VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

**Victorian Certificate of Education 2017**

# FURTHER MATHEMATICS

## Written examination 1

**Friday 2 June 2017**

**Reading time: 2.00 pm to 2.15 pm (15 minutes)**
**Writing time: 2.15 pm to 3.45 pm (1 hour 30 minutes)**

## MULTIPLE-CHOICE QUESTION BOOK

**Structure of book**

| *Section* | *Number of questions* | *Number of questions to be answered* | *Number of modules* | *Number of modules to be answered* | *Number of marks* |
|---|---|---|---|---|---|
| A – Core | 24 | 24 | | | 24 |
| B – Modules | 32 | 16 | 4 | 2 | 16 |
| | | | | | Total 40 |

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

**Materials supplied**
- Question book of 33 pages.
- Formula sheet.
- Answer sheet for multiple-choice questions.
- Working space is provided throughout the book.

**Instructions**
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

**At the end of the examination**
- You may keep this question book and the formula sheet.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

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## SECTION A – Core

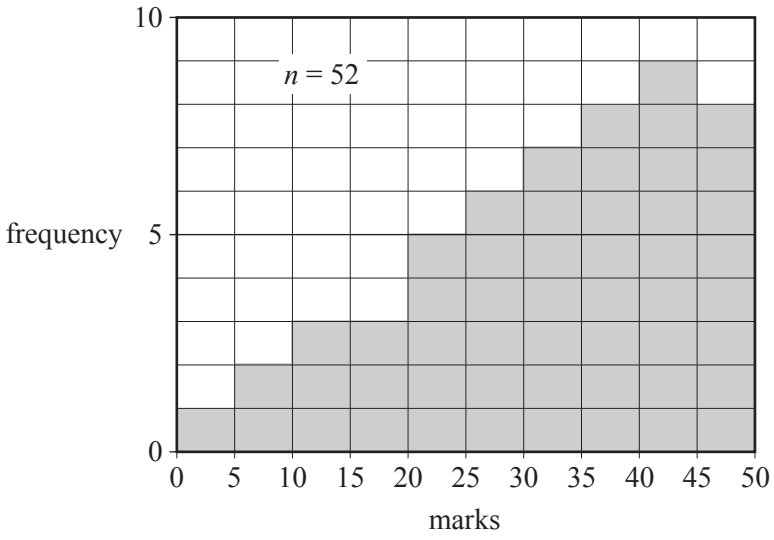
### Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1; an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

## Data analysis

*Use the following information to answer Questions 1–3.*

The histogram below shows the distribution of marks obtained by 52 students on a test.




**Question 1**

The shape of the distribution of marks is

**A.** symmetric.
**B.** bell-shaped.
**C.** positively skewed.
**D.** negatively skewed.
**E.** approximately normal.

**Question 2**

The pass mark for the test was set at 25.

The percentage of students who passed this test was closest to

**A.** 14%
**B.** 28%
**C.** 32%
**D.** 52%
**E.** 73%

**Question 3**

The median mark for the test was

**A.** greater than 20 but less than 25.

**B.** greater than 25 but less than 30.

**C.** greater than 30 but less than 35.

**D.** greater than 35 but less than 40.

**E.** greater than 40 but less than 45.

**Question 4**

The association between the *amount of solar energy* captured by a solar panel, in megajoules, and the *capital city* (Melbourne, Adelaide, etc.) in which it is captured will be investigated.

The variables *amount of solar energy* and *capital city* are

**A.** both numerical variables.

**B.** both categorical variables.

**C.** a nominal variable and a numerical variable respectively.

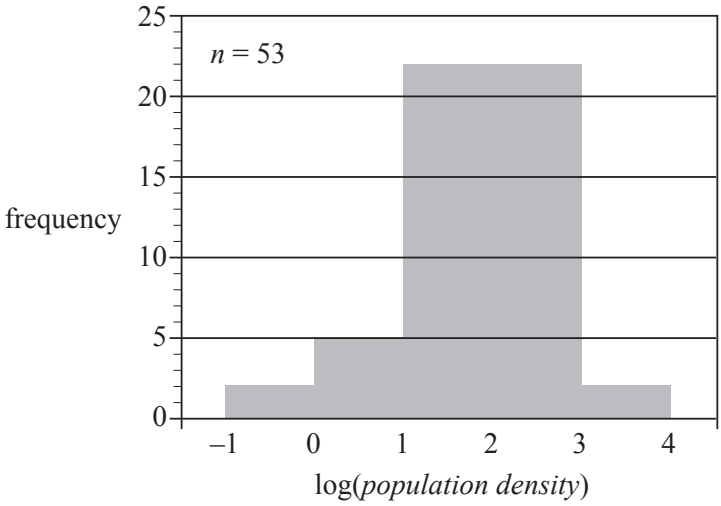
**D.** a numerical variable and a nominal variable respectively.

**E.** a numerical variable and an ordinal variable respectively.

**Question 5**

The histogram below displays the distribution of *population density*, in people per square kilometre, for 53 countries.

The horizontal scale of the histogram is log(*population density*).




Data: United Nations, Department of Economic and Social Affairs, Population Division (2015), 'World Population Prospects: The 2015 Revision' (DVD edition)

Based on the histogram, how many countries have a population density that is less than 10 people per square kilometre?

**A.** 2

**B.** 5

**C.** 7

**D.** 29

**E.** 46

**Question 6**

Which of the following could be used to identify and describe the association between the variables *height* (short, medium, tall) and *hat size* (small, medium, large)?

**A.** a histogram

**B.** a scatterplot

**C.** parallel boxplots

**D.** a segmented bar chart

**E.** a back-to-back stem plot

**Question 7**

A least squares line fitted to a scatterplot will always

**A.** maximise the number of data points lying on the line.

**B.** equalise the number of data points on either side of the line.

**C.** minimise the sum of the squares of the vertical distance from each data point to the line.

**D.** minimise the sum of the squares of the shortest distance from each data point to the line.

**E.** minimise the sum of the squares of the horizontal distance from each data point to the line.

*Use the following information to answer Questions 8 and 9.*

The beak length of small birds in a large population is approximately normally distributed with a mean of 9.5 mm and a standard deviation of 0.50 mm.

**Question 8**

Which one of the following statements relating to this population of birds is **not** true?

**A.** No bird will have a beak length that is less than 8.0 mm.

**B.** More than 99% of the birds will have a beak length that is less than 11 mm.

**C.** Approximately half of the birds will have a beak length that is less than 9.5 mm.

**D.** Approximately 2.5% of the birds will have a beak length that is greater than 10.5 mm.

**E.** Approximately 34% of the birds will have a beak length that is between 9.5 mm and 10.0 mm.

**Question 9**

A random sample of 250 of these birds is captured and the beak length of each bird is measured.

The expected number of these captured birds with beak lengths that are greater than 9 mm is closest to

**A.** 6

**B.** 13

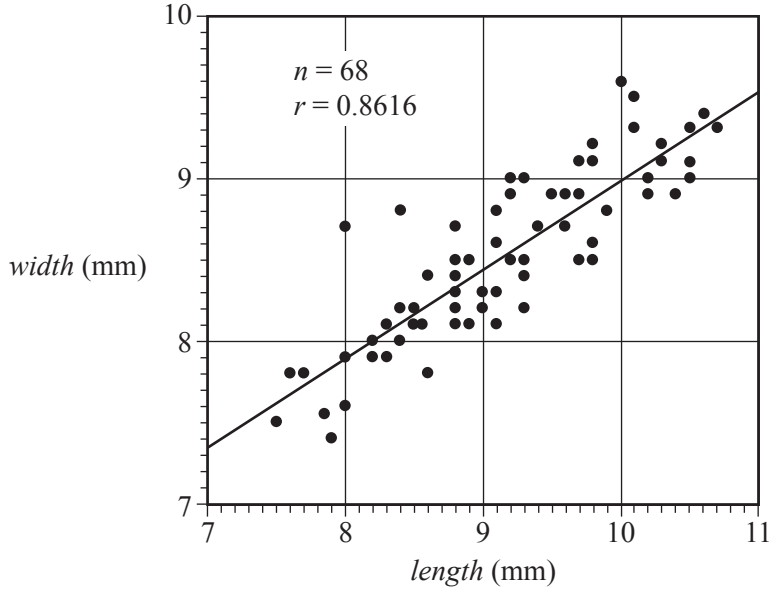
**C.** 170

**D.** 210

**E.** 244

*Use the following information to answer Questions 10 and 11.*

The scatterplot below displays the beak *length* and beak *width* of 68 birds of the same species. A least squares line has been fitted to the data.




Data: Howard Hughes Medical Institute

The correlation coefficient is $r = 0.8616$

The least squares line has been fitted to the scatterplot using beak *length* as the explanatory variable.

**Question 10**

The equation of this line is closest to

**A.** *width* = 0.56 + 3.5 × *length*

**B.** *width* = 3.5 + 0.56 × *length*

**C.** *width* = 7.4 + 0.56 × *length*

**D.** *length* = 7.4 + 0.56 × *width*

**E.** *length* = 3.5 + 0.56 × *width*

**Question 11**

Which one of the following statements is **not** true?

**A.** The slope of the least squares line is positive.

**B.** Birds with longer beaks tend to have wider beaks.

**C.** There is a strong positive linear association between beak *width* and beak *length* for these birds.

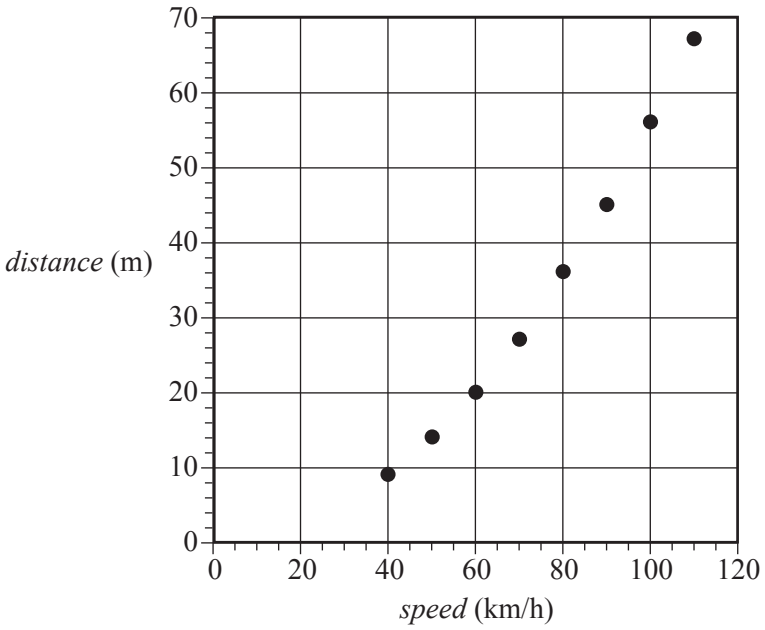
**D.** Approximately 74% of the variation in beak *width* is explained by the variation in beak *length*.

**E.** Using the least squares line to predict the beak *width* of a bird with a beak *length* of 8.1 mm would be an example of extrapolation.

**Question 12**

The table below shows the *speed*, in kilometres per hour, and the braking *distance*, in metres, of a car travelling at eight different speeds. A scatterplot has been constructed from this data.

| ***Speed*** **(km/h)** | ***Distance*** **(m)** |
|---|---|
| 40 | 9 |
| 50 | 14 |
| 60 | 20 |
| 70 | 27 |
| 80 | 36 |
| 90 | 45 |
| 100 | 56 |
| 110 | 67 |






The scatterplot shows that the association between *distance* and *speed* is non-linear.

A squared transformation is applied to the variable *speed* to linearise the data.

A least squares line is then fitted to the transformed data with *distance* as the response variable.

The equation of this least squares line is closest to

**A.** $distance = -15.6 + 180 \times speed^2$

**B.** $distance = 0.0056 + 0.092 \times speed^2$

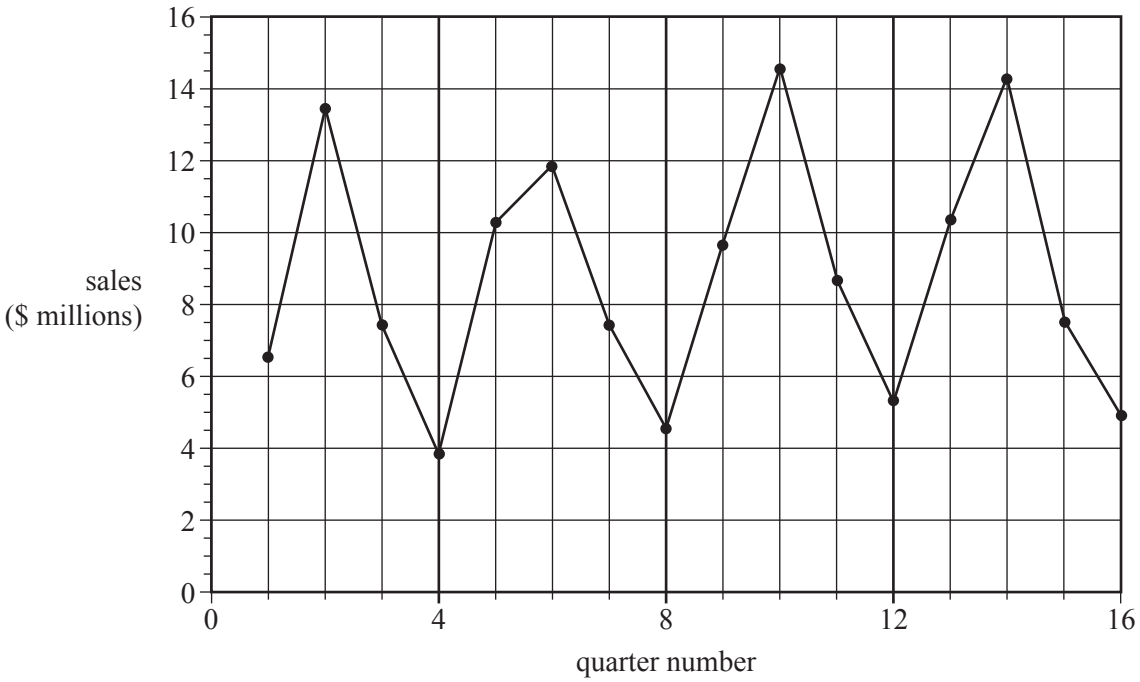
**C.** $distance = 0.092 + 0.0056 \times speed^2$

**D.** $speed^2 = 180 - 15.6 \times distance$

**E.** $speed^2 = 0.0056 + 0.092 \times distance^2$

*Use the following information to answer Questions 13 and 14.*

The time series plot below charts the quarterly sales figures, in millions of dollars, of a small manufacturing business over a period of four years.




**Question 13**

The time series plot is best described as having

**A.** no variability.

**B.** seasonality only.

**C.** irregular variation only.

**D.** a decreasing trend with seasonality.

**E.** an increasing trend with seasonality.

**Question 14**

The sales figures used to generate this time series plot are displayed in the table below.

| Year | Quarter 1 | Quarter 2 | Quarter 3 | Quarter 4 |
|---|---|---|---|---|
| 2013 | 6.5 | 13.4 | 7.4 | 3.8 |
| 2014 | 10.2 | 11.8 | 7.4 | 4.5 |
| 2015 | 9.6 | 14.5 | 8.6 | 5.3 |
| 2016 | 10.3 | 14.2 | 7.5 | 4.9 |

The four-mean smoothed sales with centring for Quarter 3 in 2015, in millions of dollars, was closest to

**A.** 8.6

**B.** 9.3

**C.** 9.5

**D.** 9.6

**E.** 9.7

*Use the following information to answer Questions 15 and 16.*

The seasonal indices (SI) for the daily earnings of a cafe in a tourist town, from Monday to Saturday, are shown in the table below. The seasonal index for Sunday is not shown.

| **Day** | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday | Sunday |
|---|---|---|---|---|---|---|---|
| **SI** | 0.65 | 0.60 | 0.74 | 0.82 | 1.12 | 1.45 | |

**Question 15**

Last Sunday, the cafe earned $3839.

The deseasonalised earnings for this day were closest to

**A.** $2370

**B.** $2500

**C.** $2650

**D.** $5570

**E.** $6220

**Question 16**

The seasonal index for Wednesday is 0.74

This tells us that Wednesday earnings tend to be

**A.** 26% less than the average daily earnings.

**B.** 26% more than the average daily earnings.

**C.** 35% less than the average daily earnings.

**D.** 35% more than the average daily earnings.

**E.** 74% less than the average daily earnings.

## Recursion and financial modelling

**Question 17**

A sequence is generated by the recurrence relation below.

$$A_0 = 2, \qquad A_{n+1} = 3A_n - 3$$

The sequence is

**A.** 2, 1, 0, –3 …

**B.** 2, 3, 0, –3 …

**C.** 2, 3, 2, 3 …

**D.** 2, 3, 3, 3 …

**E.** 2, 3, 6, 15 …

**Question 18**

Andre deposited $20 000 into a savings account earning compound interest at the rate of 3.1% per annum, compounding annually.

Which one of the following recurrence relations can be used to determine the amount in the savings account, $S_n$, after $n$ years?

**A.** $S_0 = 20\,000, \qquad S_{n+1} = S_n + 620$

**B.** $S_0 = 20\,000, \qquad S_{n+1} = 1.031 \times S_n$

**C.** $S_0 = 20\,000, \qquad S_{n+1} = 620 \times S_n$

**D.** $S_0 = 20\,000, \qquad S_{n+1} = 3.1 \times S_n + 620$

**E.** $S_0 = 20\,000, \qquad S_{n+1} = S_n + 3.1 \times 620$

**Question 19**

Consider the recurrence relation below.

$$L_0 = 2000, \qquad L_{n+1} = L_n + 80$$

This recurrence relation could be used to model a

**A.** simple interest investment of $2000 with an annual interest rate of 4%.

**B.** simple interest investment of $2000 with an annual interest rate of 8%.

**C.** simple interest investment of $2000 with an annual interest rate of 40%.

**D.** compound interest investment of $2000 with an annual interest rate of 4%.

**E.** compound interest investment of $2000 with an annual interest rate of 8%.

**Question 20**

A music school has $80 000 to invest in a perpetuity.

The interest earned from this perpetuity will provide an annual prize of $3000 to a talented musician from the school.

What annual interest rate would be required for this investment?

**A.** 0.3125%

**B.** 3.75%

**C.** 3.90%

**D.** 41.92%

**E.** 45.00%

**Question 21**

The amortisation table below shows the repayment, interest, principal reduction and balance of a reducing balance loan after the first repayment.

| Repayment number | Repayment | Interest | Principal reduction | Balance of loan |
|---|---|---|---|---|
| 0 | 0.00 | 0.00 | 0.00 | 180 000.00 |
| 1 | 850.00 | 720.00 | 130.00 | 179 870.00 |
| 2 | 850.00 | | | |

What amount of interest is paid with Repayment number 2?

**A.** $608.56
**B.** $609.44
**C.** $717.12
**D.** $719.48
**E.** $720.00

**Question 22**

Vusa has invested $420 000 in an annuity that pays interest at the rate of 3.6% per annum, compounding monthly.

After the interest has been added each month, Vusa immediately receives a payment from the annuity.

The value of Vusa's investment is $372 934.71 after three years.

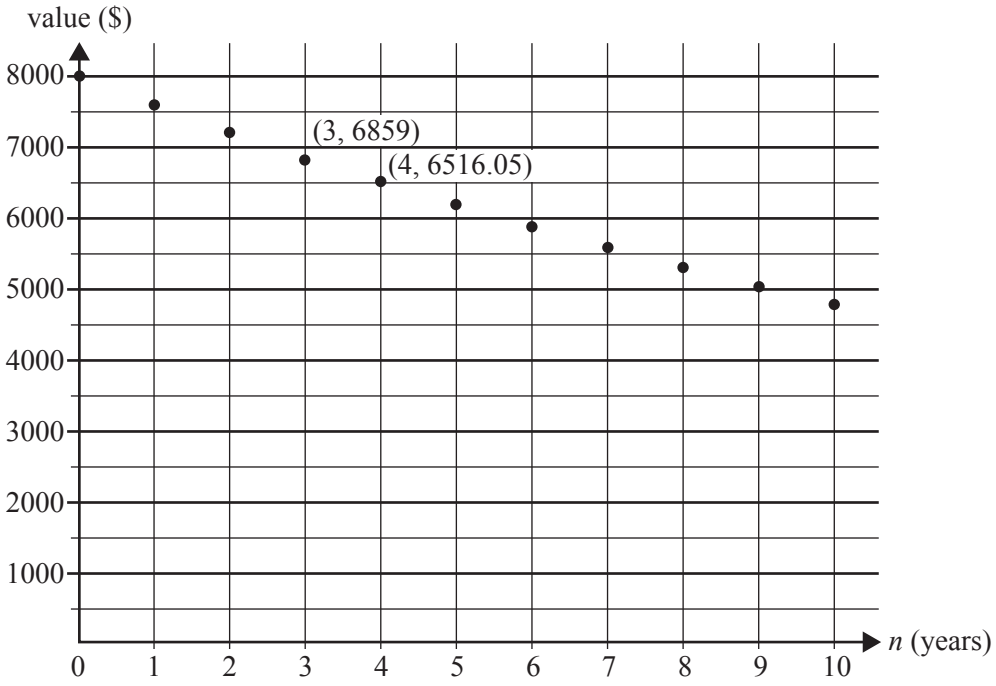
The monthly payment that Vusa receives from the annuity is closest to

**A.** $1260
**B.** $1310
**C.** $2500
**D.** $15 120
**E.** $16 900

**Question 23**

The value of a piano is depreciated using the reducing balance method.

The graph below shows the value of the piano as it depreciates over a period of 10 years.




Let $P_n$ be the value of the piano after $n$ years.

A recurrence relation that could be used to determine $P_n$ is

**A.** $P_0 = 8000, \quad P_{n+1} = 0.95 \times P_n$

**B.** $P_0 = 8000, \quad P_{n+1} = 342.95 \times P_n$

**C.** $P_0 = 8000, \quad P_{n+1} = 1.05 \times P_n - 3$

**D.** $P_0 = 8000, \quad P_{n+1} = 0.95 \times P_n - 342.95$

**E.** $P_0 = 8000, \quad P_{n+1} = P_n - 342.95$

**Question 24**

Geoff has a compound interest investment that earns interest compounding monthly.

The balance of Geoff's compound interest investment was $4418.80 after six months.

The balance of Geoff's compound interest investment was $4862.80 after two years.

The amount of money that Geoff initially invested is closest to

**A.** $4000

**B.** $4015

**C.** $4280

**D.** $4370

**E.** $4715

**END OF SECTION A**
**TURN OVER**

## SECTION B – Modules

### Instructions for Section B

Select **two** modules and answer **all** questions within the selected modules in pencil on the answer sheet provided for multiple-choice questions.

Show the modules you are answering by shading the matching boxes on your multiple-choice answer sheet **and** writing the name of the module in the box provided.

Choose the response that is **correct** for the question.

A correct answer scores 1; an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

**Contents** **Page**

## Module 1 – Matrices

Before answering these questions, you must **shade** the 'Matrices' box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

**Question 1**

$\begin{bmatrix} 4 & 2 \\ 1 & 5 \end{bmatrix} + 3\begin{bmatrix} 4 & 2 \\ 1 & 5 \end{bmatrix}$ is equal to

**A.** $\begin{bmatrix} 8 & 4 \\ 2 & 10 \end{bmatrix}$

**B.** $\begin{bmatrix} 11 & 1 \\ 5 & 13 \end{bmatrix}$

**C.** $\begin{bmatrix} 16 & 8 \\ 4 & 20 \end{bmatrix}$

**D.** $\begin{bmatrix} 24 & 12 \\ 6 & 30 \end{bmatrix}$

**E.** $\begin{bmatrix} 38 & 34 \\ 31 & 40 \end{bmatrix}$

**Question 2**

The cost of fruit at a stall, in dollars per kilogram, is shown in the table below.

| | |
|---|---|
| **Apples** | \$2.50 |
| **Pears** | \$3.20 |
| **Bananas** | \$1.90 |

Sean wants to buy 2 kg of apples, 1 kg of pears and 3 kg of bananas.

Which one of the following matrix products will result in a matrix that contains the total cost of Sean's fruit purchase, in dollars?

**A.** $[2 \quad 1 \quad 3]\begin{bmatrix} 2.50 \\ 3.20 \\ 1.90 \end{bmatrix}$

**B.** $[2 \quad 1 \quad 3][2.50 \quad 3.20 \quad 1.90]$

**C.** $\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}\begin{bmatrix} 2.50 \\ 3.20 \\ 1.90 \end{bmatrix}$

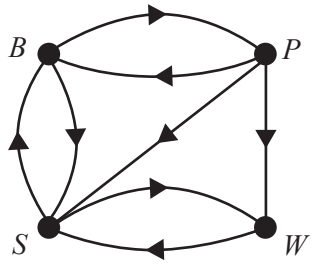
**D.** $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} 2.50 \\ 3.20 \\ 1.90 \end{bmatrix}$

**E.** $\begin{bmatrix} 2.50 \\ 3.20 \\ 1.90 \end{bmatrix}[2 \quad 1 \quad 3]$

**Question 3**

Peter ($P$), Sally ($S$) and Ben ($B$) are managers in a business.

The diagram below shows the direct communication that is possible between these managers and a worker, Whitney ($W$).




For example, the arrow from $S$ to $W$ indicates that Sally is able to communicate directly with Whitney.

A communication matrix can represent the direct communication that is possible between the managers and Whitney. The elements in the matrix are such that:

- '1' indicates that direct communication from one person to another is possible
- '0' indicates that direct communication is not possible.

This communication matrix could be

**A.**
$$\begin{array}{c} \\ from \end{array}\begin{array}{c} \begin{array}{cccc} to & & & \\ P & S & B & W \end{array} \\ \begin{array}{c} P \\ S \\ B \\ W \end{array}\begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

**B.**
$$\begin{array}{c} \\ from \end{array}\begin{array}{c} \begin{array}{cccc} to & & & \\ P & S & B & W \end{array} \\ \begin{array}{c} P \\ S \\ B \\ W \end{array}\begin{bmatrix} 0 & 1 & 2 & 1 \\ 1 & 0 & 2 & 2 \\ 2 & 2 & 0 & 1 \\ 1 & 2 & 1 & 0 \end{bmatrix} \end{array}$$

**C.**
$$\begin{array}{c} \\ from \end{array}\begin{array}{c} \begin{array}{cccc} to & & & \\ P & S & B & W \end{array} \\ \begin{array}{c} P \\ S \\ B \\ W \end{array}\begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

**D.**
$$\begin{array}{c} \\ from \end{array}\begin{array}{c} \begin{array}{cccc} to & & & \\ P & S & B & W \end{array} \\ \begin{array}{c} P \\ S \\ B \\ W \end{array}\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

**E.**
$$\begin{array}{c} \\ from \end{array}\begin{array}{c} \begin{array}{cccc} to & & & \\ P & S & B & W \end{array} \\ \begin{array}{c} P \\ S \\ B \\ W \end{array}\begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

**Question 4**

The element in row $i$ and column $j$ of matrix $M$ is $m_{ij}$.

The elements in matrix $M$ are determined using the rule $m_{ij} = 2i + j$.

Matrix $M$ could **not** be

**A.** [3]

**B.** $\begin{bmatrix} 3 & 4 & 5 \end{bmatrix}$

**C.** $\begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$

**D.** $\begin{bmatrix} 3 \\ 4 \\ 5 \\ 6 \end{bmatrix}$

**E.** $\begin{bmatrix} 3 & 4 & 5 \\ 5 & 6 & 7 \\ 7 & 8 & 9 \end{bmatrix}$

**Question 5**

Yvette needs to buy a total of nine pens and markers.

Five pens and four markers will cost \$31.00

Four pens and five markers will cost \$32.00

Let $p$ be the cost of a pen.

Let $m$ be the cost of a marker.

Consider the following matrix equations.

$$\begin{bmatrix} p \\ m \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}^{-1} \begin{bmatrix} 31 \\ 32 \end{bmatrix} \qquad \begin{bmatrix} p \\ m \end{bmatrix} = \begin{bmatrix} 32 \\ 31 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}^{-1}$$

$$\begin{bmatrix} p \\ m \end{bmatrix} = -\frac{1}{9} \begin{bmatrix} 4 & -5 \\ -5 & 4 \end{bmatrix} \begin{bmatrix} 32 \\ 31 \end{bmatrix} \qquad \begin{bmatrix} p \\ m \end{bmatrix} = \begin{bmatrix} \frac{5}{9} & -\frac{4}{9} \\ -\frac{4}{9} & \frac{5}{9} \end{bmatrix} \begin{bmatrix} 32 \\ 31 \end{bmatrix}$$

How many of the matrix equations above could Yvette solve to get a matrix that contains the price of a pen and the price of a marker?

**A.** 0

**B.** 1

**C.** 2

**D.** 3

**E.** 4

**Question 6**

Students at a school choose an afternoon activity every week.

They can choose either sport ($S$), art ($A$) or music ($M$).

The table below shows the number of students who chose sport, art and music in Week 1 of the school term.

| **Sport (*S*)** | **Art (*A*)** | **Music (*M*)** |
|---|---|---|
| 150 | 85 | 35 |

The students are expected to change the activity they choose from week to week as shown in the transition matrix $P$ below.

$$P = \begin{array}{c} \\ S \\ A \\ M \end{array} \begin{array}{c} \begin{array}{ccc} \textit{this week} & & \\ S & A & M \end{array} \\ \begin{bmatrix} 0.80 & 0.20 & 0.05 \\ 0.10 & 0.70 & 0.15 \\ 0.10 & 0.10 & 0.80 \end{bmatrix} \end{array} \begin{array}{l} S \\ A \\ M \end{array} \quad \textit{next week}$$

Which one of the following statements is **true** for this situation?

**A.** 30% of the students will never choose art.

**B.** The number of students who choose music will decrease every week.

**C.** The number of students who choose sport in one week will always be 20% less than the number of students who chose sport in the previous week.

**D.** In Week 3 of the school term, the number of students who choose music will be less than half the number of students who choose art.

**E.** In the long term, the number of students who choose music will be more than the number of students who choose art.

**Question 7**

A badminton competition is held between four players, Amanda ($A$), Ben ($B$), Carlos ($C$) and Darius ($D$).

In the competition, each player competes in one game with each of the other three players.

The matrix $S^2$ below shows the two-step dominance that each player has over the other players.

$$S^2 = \textit{winner} \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} \textit{loser} & & & \\ A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

From the matrix above, which one of the following events must have occurred during the competition?

**A.** Ben beat Amanda.

**B.** Ben beat Carlos.

**C.** Carlos beat Amanda.

**D.** Darius beat Amanda.

**E.** Darius beat Carlos.

**Question 8**

Matrix $A$ is an $n \times n$ matrix where $n > 1$.

Matrix $R$ is a row matrix.

Matrix $C$ is a column matrix.

Which one of the matrix products below could result in a $1 \times 1$ matrix?

**A.** $ACR$

**B.** $ARC$

**C.** $CAR$

**D.** $RAC$

**E.** $RCA$

## Module 2 – Networks and decision mathematics

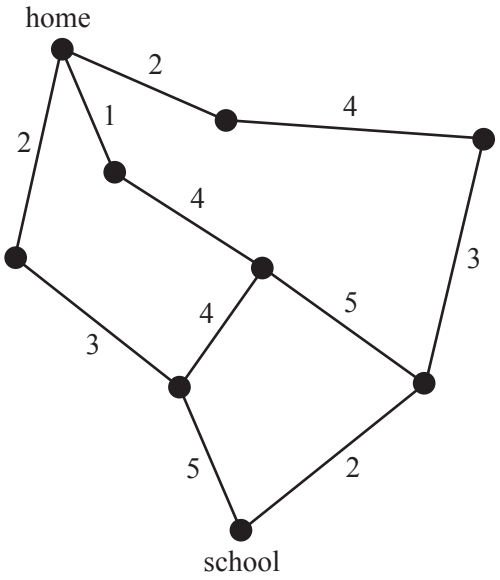
Before answering these questions, you must **shade** the 'Networks and decision mathematics' box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

**Question 1**

Hunter rides his bike to school each day.

The edges of the network below represent the roads that Hunter can use to ride to school.

The numbers on the edges give the distance, in kilometres, along each road.




What is the shortest distance that Hunter can ride between home and school?

**A.** 10 km

**B.** 11 km

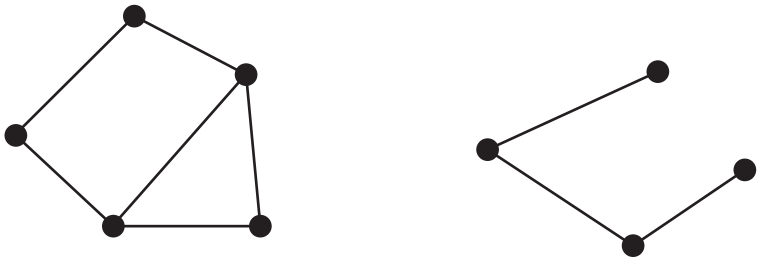
**C.** 12 km

**D.** 14 km

**E.** 23 km

**Question 2**

Consider the two graphs below.



Which one of the following statements is **not** true?

**A.** Each graph is connected.

**B.** Each graph contains an Eulerian trail.

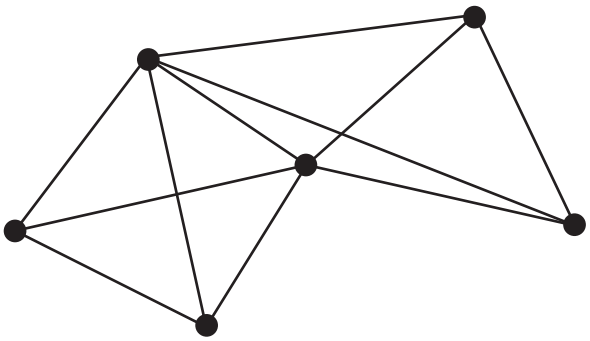
**C.** Each graph contains a Hamiltonian cycle.

**D.** Each graph has at least one vertex of degree two.

**E.** The sum of the degrees of the vertices for each graph is even.

**Question 3**

The graph shown below is planar.



How many faces does this graph have?

**A.** 5

**B.** 6

**C.** 7

**D.** 8

**E.** 9

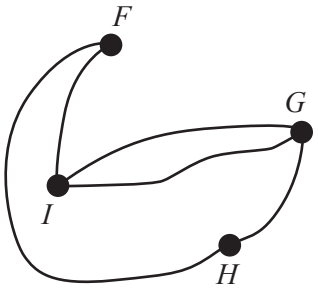
**Question 4**

The matrix below shows the number of road connections between towns $F$, $G$, $H$ and $I$.

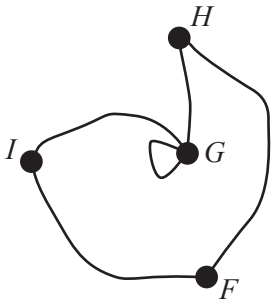
$$\begin{array}{c} \\ F \\ G \\ H \\ I \end{array} \begin{array}{c} \begin{array}{cccc} F & G & H & I \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \\ 1 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 \end{bmatrix} \end{array}$$

Which one of the following graphs shows all of these road connections?

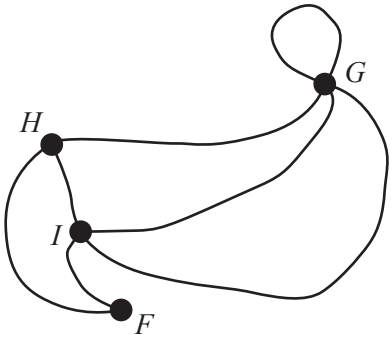
**A.**



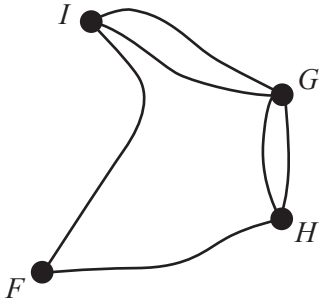

**B.**



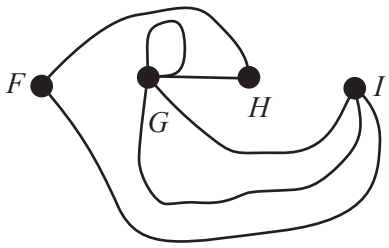

**C.**




**D.**



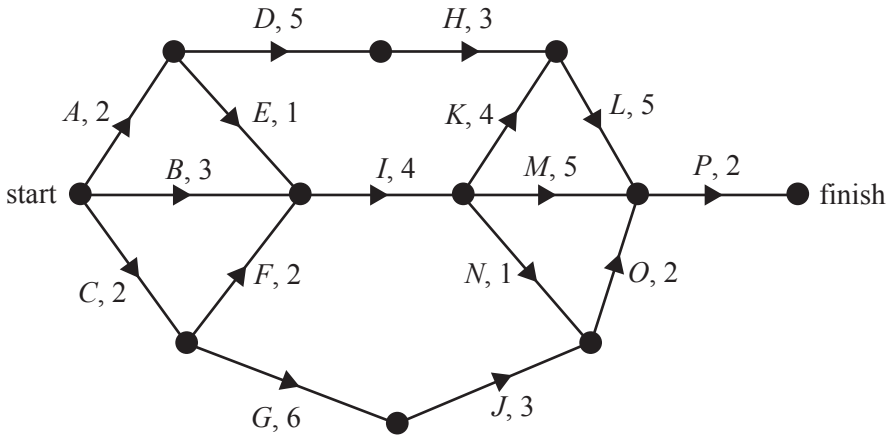

**E.**



*Use the following information to answer Questions 5 and 6.*

The activity network below shows the sequence of activities required to complete a project.

The number next to each activity in the network is the time it takes to complete that activity, in days.




**Question 5**

Beginning with Activity *C*, the number of paths from start to finish is

**A.** 1

**B.** 2

**C.** 3

**D.** 4

**E.** 5

**Question 6**

What is the latest starting time for Activity *I*, in days, so that the project is completed in the shortest time possible?

**A.** 3

**B.** 4

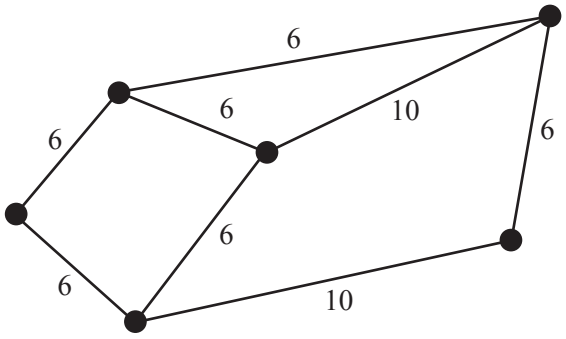
**C.** 5

**D.** 6

**E.** 7

**Question 7**

Consider the weighted graph shown below.




How many different minimal spanning trees are possible?

**A.** 2
**B.** 3
**C.** 4
**D.** 5
**E.** 6

**Question 8**

Abbey, Barb, Cathal and Dinh are four workers at a business.

Each worker will perform one duty.

The time for each worker to complete duties 1, 2, 3 and 4, in minutes, is shown in the table below.

| | **Duty 1** | **Duty 2** | **Duty 3** | **Duty 4** |
|---|---|---|---|---|
| **Abbey** | 6 | 5 | 6 | 8 |
| **Barb** | 9 | 12 | 10 | 6 |
| **Cathal** | 8 | 7 | 4 | 8 |
| **Dinh** | 5 | 3 | 6 | 4 |

The minimum total time for all duties is 19 minutes, with Dinh performing Duty 2.

Before the duties are performed, it is found that Dinh will require 7 minutes for Duty 2 rather than 3 minutes.

If the duties are allocated again, the minimum total time for all duties will

**A.** remain the same.
**B.** increase by 1 minute.
**C.** increase by 2 minutes.
**D.** increase by 3 minutes.
**E.** increase by 4 minutes.

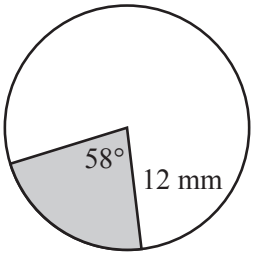
## Module 3 – Geometry and measurement

Before answering these questions, you must **shade** the 'Geometry and measurement' box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

**Question 1**

A sector is cut from a circle of radius 12 mm.

This sector is shown shaded in the diagram below.




The angle at the point of the sector is 58°.

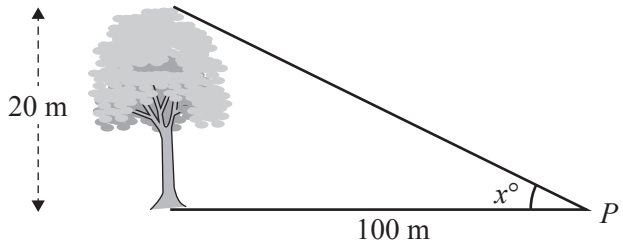
The area of the sector, in square millimetres, is closest to

**A.** 1
**B.** 6
**C.** 46
**D.** 73
**E.** 146

**Question 2**

A vertical tree of height 20 m stands on horizontal ground.

The tree is 100 m away from point $P$, as shown in the diagram below.




The value of the angle $x$, shown in the diagram above, is closest to

**A.** 3°
**B.** 11°
**C.** 12°
**D.** 78°
**E.** 79°

**Question 3**

Which of the following pairs of cities has the largest time difference between them?

**A.** Belfast (55° N, 6° W) and Johannesburg (26° S, 28° E)

**B.** Havana (23° N, 82° W) and Lagos (7° N, 3° E)

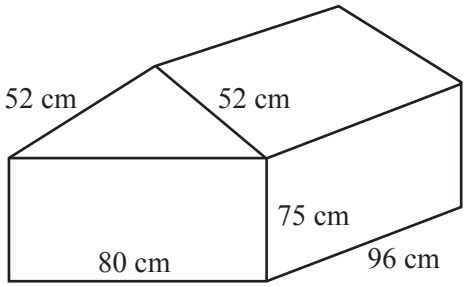
**C.** Helsinki (60° N, 25° E) and Manila (15° N, 121° E)

**D.** Malang (8° S, 113° E) and Rockhampton (23° S, 151° E)

**E.** Moscow (56° N, 38° E) and Atlanta (34° N, 84° W)

**Question 4**

Paula has built a model house using a triangular prism on top of a rectangular box. The dimensions of the model house are shown on the diagram below.




Paula will paint the outside walls and the roof of the model house.

The area that will be painted, in square centimetres, is closest to

**A.** 12 600

**B.** 26 400

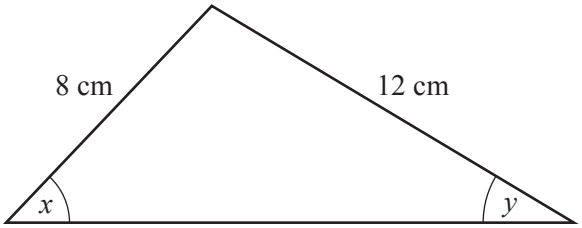
**C.** 36 400

**D.** 37 700

**E.** 39 000

**Question 5**

A triangle has one side of length 8 cm and another side of length 12 cm, as shown in the diagram below.
The angles $x$ and $y$ are also shown.




The angle $x$ is such that $\sin(x) = \frac{2}{5}$.
What is the value of $\sin(y)$?

**A.** $\frac{3}{5}$

**B.** $\frac{4}{5}$

**C.** $\frac{5}{3}$

**D.** $\frac{4}{15}$

**E.** $\frac{7}{15}$

**Question 6**

City A has latitude 25° N and longitude 50° E.
City B has latitude 35° S and longitude 50° E.
Assume that the radius of Earth is 6400 km.
The shortest distance along the meridian between A and B, in kilometres, is closest to

**A.** 1117
**B.** 2793
**C.** 3910
**D.** 6400
**E.** 6702

**Question 7**

The latitude and longitude of two cities, A and B, are shown in the table below.

| City | Latitude | Longitude |
|---|---|---|
| A | 50° N | 85° E |
| B | 50° N | 112° W |

Assume that the radius of Earth is 6400 km.

The shortest distance along the parallel between A and B, in kilometres, can be found from which one of the following calculations?

**A.** $\frac{163}{360} \times 2 \times \pi \times 6400 \times \sin 40°$

**B.** $\frac{197}{360} \times 2 \times \pi \times 6400 \times \sin 40°$

**C.** $\frac{197}{360} \times \pi \times 6400 \times \sin 40°$

**D.** $\frac{197}{360} \times 2 \times \pi \times 6400 \times \sin 50°$

**E.** $\frac{163}{360} \times 2 \times \pi \times 6400 \times \sin 50°$

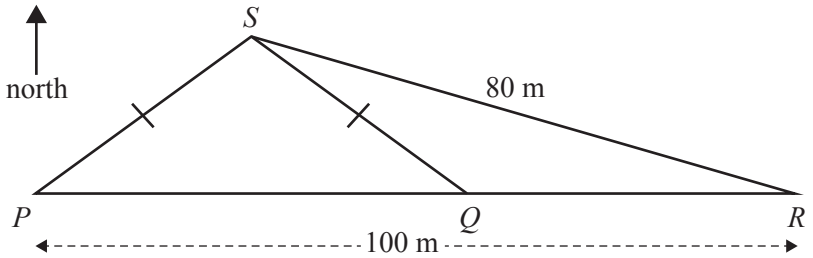
**Question 8**

A triangle has vertices *P*, *S* and *R*.

Point *R* lies due east of point *P*.

Point *Q* lies on side *PR*.

The length of side *SP* is the same as the length of side *SQ*, as shown in the diagram below.




The length of side *SR* is 80 m.

The length of side *PR* is 100 m.

The bearing of *Q* from *S* is 120°.

The distance between *Q* and *R*, in metres, is closest to

**A.** 26

**B.** 43

**C.** 58

**D.** 74

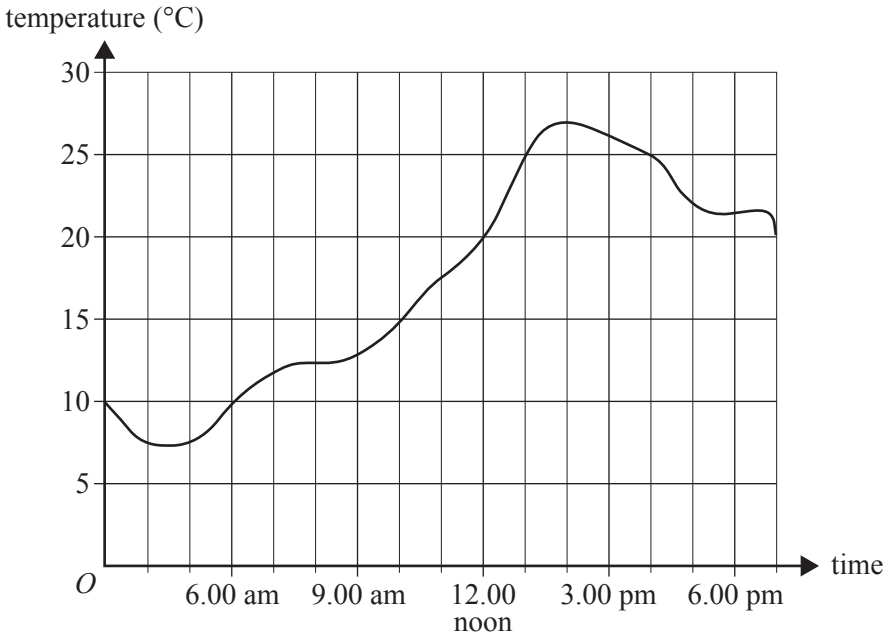
**E.** 100

## Module 4 – Graphs and relations

Before answering these questions, you must **shade** the 'Graphs and relations' box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

**Question 1**

The graph below shows the temperature, in degrees Celsius, over the course of a day.




The time that the temperature first reached 25 °C was closest to

**A.** 12.00 noon

**B.** 12.30 pm

**C.** 1.00 pm

**D.** 1.30 pm

**E.** 2.00 pm

**Question 2**

Players at a football club pay a fee of $130 each year.

They also pay a fee of $12 for every game they play in that year.

Last year, Jenny paid a total of $262 in fees at this football club.

How many games did Jenny play last year?

**A.** 10

**B.** 11

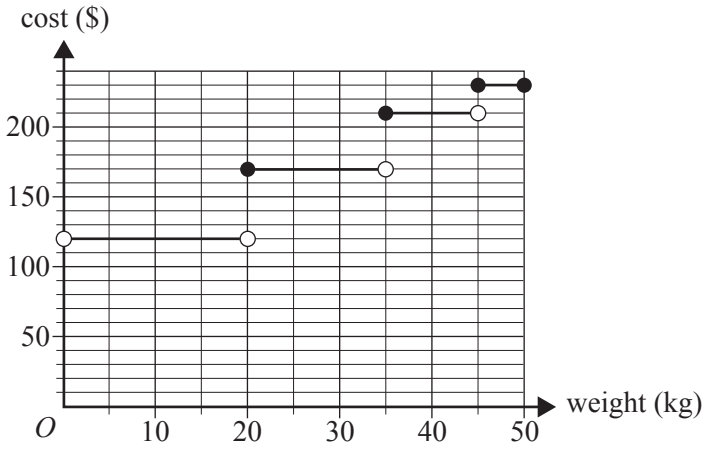
**C.** 12

**D.** 13

**E.** 14

**Question 3**

The graph below shows the cost of sending a package by priority airmail, in dollars, according to the weight of the package.




Manu ships three separate packages:

- one 15 kg package
- one 35 kg package
- one 48 kg package

What is the total cost of sending all three packages?

**A.** $120

**B.** $230

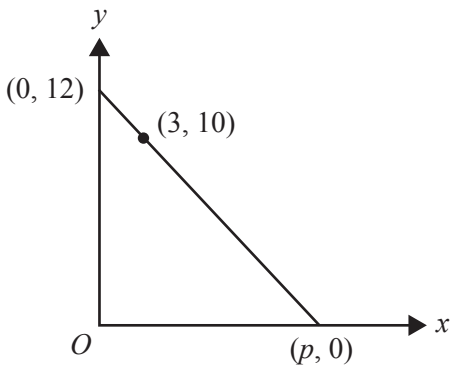
**C.** $360

**D.** $520

**E.** $560

**Question 4**

The graph below shows a straight line that touches the $x$-axis at the point $(p, 0)$.




What is the value of $p$?

**A.** 12

**B.** 14

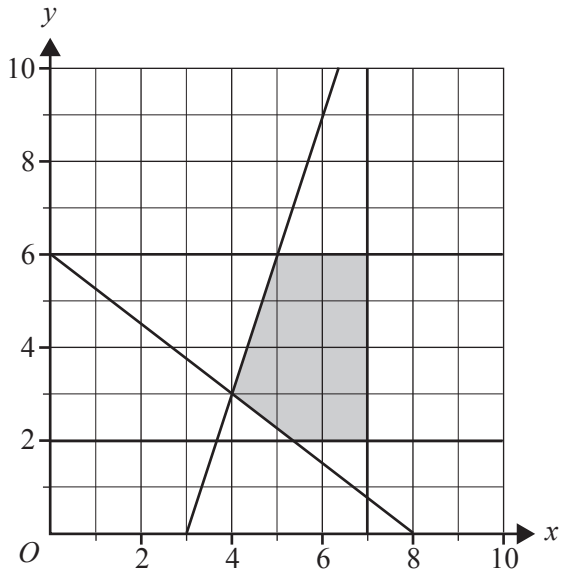
**C.** 16

**D.** 18

**E.** 22

**Question 5**

The shaded region on the graph below shows the feasible region defined by five inequalities.




One of the inequalities used to define this feasible region could be

**A.** $4x + 3y \leq 24$

**B.** $4x + 3y \geq 24$

**C.** $3x - y \leq 9$

**D.** $3x - y \geq 9$

**E.** $2 \leq x \leq 6$

**Question 6**

A bookstore is having a sale.

Each paperback book costs $8 and each hardback book costs $14.

Jane bought 12 books for a total of $126.

The number of hardback books that Jane bought is

**A.** 5

**B.** 7

**C.** 8

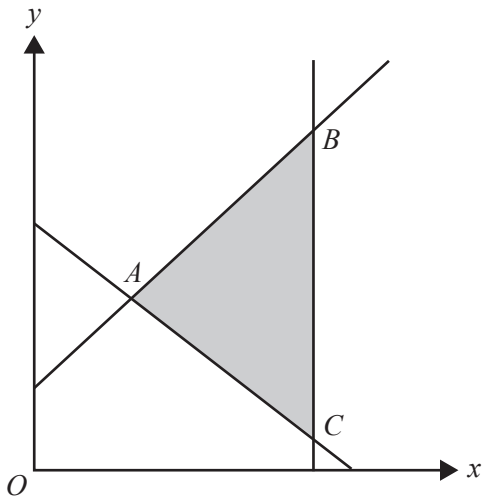
**D.** 12

**E.** 14

**Question 7**

The shaded area in the graph below shows the feasible region for a linear programming problem defined by the following inequalities.

Inequality 1 $y \leq 0.8x + 2$
Inequality 2 $y \geq -0.7x + 6$
Inequality 3 $x \leq 7$




$A$, $B$ and $C$ are the corner points of this region.

All of the points along the line $AB$ give the maximum value of an objective function $Z$.

This objective function could have the equation

**A.** $Z = -56x + 80y$

**B.** $Z = -52x + 65y$

**C.** $Z = 52x + 65y$

**D.** $Z = 56x - 80y$

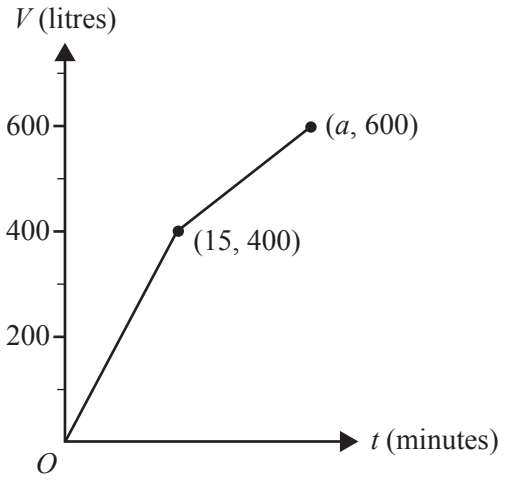
**E.** $Z = 56x + 80y$

**Question 8**

A tank is being filled with water by two hoses.

After 15 minutes, one of the hoses is removed.

The graph below shows the volume of water, $V$ litres, in the tank at time $t$ minutes.




The tank has a capacity of 600 litres and is full after $a$ minutes.

If both hoses had continued to fill the tank, it would have reached its capacity nine minutes earlier than this.

The rate at which the tank is filling, in litres per minute, over the time interval from 15 minutes to $a$ minutes, is closest to

**A.** 12

**B.** 13

**C.** 22

**D.** 23

**E.** 27

**END OF MULTIPLE-CHOICE QUESTION BOOK**

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

**Victorian Certificate of Education**
**2017**

# FURTHER MATHEMATICS

## Written examination 1

### FORMULA SHEET

**Instructions**

This formula sheet is provided for your reference.
A multiple-choice question book is provided with this formula sheet.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

# Further Mathematics formulas

## Core – Data analysis

| | |
|---|---|
| standardised score | $z = \frac{x - \bar{x}}{s_x}$ |
| lower and upper fence in a boxplot | lower $Q_1 - 1.5 \times IQR$ upper $Q_3 + 1.5 \times IQR$ |
| least squares line of best fit | $y = a + bx$, where $b = r\frac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$ |
| residual value | residual value = actual value – predicted value |
| seasonal index | seasonal index = $\frac{\text{actual figure}}{\text{deseasonalised figure}}$ |

## Core – Recursion and financial modelling

| | |
|---|---|
| first-order linear recurrence relation | $u_0 = a$, $u_{n+1} = bu_n + c$ |
| effective rate of interest for a compound interest loan or investment | $r_{effective} = \left[\left(1 + \frac{r}{100n}\right)^n - 1\right] \times 100\%$ |

## Module 1 – Matrices

| | |
|---|---|
| determinant of a 2 × 2 matrix | $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$ |
| inverse of a 2 × 2 matrix | $A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, where $\det A \neq 0$ |
| recurrence relation | $S_0$ = initial state, $S_{n+1} = TS_n + B$ |

## Module 2 – Networks and decision mathematics

| | |
|---|---|
| Euler's formula | $v + f = e + 2$ |

## Module 3 – Geometry and measurement

| | |
|---|---|
| area of a triangle | $A = \frac{1}{2}bc\sin(\theta^\circ)$ |
| Heron's formula | $A = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{1}{2}(a+b+c)$ |
| sine rule | $\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$ |
| cosine rule | $a^2 = b^2 + c^2 - 2bc\cos(A)$ |
| circumference of a circle | $2\pi r$ |
| length of an arc | $r \times \frac{\pi}{180} \times \theta^\circ$ |
| area of a circle | $\pi r^2$ |
| area of a sector | $\pi r^2 \times \frac{\theta^\circ}{360}$ |
| volume of a sphere | $\frac{4}{3}\pi r^3$ |
| surface area of a sphere | $4\pi r^2$ |
| volume of a cone | $\frac{1}{3}\pi r^2 h$ |
| volume of a prism | area of base × height |
| volume of a pyramid | $\frac{1}{3}$ × area of base × height |

## Module 4 – Graphs and relations

| | |
|---|---|
| gradient (slope) of a straight line | $m = \frac{y_2 - y_1}{x_2 - x_1}$ |
| equation of a straight line | $y = mx + c$ |

**END OF FORMULA SHEET**