2017 VCE Mathematical Methods 2 (NHT) examination report

Specific information

This report provides sample answers or an indication of what answers may have included. Unless otherwise stated, these are not intended to be exemplary or complete responses.

Section A – Multiple-choice questions

Question	Answer
1	A
2	E
3	C
4	D
5	C
6	B
7	A
8	D
9	C
10	A
11	C
12	B
13	B
14	E
15	E
16	B
17	C
18	D
19	B
20	D

Section B

Question 1a.

$T(t)=19+6\sin\left(\frac{\pi}{12}(t-8)\right)$, range of function is $[-6+19,\ 6+19]$ = $[13,\ 25]$, minimum temperature is 13 °C, maximum temperature is 25 °C

Question 1b.
$T(6)=16$ °C

Question 1c.
$T(8)=19$ °C

Question 1d.

Solve $T(t) \geq 19$ °C, $8 \leq t \leq 20$, $20-8=12$ hours

Question 1e.

Average rate of change = $\dfrac{T(12)-T(8)}{12-8} = \dfrac{3\sqrt{3}}{4}$ °C/hr

Question 1fi.

$T'(t) = \dfrac{\pi}{2}\cos\left(\dfrac{\pi}{12}(t-8)\right)$ or $T'(t) = -\dfrac{\pi}{2}\cos\left(\dfrac{\pi}{12}t + \dfrac{\pi}{3}\right)$, or equivalent

Question 1fii.
Find the minimum of the derivative, decreasing most rapidly at 8.00 pm or 20 hours.

Question 2a.

$f: R \to R$, where $f(x) = (x-2)^2(x-5)$, $f'(x) = 3(x-4)(x-2)$, or equivalent

Question 2b.

Solve $f'(x) < 0$, $2 < x < 4$

Question 2ci.

$f(1) = -4$, $f(5) = 0$, $\dfrac{f(5)-f(1)}{5-1} = 1$

Question 2cii.

Midpoint $\left(\dfrac{5+1}{2}, \dfrac{-4+0}{2}\right) = (3, -2)$, $f(3) = -2$ hence midpoint lies on the graph of $y = f(x)$

Question 2ciii.

Solve $f'(x) = 1$, $x = \dfrac{9+2\sqrt{3}}{3}$ or $x = \dfrac{9-2\sqrt{3}}{3}$

Question 2d.

$g: R \to R$, where $g(x) = (x-2)^2(x-a)$, $g'(x) = 0$, $x = 2$ or $x = \dfrac{2(a+1)}{3}$,

$p = \dfrac{2}{3}$, $g\left(\dfrac{2(a+1)}{3}\right) = -\dfrac{4}{27}(a-2)^3$, $q = -\dfrac{4}{27}$

Question 2e.
$g'(a) = (a-2)^2$, $(a-2)^2 \geq 0$, when $a = 2$, $g'(x) = 0$, gradient of the tangent is positive for $a \in R \setminus \{2\}$

Question 2fi.

$g'(x) = (a-2)^2$, $\left(\dfrac{8-a}{3}, -\dfrac{4}{27}(a-2)^3\right)$

Question 2fii.

$Q\left(\frac{2(a+1)}{3}, -\frac{4}{27}(a-2)^3\right)$ and $\left(\frac{8-a}{3}, -\frac{4}{27}(a-2)^3\right)$,

distance = $\sqrt{\left(-\frac{4}{27}(a-2)^3 - -\frac{4}{27}(a-2)^3\right)^2 + \left(\frac{8-a}{3} - \frac{2(a+1)}{3}\right)^2} = a-2$

Question 3a.

$X_A \sim N\left(11, \left(\frac{1}{4}\right)^2\right), \Pr\left(X_A > 10.5\right) = 0.977$, correct to three decimal places

Question 3b.

$\mathrm{E}\left(X_B\right) = \int_0^{12} x f(x)dx = 7.75 \text{ hours}$, correct to two decimal places

Question 3c.

$\mathrm{sd}\left(X_B\right) = \sqrt{\int_0^{12} x^2 f(x)dx - \left(\int_0^{12} x f(x)dx\right)^2} = 2.31 \text{ hours}$, correct to two decimal places

Question 3d.

$\Pr\left(X_B > 10.5\right) = \int_{10.5}^{12} f(x)dx = 0.1134$, correct to four decimal places

Question 3e.

Pr(boxes mislabelled) = $\Pr\left(A \cap (X_A < 10.5)\right) + \Pr\left(\mathrm{B} \cap (X_B > 10.5)\right)$

= 0.5 × 0.0228 + 0.5 × 0.1134

= 0.068, correct to three decimal places

Question 3f.

$\Pr\left(\mathrm{B}|\text{mislabelled}\right) = \frac{\Pr\left(\mathrm{B} \cap \text{mislabelled}\right)}{\Pr\left(\text{mislabelled}\right)} = \frac{0.5 \times 0.1134}{0.0681} = 0.833$, correct to three decimal places

Question 3g.

$X_1 \sim \mathrm{Bi}\left(26, 0.05\right)$ or $1 - 0.95^{26}, \Pr\left(X_1 \geq 1\right) = 0.7365$, correct to four decimal places

Question 3h.

$X_2 \sim \mathrm{Bi}\left(100, 0.05\right), \ \Pr(\hat{P}_A > 0.04 \mid \hat{P}_A < 0.08) = \frac{\Pr(5 \leq X_1 \leq 7)}{\Pr(X_1 \leq 7)} = \frac{0.4361}{0.8720} = 0.5000$,

correct to four decimal places

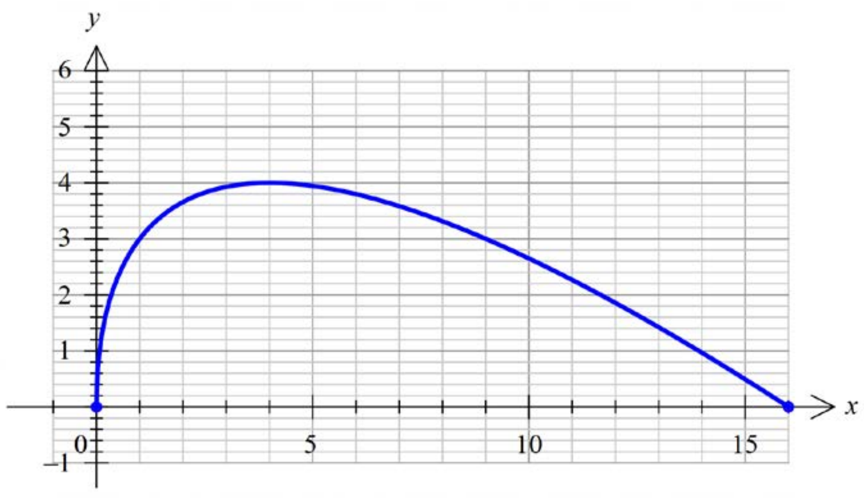
Question 3i.

A 90% confidence interval for the population proportion from this sample is (0.02, 0.10), correct to two decimal places

Question 4a.

$f: [0,16] \to R, f(x) = 4\sqrt{x} - x$, maximum occurs when $x = 4$ and is $f(4) = 4$

Question 4b.



Question 4ci.

$A=\int_0^{16} f(x)dx=\frac{128}{3}$ square units

Question 4cii.

$A_{\Delta OCX}=\frac{16\times f(c)}{2}$, maximum occurs when $c=4$ and the maximum area is 32 square units

Question 4d.

$4\sqrt{b}-b=4\sqrt{a}-a$, $4\sqrt{b}-4\sqrt{a}=b-a$, $4\left(\sqrt{b}-\sqrt{a}\right)=\left(\sqrt{b}-\sqrt{a}\right)\left(\sqrt{b}+\sqrt{a}\right)$, $\sqrt{b}=4-\sqrt{a}$,

$b=\left(\sqrt{a}-4\right)^2$

Question 4ei.

Area of rectangle $A_R=(b-a)f(a)=\left(\left(4-\sqrt{a}\right)^2-a\right)\left(4\sqrt{a}-a\right)=8\left(2-\sqrt{a}\right)\left(4\sqrt{a}-a\right)$ or

$8a^{\frac{3}{2}}-48a+64\sqrt{a}$ square units

Question 4eii.

$A_R'(a)=0$ or find maximum of A_R, $a=\frac{8}{3}\left(2-\sqrt{3}\right)$, $b=\frac{8}{3}\left(2+\sqrt{3}\right)$

Question 4eiii.

$A_R\left(\frac{8}{3}\left(2-\sqrt{3}\right)\right)=\frac{128\sqrt{3}}{9}$ square units

Question 4fi.

$A_T=\frac{1}{2}(16+(b-a))\times f(a)$, $A_T{}'(a)=0$, $a=\frac{16}{9}$, $A_T=\frac{1024}{27}$ square units

Question 4fii.

$$\frac{A_T}{A} = \frac{\frac{1024}{27}}{\frac{128}{9}} = \frac{8}{9}$$