmatrix}$$

S_2 gives the greatest number of adult trout.

It was not sufficient to assume that the number of adult trout was decreasing without checking the values given by S_2 and S_3.

A common incorrect answer was 800, taken directly from S_0. This may have been based on observing that S_1 had fewer (650) adult trout and S_4 had ever fewer (331.25) adult trout. The assumption that this meant there was decrease from S_0 through to S_4 should have been checked.

2d.
Add 10 000 eggs, remove 3000 baby trout and add 150 adult trout.

$$S_0 - S_1 = \begin{bmatrix} 10\,000 \\ 1000 \\ 800 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 4000 \\ 650 \\ 7150 \end{bmatrix} = \begin{bmatrix} 10\,000 \\ -3000 \\ 150 \\ -7150 \end{bmatrix}$$

Many students did not answer this question.

Questions 2ei.–2eii.

Marks	0	1	2	3	Average
%	34	27	16	22	1.3

2ei.

$$S_1 = \begin{bmatrix} 200\,000 \\ 4000 \\ 650 \\ 7150 \end{bmatrix}$$

$$S_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.40 & 0 & 0 & 0 \\ 0 & 0.25 & 0.50 & 0 \\ 0.60 & 0.75 & 0.50 & 1.0 \end{bmatrix}\begin{bmatrix} 10\,000 \\ 1000 \\ 800 \\ 0 \end{bmatrix} + 500 \times \begin{bmatrix} 0 & 0 & 0.50 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 10\,000 \\ 1000 \\ 800 \\ 0 \end{bmatrix} = \begin{bmatrix} 200\,000 \\ 4000 \\ 650 \\ 7150 \end{bmatrix}$$

2eii.
162 500

$$S_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.40 & 0 & 0 & 0 \\ 0 & 0.25 & 0.50 & 0 \\ 0.60 & 0.75 & 0.50 & 1.0 \end{bmatrix} \times S_1 + 500 \times \begin{bmatrix} 0 & 0 & 0.50 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \times S_1$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.40 & 0 & 0 & 0 \\ 0 & 0.25 & 0.50 & 0 \\ 0.60 & 0.75 & 0.50 & 1.0 \end{bmatrix}\begin{bmatrix} 200\,000 \\ 4000 \\ 650 \\ 7150 \end{bmatrix} + 500 \times \begin{bmatrix} 0 & 0 & 0.50 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 200\,000 \\ 4000 \\ 650 \\ 7150 \end{bmatrix} = \begin{bmatrix} 162\,500 \\ 80\,000 \\ 1325 \\ 130\,475 \end{bmatrix}$$

Using the equation given, the state matrix for each year is calculated by using the state matrix from the year before. Many students instead found $S_2 = T^2 S_0 + 500 M S_0$ rather than using S_1 to determine S_2.

Of those who did a correct calculation, some left the matrix S_2 as their answer. The number of eggs had to be extracted from matrix S_2 for full marks.