2016 VCE Further Mathematics 1 examination report

General comments

The majority of students were well prepared for Further Mathematics examination 1 in 2016.

Specific information

The tables below indicate the percentage of students who chose each option. The correct answers are indicated by shading.

The statistics in this report may be subject to rounding resulting in a total more or less than 100 per cent.

Section A – Core

In 2016, the Core section comprised two components: Data analysis (Questions 1–16) and Recursion and financial modelling (Questions 17–24).

Question	% A	% B	% C	% D	% E	% No Answer
1	12	83	2	2	1	0
2	8	31	10	45	5	0
3	87	9	1	2	1	0
4	1	6	7	19	66	0
5	7	13	58	11	10	1
6	11	3	6	71	9	0
7	29	12	7	6	45	1
8	4	21	45	4	25	0
9	7	17	62	9	6	0
10	5	54	22	16	2	1
11	54	10	18	12	5	1
12	11	8	3	67	11	0
13	5	3	72	15	6	0
14	79	6	3	7	4	0
15	6	5	6	11	72	0
16	10	16	12	54	7	1
17	2	1	3	3	89	0
18	7	5	77	9	2	0
19	74	14	2	6	4	0
20	4	10	14	67	5	0
21	2	50	4	24	19	0
22	9	54	19	7	10	1
23	13	12	11	58	5	1
24	14	12	30	20	23	1

Data analysis (Questions 1–16)

Core – Data analysis was generally answered well.

Students generally correctly answered questions that required the routine application of a skill in a familiar context.

Students struggled to answer questions that required the application of conceptual knowledge to obtain an answer, in particular Questions 2 and 8. It was also clear that Question 7, which required the interpretation of a histogram with a log scale, was challenging for many students.

Question 2

Students were asked to classify two variables in terms of the type of data they generated. The variables were *blood pressure* (low, normal, high) and *age* (under 50 years, 50 years or over). Both variables are ordinal. However, many students incorrectly identified the variable *age* (under 50 years, 50 years or over) as nominal. The variable *age* (under 50 years, 50 years or over) is ordinal because the process of allocating each of the people to one of these two categories 'under 50 years' or '50 years or over' orders the group of people by age.

Question 7

Students were asked to interpret a histogram displaying the distribution of the number of billionaires in 53 countries. This question required students to recognise that, on a $\log_{10}$ scale, 1 is plotted as 0, because, by definition, $1 = 10^0$ means that $\log_{10}(1) = 0$.

A common error was to ignore the log scale to arrive at the answer 1 (option A).

Question 8

Parallel boxplots are one of the statistical graphs that can be used to investigate the association between a numerical variable and a categorical variable. In this question, the variable *monthly rainfall* (in mm) was numerical, so the unknown second variable must be categorical. *Month of the year* (January, February, March, etc.), option C, was the only categorical variable in the list of options provided.

Recursion and financial modelling (Questions 17–24)

Core: Recursion and financial modelling was reasonably well answered.

Students generally correctly answered questions that required a routine application of a skill or the application of knowledge in familiar circumstances. The most significant student weakness was in the incorrect use of a financial solver to solve an annuity problem presented in non-routine form (Question 24).

Question 24

Students were asked to determine the monthly repayment received from an annuity, given the annual interest rate and the value of the annuity at two points in time. In terms of using a financial solver to answer this question, students needed to:

- recognise that the value of the annuity after 5 years could be treated as the start of a new annuity with a value of $130 784.93, while the value of the annuity after 10 years, $66 992.27, could be regarded as the future value of the annuity after a further 5 years of payments
- correctly apply a sign convention appropriate to an annuity.

Inputting the following values into a financial solver:

N = 5 × 12 = 60

I = 5.2
PV = –130 784.93
PMT =?
FV = 66 992.27
P/Y = 12
C/Y = 12
gives PMT = 1500 (option C).

A common error was to choose option E, which corresponded to both the PV and FV being incorrectly allocated the same sign.

Module 1 – Matrices

Question	% A	% B	% C	% D	% E	% No Answer
1	4	3	81	11	1	0
2	1	7	3	86	3	0
3	5	9	15	7	63	1
4	31	34	8	13	13	0
5	15	11	51	9	13	2
6	1	86	7	2	5	0
7	2	5	30	52	10	0
8	42	12	21	10	14	1

The questions in Module 1 – Matrices were generally well answered. However, Questions 4 and 8 proved challenging for the majority of students.

Question 4

In this question, students were required to identify the matrix product that calculates two quantities: the total number of coins **and** the total value of the coins saved in a moneybox.

Inspection of the five matrix products shows that only option B calculates the required information.

$$\begin{bmatrix} 15 & 32 & 48 & 24 \end{bmatrix} \begin{bmatrix} 1 & 5 \\ 1 & 10 \\ 1 & 20 \\ 1 & 50 \end{bmatrix} \quad \text{(option B)}$$

$$= \begin{bmatrix} 15\times1+32\times1+48\times1+24\times1 & 15\times5+32\times10+48\times20+24\times50 \end{bmatrix}$$

$$= \begin{bmatrix} \text{total number of coins} & \text{total value of coins} \end{bmatrix}$$

Question 8

This question provided students with a one-step dominance matrix for four teams playing in a bowling tournament. When one- and two-step dominances were used to rank the teams, team *C* was ranked number one. However, if the result in only one match was reversed, team *A* would have been ranked number one. The students were asked to identify this match.

A routine way of solving this problem was to construct a matrix whose row sums gave the total dominance (one- and two-step) scores for each player as shown below.

$$\begin{matrix} & A & B & C & D \\ A \\ B \\ C \\ D \end{matrix}\begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}+\begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}^2=\begin{bmatrix} 0 & 1 & 1 & 1 \\ 2 & 0 & 1 & 1 \\ 2 & 1 & 0 & 2 \\ 1 & 0 & 1 & 0 \end{bmatrix}\begin{matrix} \textit{total} \\ 3 \\ 4 \\ 5 \\ 2 \end{matrix}$$

This process could then be repeated for each of the options given until the desired result was obtained. An inspection of the five options given shows that reversing the result of the match team *A* versus team *B*, makes team *A* the winner of the tournament, with a total dominance score of 5, as below.

$$\begin{matrix} & A & B & C & D \\ A \\ B \\ C \\ D \end{matrix}\begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}+\begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}^2=\begin{bmatrix} 0 & 2 & 1 & 2 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 2 \\ 1 & 1 & 1 & 0 \end{bmatrix}\begin{matrix} \textit{total} \\ 5 \\ 2 \\ 4 \\ 3 \end{matrix}$$

Module 2 – Networks and decision mathematics

Question	% A	% B	% C	% D	% E	% No Answer
1	1	97	1	1	1	0
2	11	18	54	14	3	0
3	11	49	35	1	3	0
4	7	9	15	17	52	1
5	58	8	13	8	12	1
6	2	5	74	14	4	0
7	13	45	14	12	15	1
8	11	30	9	27	21	1

The majority of questions in Module 2 were well answered.

No particular content area weaknesses were revealed. However, Questions 3, 7 and 8 proved to be challenging for most students.

Question 3

Students were asked to find the number of edges that could be removed from a complete graph with five vertices so that the graph remains connected but with the minimum number of edges.

The key to answering this question was to recognise that the answer would be a spanning tree, which, for a graph with five vertices, would have four edges. As the original graph had 10 edges, the number of edges to be removed was 10 – 4 = 6 (option C).

Question 7

A systematic way of answering this question is as follows.

Use the information in the directed graph to write down the five paths in the graph and their lengths:

1. *A-D-I-L* 19
2. *B-E-I-L* 20 (critical path)
3. *B-F-J-L* 17
4. *C-G-J-L* 13
5. *C-H-K-L* 19

Inspection of the five paths shows that reducing the time for activity *E* on this path by one hour (option B) reduces the length of the original critical path from 20 to 19, so that there are now three critical paths of length 19. See below.

1. *A-D-I-L* 19
2. *B-E-I-L* ~~20~~ 19 (E reduced by one hour)
3. *B-F-J-L* 17
4. *C-G-J-L* 13
5. *C-H-K-L* 19

Question 8

This question required students to use the Hungarian algorithm to find the allocation of five students to five jobs to ensure that the total time to complete the five jobs was a minimum. Correct performance of the Hungarian algorithm shows that there are two optimum allocations.

Allocation 1:

Child	**Alan**	**Brianna**	**Chamath**	**Deidre**	**Ewen**
Allocated job	2	5	1	3	4
Completion time	5 min	4 min	5 min	5 min	5 min

Allocation 2:

Child	**Alan**	**Brianna**	**Chamath**	**Deidre**	**Ewen**
Allocated job	2	3	1	5	4
Completion time	5 min	5 min	5 min	4 min	5 min

Thus the first child to finish their allocated job could be Brianna or Deidre (option B), depending on which allocation was used.

Module 3 – Geometry and measurement

Question	% A	% B	% C	% D	% E	% No Answer
1	87	4	3	1	3	0
2	1	3	2	92	1	0
3	9	10	6	4	69	1
4	36	13	17	7	26	1
5	70	10	13	4	2	1
6	2	7	11	28	51	1
7	4	10	65	13	7	1
8	15	30	14	30	11	1

The majority of questions in Module 3 were very well answered.

Students generally correctly answered questions from most curriculum areas, although Questions 4 and 8 revealed content area weaknesses worthy of further attention.

Question 4

Students were asked to work with the time of sunrise at two different locations in the same time zone, one in the east (Mallacoota 38° S, 150° E), the other in the west (Portland 38° S, 142° E).

The solution is as follows:

Time difference between sunrise in the two locations = (150 – 142)/15 × 60 = 32 minutes

The sun rises in Mallacoota at 6.03 am.

Thus, the time of sunrise in Portland is 6.03 + 0.32 = 6.35 am (option E). The time difference is added because, within a time zone, the sun rises first in the east.

Question 8

The solution to this question involved calculating an area scaling factor to take into account the different areas of the flags, and a linear scaling factor to take into account the different number of black and white flags in the string of flags. The correct answer was option D: $2^2 \times (4/3) = 16/3$.

Many students, in choosing option B, ignored the fact that there were different numbers of black and white flags in the string of flags.

Module 4 – Graphs and relations

Question	% A	% B	% C	% D	% E	% No Answer
1	1	94	4	1	0	0
2	94	2	1	2	1	0
3	4	3	13	77	2	1
4	4	11	14	61	10	1
5	7	21	29	3	39	1
6	20	38	16	9	17	1
7	14	7	5	41	32	1
8	7	26	49	10	7	1

The questions in Module 4 were generally less well answered than the questions in other modules. The questions not answered well by students were Questions 5–8.

Question 5

Students were required to apply the sliding-line technique to determine the maximum value of an objective function for a given feasible region. The correct application of the sliding-line technique shows that the maximum occurred at point *C* (option C). The majority of students either had no understanding of the sliding-line technique or applied it inaccurately to conclude that the maximum occurred at any point on line segment *BC* (option E).

Question 6

A systematic approach to answering this question is as follows:

Option A can immediately be rejected because the original graph is non-linear.

Option B, which represents the relationship $y = (6/1)x^2 = 6x^2$, cannot be correct because the point (2, 12) does not lie on the graph $y = 6x^2$ ($6 \times 2^2 = 24 \neq 12$).

Option C, which represents the relationship $y = (3/2)x^2 = 1.5x^2$, cannot be correct because the point (2, 12) does not lie on the graph $y = 1.5\ x^2$ ($1.5 \times 2^2 = 6 \neq 12$).

Option D, which represents the relationship $y = (6/1)x^3 = 6x^3$, is incorrect because the point (2, 12) does not lie on the graph $y = 6\ x^3$ ($6 \times 2^3 = 48 \neq 12$).

Option E, which represents the relationship $y = (3/2)x^3 = 1.5x^3$, is correct because the point (2, 12) lies on the graph $y = 1.5x^3$ ($1.5 \times 2^3 = 12$).

Question 7

Students were asked to identify an inequality that could be used to represent the following constraint from a linear programming problem.

Simon grows cucumbers and zucchinis.

Let *x* be the number of cucumbers that are grown.

Let *y* be the number of zucchinis that are grown.

For every two cucumbers that are grown, Simon grows at least three zucchinis.

This inequality translates into everyday language as: ‘For every cucumber grown, Simon grows at least one-and-a-half zucchinis.’

Thus the correct response was option E: $y \geq \frac{3x}{2}$.

Question 8

In this question, students were given a rule that specified a segmented graph that described Megan’s walk from her house to a shop.

The rule contained two unknowns, *k* and *a*.

$$\text{distance} = \begin{cases} 100t & 0 \leq t \leq 6 \\ 600 & 6 < t \leq a \\ kt & a < t \leq 10 \end{cases}$$

While the question asked for the value of *a*, this could not be found without first finding the value of *k*.

Calculating *k*: Given that Megan took 10 minutes to walk the 800 metres from her house to the shop, we can write $k \times 10 = 800$, so $k = 80$.

Calculating *a*: From the graph and using $k = 80$ we have $80a = 600$, so $a = 7.5$ minutes (option B).