2015 VCE Further Mathematics 1 examination report

General comments

The majority of students were generally well prepared for Further Mathematics examination 1 in 2015.

Specific information

The tables below indicate the percentage of students who chose each option. The correct answer is indicated by shading.

The statistics in this report may be subject to rounding resulting in a total more or less than 100 per cent.

Section A
Core: Data analysis

Question	% A	% B	% C	% D	% E	% No Answer
1	4	12	2	12	70	0
2	3	88	1	7	1	0
3	5	2	12	14	66	0
4	65	9	16	7	3	0
5	71	14	9	4	2	1
6	50	46	1	2	1	0
7	4	8	6	68	14	0
8	10	15	47	16	13	0
9	62	7	13	4	14	1
10	5	8	69	13	5	1
11	69	6	9	5	11	1
12	2	13	13	63	8	0
13	8	10	17	47	16	1

The Core section was generally well answered.

Most students correctly answered questions that required a routine application of a skill in a familiar circumstance (for example, Questions 1, 3, 4 and 5).

Fewer students correctly answered questions that required the application of conceptual understanding to obtain an answer (for example, Questions 6, 8 and 13).

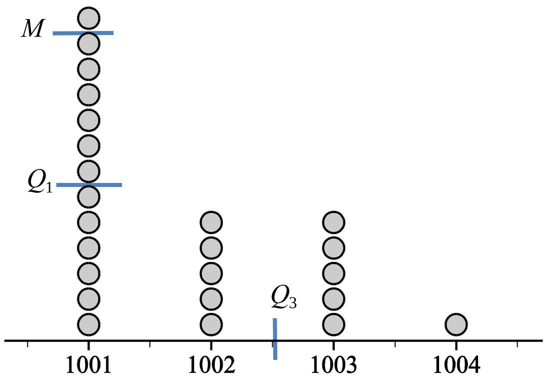
Question 6

Students were asked to construct a five-number summary from a boxplot with outliers. Many students incorrectly selected option A because they failed to recognise that, when a boxplot displays outliers, these values cannot be ignored when determining the minimum and maximum values in the distribution.

Question 8

Students were asked which one of the boxplots best represented the given dot plot.

The most efficient and effective way of answering this question was to recognise that a dot plot automatically orders the data value numerically. The key values needed to construct a boxplot can be readily obtained by inspection.



Firstly, the minimum and maximum values correspond to the first and last dots in the plot. Thus, minimum value = 1001 and the maximum value = 1004

Secondly, the median value is the middle value. For 24 data points it lies between the twelfth and thirteenth values, which are both 1001. Thus, M =1001

The first quartile is the median of the bottom 12 values, which lies between the sixth and seventh values, which are both 1001, thus, Q_1 = 1001

The third quartile is the median of the top 12 values, which lies between the sixth and seventh values, which are 1002 and 1003, thus, $Q_3 = \dfrac{1002+1003}{2} = 1002.5$

Finally, the upper, outer fence needs to be located to determine whether any of the larger values are outliers.

upper fence $= Q_3 + 1.5 \times IQR$

$= 1002.5 + 1.5 \times (Q_3 - Q_1)$

$= 1002.5 + 1.5 \times (1002.5 - 0)$

$= 1006.25$

Thus, as the largest data value, 1004, is less than the upper fence, there are no outliers.

Question 13

Students were asked to fit a least squares regression line to some sales data that first had to be deseasonalised.

To answer the question, the sales data in Table 1 needed to first be deseasonalised using the seasonal indices in Table 1 as shown below.

Quarter number	1	2	3	4
Sales (tractors sold)	2800	1032	875	759
Deseasonalised sales	1750 (= 2800/1.6)	1720 (= 1032/0.6)	1250 (= 875/0.7)	690 (= 759/1.1)

Using technology to fit a least squares regression line to the *deseasonalised sales* using *quarter number* as the independent (explanatory) variable led to the equation:

$$\textit{deseasonalised sales} = 2300 - 370 \times \textit{quarter number}$$

Module 1: Number patterns

Question	% A	% B	% C	% D	% E	% No Answer
1	1	1	2	97	0	0
2	2	3	19	74	2	0
3	8	5	23	5	58	0
4	29	7	3	2	59	0
5	2	86	3	8	1	0
6	6	13	14	57	9	1
7	8	17	48	21	5	1
8	22	25	38	7	7	1
9	7	9	19	47	17	1

The questions in Module 1: Number patterns were generally well answered.

Questions that required a routine application of arithmetic and geometric sequences in familiar circumstances – for example, Questions 1, 2 and 5 – were generally answered well. Students demonstrated a general competence with difference equations (recurrence relations) and their applications.

Many students found Question 8 challenging. A routine way of answering this question is to write out the first few terms of the sequences generated by each of the five given rules and to identify the sequence that was not geometric. Doing this showed that option A, $A_{n+1} = n \quad A_0 = 1$, generates the sequence 1, 0, 1, 2 …, which is clearly not geometric.

Module 2: Geometry and trigonometry

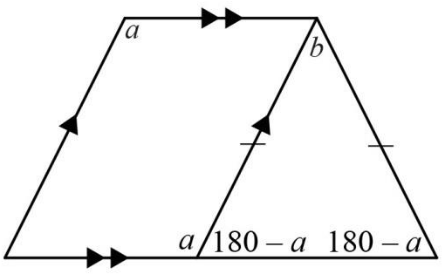
Question	% A	% B	% C	% D	% E	% No Answer
1	1	2	86	9	2	0
2	69	18	6	5	2	0
3	83	6	6	2	3	0
4	11	15	6	17	51	0
5	6	5	57	25	6	1
6	11	7	74	6	3	1
7	2	22	9	61	5	1
8	6	30	21	18	25	1
9	21	41	20	6	11	1

The questions in Module 2 were well answered.

Questions that required a routine application of geometric and trigonometric techniques in a range of contexts, including the use of bearings, were generally answered well. However, it was clear that Questions 8 and 9 were challenging for most students.

Question 8

Students were given five statements concerning the relationships between the angles in the diagram and asked to identify which of the statements was always true.

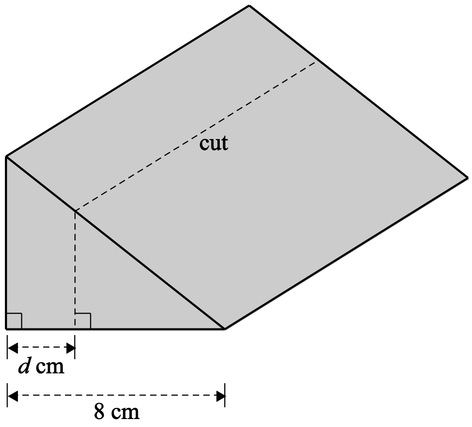


From the diagram, the isosceles triangle has base angles of size $(180 - a)°$.

Thus, $b + 2(180 - a) = 180$ or $2a - b = 180$ (option E).

Question 9

To answer Question 9 correctly, students needed to recognise that only two dimensions of the large wedge of cheese were halved when the smaller wedge of cheese was cut from the larger block of cheese with a single cut. See the following diagram. Thus, the scaling factor was, $k^2 = \frac{1}{2}$ or $k = \frac{1}{\sqrt{2}}$.



From the diagram, the right-angled triangle with the dotted line is similar to the large right-angled triangle, so $\frac{8}{d-8}=k=\frac{1}{\sqrt{2}}$ or $d=2.341...$ (option B).

The word ‘similar’ was used in the stem of this question in its everyday sense. However, some students interpreted this word geometrically, so option A, 1.7, was also accepted.

Module 3: Graphs and relations

Question	% A	% B	% C	% D	% E	% No Answer	Comments
1	23	76	1	0	0	0	
2	3	4	5	11	76	1	
3	20	1	78	0	1	0	
4	62	14	10	6	7	1	
5	9	4	40	37	9	0	
6	8	62	17	9	4	1	
7	5	15	63	12	4	1	
8	4	13	9	11	62	1	
9	16	16	58	6	4	1	

The questions in Module 3 were well answered. Students were generally able to answer questions from most curriculum areas, although most students were challenged when asked to translate a constraint written in everyday language into its mathematical equivalent in Question 5.

Question 5

Students were asked to identify an inequality that could be used to represent the following constraint from a linear programming problem.

Let x be the number of students

y be the number of teachers.

For an overnight school excursion there must be at least one teacher for every 15 students.

The correct response was option D, y ³ $\frac{x}{15}$

Students who chose this option could have quickly tested that this inequality also makes sense when translated into everyday language. For example, if there are 30 students, there will be at least 30/15 = 2 teachers, a reasonable number. However, many students incorrectly chose option C, y ³ $15x$

This inequality translates into everyday language as: ‘There are at least 15 teachers for each student’. Or, if there are 30 students, there will be at least 30 × 15 = 450 teachers, which is not reasonable. Students should check whether their response is reasonable when they are not sure of the answer.

Module 4: Business-related mathematics

Question	% A	% B	% C	% D	% E	% No Answer
1	3	1	83	2	10	0
2	9	8	9	64	9	0
3	4	6	77	8	4	0
4	5	6	18	11	60	1
5	3	19	68	5	4	1
6	13	23	11	46	5	2
7	9	4	10	60	16	1
8	7	22	33	15	23	1
9	19	19	9	37	15	1

Student performance in Module 4: Business-related mathematics was similar to the performance of students in other modules.

Questions that required the routine application of percentage change, the principles of simple and compound interest, and straight line and reducing balance depreciation were generally answered well. Questions that required the routine application of a financial solver for solution were also relatively well done.

Results for Questions 8 and 9 showed that, while students are generally able to solve standard one-step problems involving the use of the financial solver on their calculator, many are unable to solve problems that involve repeated use of the technology.

Question 8

The following is a possible solution strategy.

Use a financial solver to find the quarterly repayment and then reapply this information to find the balance of the loan after six years. This requires two applications on the financial solver.

Step 1: find the quarterly repayments to fully repay the loan in 6 years.

N = 6 × 4 = 24

I = 9

PV = 8400

PMT = ?

FV = 0

P/Y = 4

C/Y = 4

Which gives PMT = –456.7939…

Step 2: Use this amount to find the amount still owing (the balance of the loan) after 3 years

N = 3 × 4 = 12

I = 9

PV = 8400

PMT –456.7939… (this value did not need to be re-entered)

FV = ?

P/Y = 4

C/Y = 4

Which gives FV = –4757.4076 …

Thus, the balance of the loan (the amount still owing) is $4757.41 (to the nearest cent).

Step 3: Use this amount to find the percentage decrease

Percentage decrease = $\frac{(8400 - 4757.41)}{8400} \times 100\% = 43.36...\%$ (option C)

Question 9

Question 9 assessed students' knowledge of the properties of a reducing balance loan.

This question was best answered by systematically testing each of the options against the information given in the stem.

A. His first payment reduces the loan by less than $1950.

True: Part of each $1950 payment goes towards paying interest. What remains, which is less than $1950, goes towards reducing the amount owed.

B. His second payment reduces the loan by more than the first payment.

True: Because it is a reducing balance loan, the amount of money owing reduces with each payment. Thus, the part of each payment that goes towards paying interest decreases with each payment and the amount going to repay the loan increases.

C. Repaying more than $1950 per month will reduce the term of the loan.

True: The extra amount goes to repaying the amount owed, not the interest, so the amount owed decreases more rapidly.

D. His final payment will be less than $1760.

Not true: To test this statement it was necessary to calculate the value of the final payment. Using a financial solver, the number of months required to repay the loan at $1950 per month is 106.9 … months. To calculate the final payment, calculate the future value of the loan after 106 payments and add interest: 1759.88 + (6.18/12/100) × 1759.88 = $1768.94. This amount ($1768.94) is greater than $1760, so D is **not** true.

E. His final payment includes interest.

True: Whenever money is still owed to the lender, interest must be paid.

Module 5: Networks and decision mathematics

Question	% A	% B	% C	% D	% E	% No Answer
1	1	2	94	3	1	0
2	1	5	85	5	3	0
3	6	4	1	7	83	0
4	1	11	67	4	17	0
5	5	1	1	92	1	0
6	59	3	20	11	6	0
7	4	76	12	6	2	1
8	59	21	11	8	1	0
9	20	56	7	12	4	1

Module 5: Networks and decision mathematics questions was well answered.

Questions that required knowledge of the general properties of graphs and their applications across the curriculum were generally answered well.

Module 6: Matrices

Question	% A	% B	% C	% D	% E	% No Answer
1	3	85	4	2	6	0
2	5	2	3	2	88	0
3	9	51	23	15	3	1
4	7	12	5	16	59	0
5	4	6	13	72	5	1
6	8	15	16	56	5	1
7	43	13	29	8	6	1
8	43	26	17	7	6	1
9	15	23	40	12	9	1

The questions in the Matrices module were generally well answered.

Questions that required knowledge of the general properties of matrices and their applications across the curriculum were generally answered well.

However, Questions 7, 8 and 9 required a little more in-depth thinking and were less well answered.

Question 7

In this question students were given that the relationship between two matrices was $wP = Q$, where w is a scalar. The task was to identify the relationship between Q^{-1} and P^{-1}.

Intuitively, the answer is $Q^{-1} = \frac{1}{w}P^{-1}$ if every element in a matrix is multiplied by the same scalar then every element in its inverse will be divided by that scalar (a number). Formally we can verify that the inverse of wP is $\frac{1}{w}P^{-1}$ as follows:

$$wP\left(\frac{1}{w}P^{-1}\right) = \frac{w}{w}PP^{-1} = 1.\,I = I$$

Question 8

To answer Question 8, students needed to first use the rule $x_{ij} = i - j$ for ijth term of the matrix X to determine the matrix and then equate it to one of five given matrix expressions.

Using the rule, $X = \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \end{bmatrix}$, which is equivalent to the matrix expression given in option A.

A: $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \end{bmatrix} = X$

Question 9

In answering this question many students did not take into account that the transition matrix changed from T_1 to T_2 after week 5.

The following is one approach to answering this question.

Step 1: Find an expression for S_5 in terms of T_1 and S_1

$S_2 = T_1S_1$
$S_3 = T_1S_2 = T_1\,(T_1S_1) = T_1^{\,2}\,S_1$
so $S_5 = T_1^{\,4}S_1$

Step 2: The transition matrix changes to T_2 after week 5, so using S_5 as the new starting point:

$S_6 = T_2S_5$ or $S_6 = T_2(T_1^{\,4}S_1)$

so $S_8 = T_2^{\,3}(T_1^{\,4}S_1)$ (option C)