VICTORIAN CURRICULUM
AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2018

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER									Letter

FURTHER MATHEMATICS

Written examination 2

Thursday 31 May 2018

Reading time: 2.00 pm to 2.15 pm (15 minutes)
Writing time: 2.15 pm to 3.45 pm (1 hour 30 minutes)

QUESTION AND ANSWER BOOK

Structure of book

Section A – Core	*Number of questions*	*Number of questions to be answered*	*Number of marks*
	9	9	36
Section B – Modules	*Number of modules*	*Number of modules to be answered*	*Number of marks*
	4	2	24
			Total 60

- Students are to write in blue or black pen.
- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 34 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination
- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION A – Core

Instructions for Section A

Answer **all** questions in the spaces provided.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.

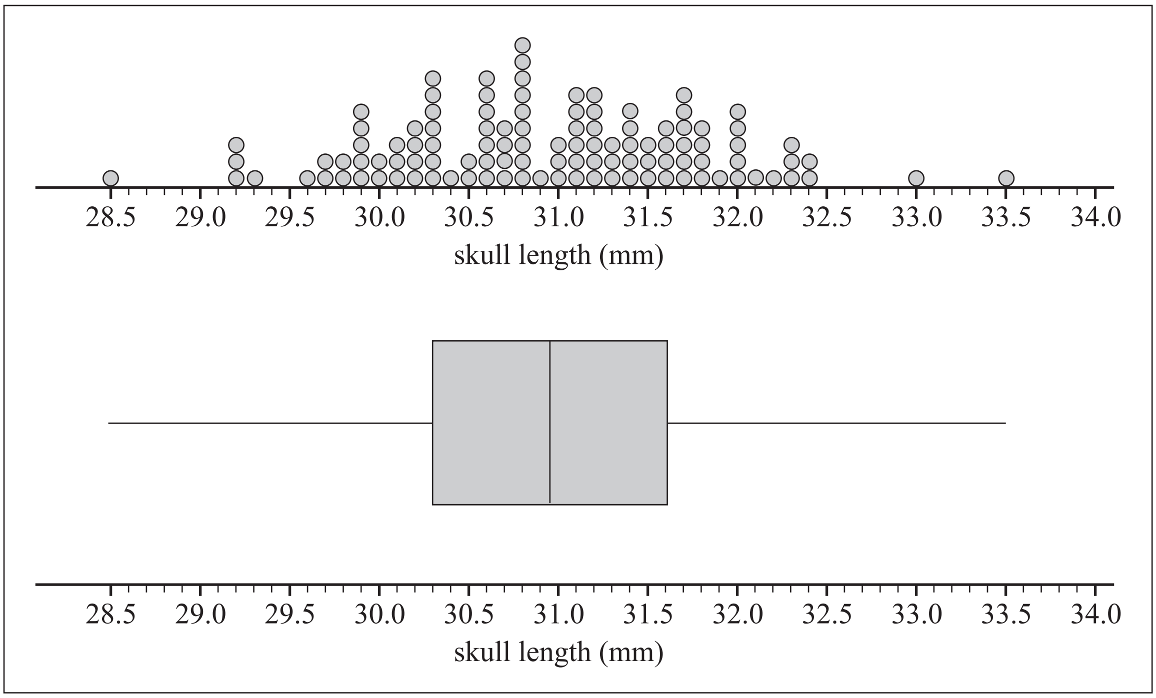
In 'Recursion and financial modelling', all answers should be rounded to the nearest cent unless otherwise instructed.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Data analysis

Question 1 (3 marks)

The dot plot and boxplot below display the distribution of *skull length*, in millimetres, for a sample of the same species of bird.



a. Write down the modal skull length. 1 mark

b. Use information from the plots above to show why the bird with a skull length of 33.5 mm is **not** plotted as an outlier in the boxplot. 2 marks

Question 2 (3 marks)

The *weight* of a species of bird is approximately normally distributed with a mean of 71.5 g and a standard deviation of 4.5 g.

a. What is the standardised weight (z score) of a bird weighing 67.9 g? 1 mark

b. Use the 68–95–99.7% rule to estimate

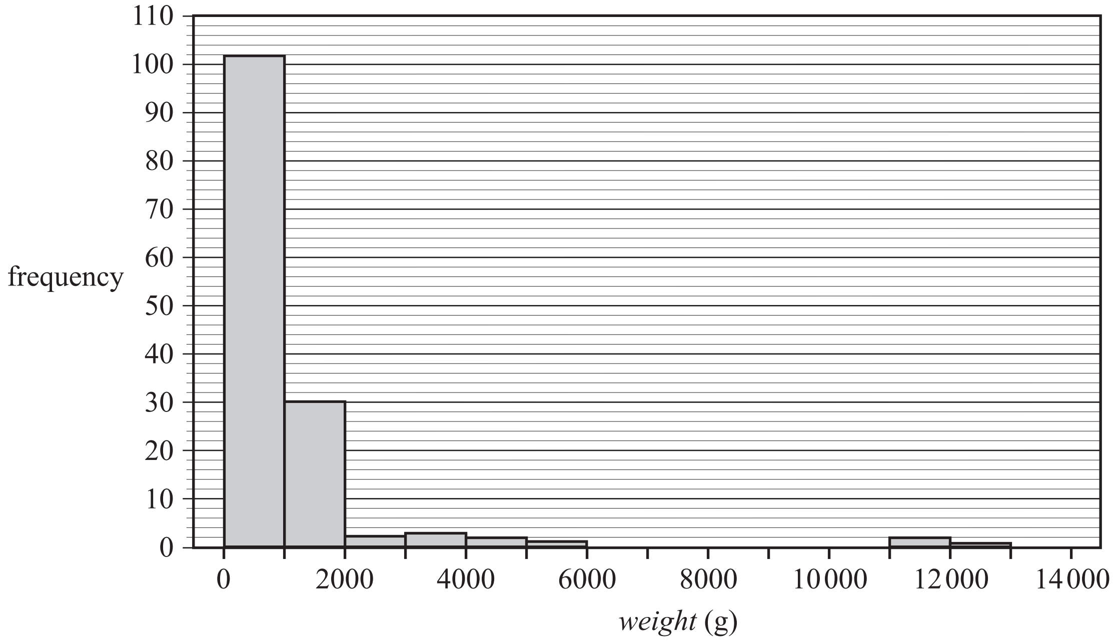
i. the expected percentage of these birds that weigh less than 67 g 1 mark

ii. the expected number of birds that weigh between 62.5 g and 76.0 g in a flock of 200 of these birds. 1 mark

Question 3 (3 marks)

Histogram 1 below displays the *weight* distribution of 143 birds of different species living in a small zoo.

Histogram 1



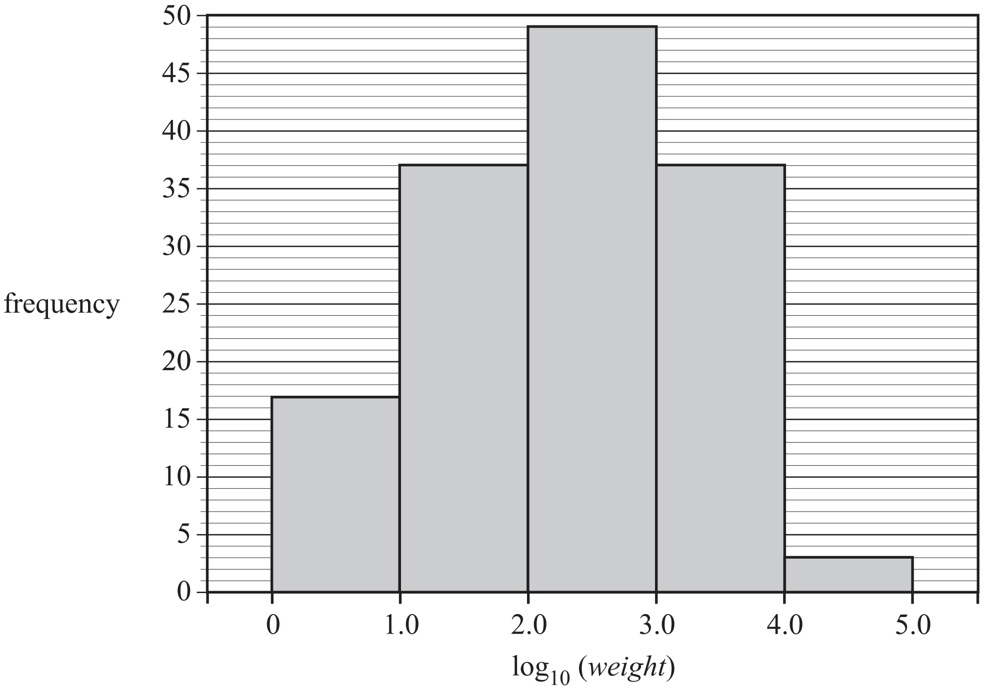
a. Describe the shape of the distribution displayed in Histogram 1. Note the number of possible outliers, if any. 1 mark

b. What percentage of these birds weigh less than 1000 g?

Round your answer to one decimal place. 1 mark

c. Histogram 2 below displays the *weight* distribution of the same 143 birds plotted on a $\log_{10}$ scale.

Histogram 2



How many of these birds weigh between 10 g and 100 g? 1 mark

Question 4 (4 marks)

A sample of 96 birds are grouped according to their *beak size* (small, medium, large).

The percentage of birds in each group is calculated. The results are displayed in Table 1 below.

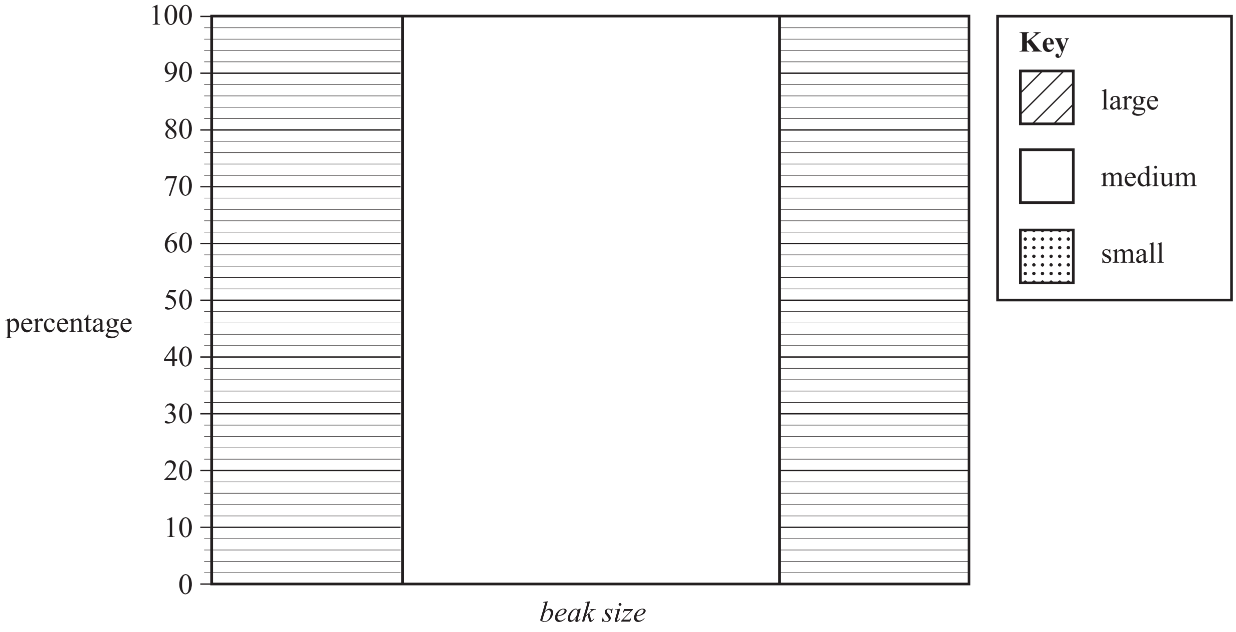
Table 1

Beak size	**Percentage (%)**
small	25
medium	44
large	31
Total	100

a. How many of the 96 birds have small beaks? 1 mark

b. Use the percentages in Table 1 to construct a percentaged segmented bar chart.

A template is provided below to assist you in completing this task. Use the key to indicate the segment of your bar chart that corresponds to each beak size. 1 mark



c. In order to investigate a possible association between *beak size* and *sex*, the same birds are grouped by both their *beak size* (small, medium, large) and their *sex* (male, female). The results of this grouping are shown in Table 2.

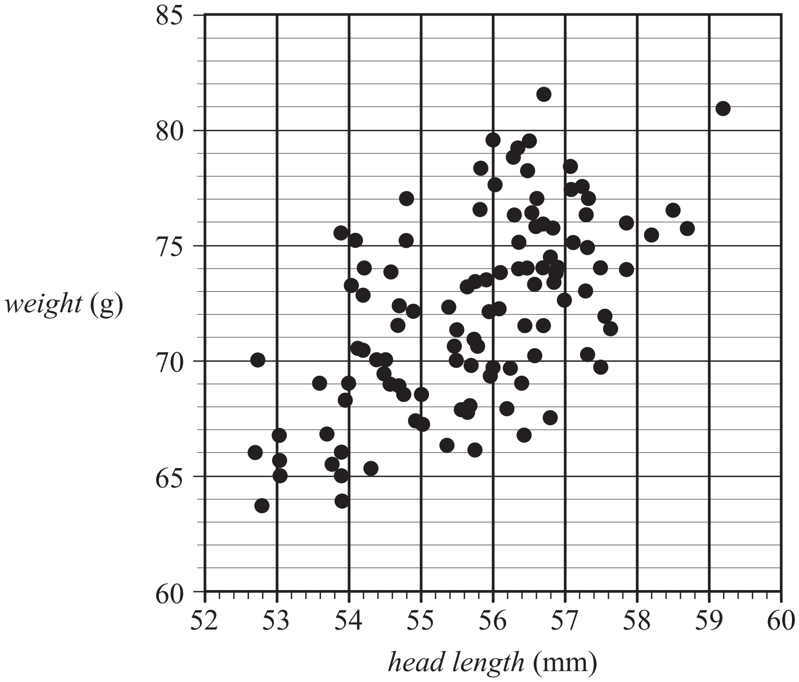
Table 2

	Sex	
Beak size	**Male**	**Female**
small	1	23
medium	26	16
large	27	3
Total	54	42

Does the information provided above support the contention that *beak size* is associated with *sex*? Justify your answer by quoting appropriate percentages. It is sufficient to consider one *beak size* only when justifying your answer. 2 marks

Question 5 (7 marks)

The scatterplot below shows the *weight*, in grams, and the *head length*, in millimetres, of 110 birds.



The equation of the least squares line fitted to this data is

$$weight = -24.83 + 1.739 \times head\ length$$

a. Draw this least squares line on the **scatterplot above**. 1 mark

(*Answer on the scatterplot above.*)

b. Use the equation to predict the *weight*, in grams, of a bird with a *head length* of 49.0 mm. Round your answer to one decimal place. 1 mark

c. Is the prediction made in **part b.** an example of interpolation or extrapolation? Explain your answer briefly. 1 mark

d. When the least squares line is used to predict the *weight* of a bird with a *head length* of 59.2 mm, the residual value is 2.78

Calculate the actual weight of this bird.

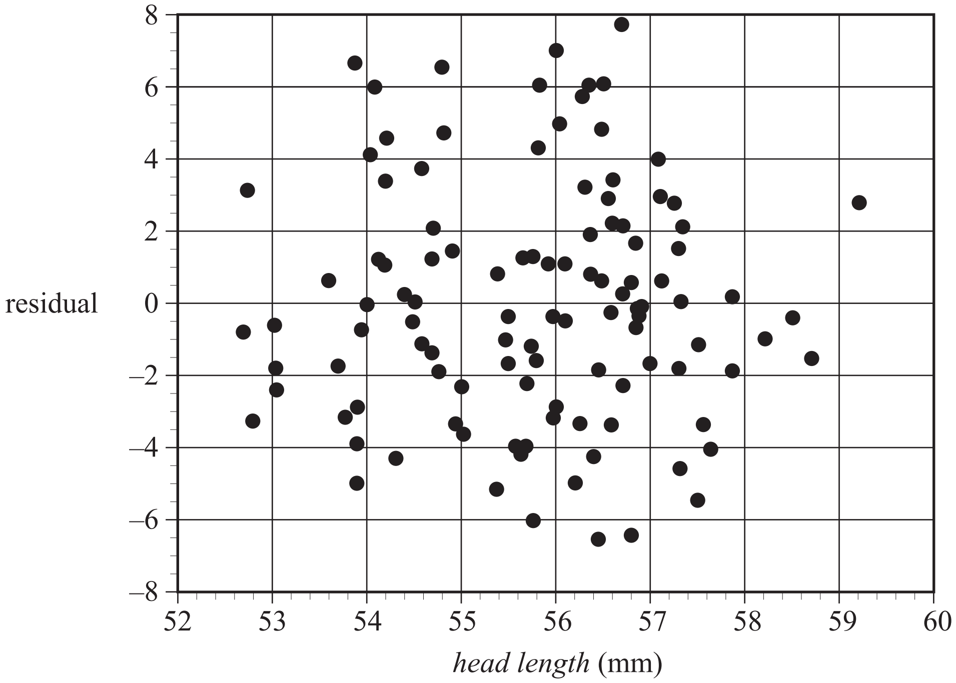
Round your answer to one decimal place. 2 marks

e. Pearson's correlation coefficient, *r*, is equal to 0.5957

Given this information, what percentage of the variation in the *weight* of these birds is **not** explained by the variation in *head length*?

Round your answer to one decimal place. 1 mark

f. The residual plot obtained when the least squares line is fitted to the data set is shown below.



What does the residual plot indicate about the association between *head length* and *weight* for these birds? 1 mark

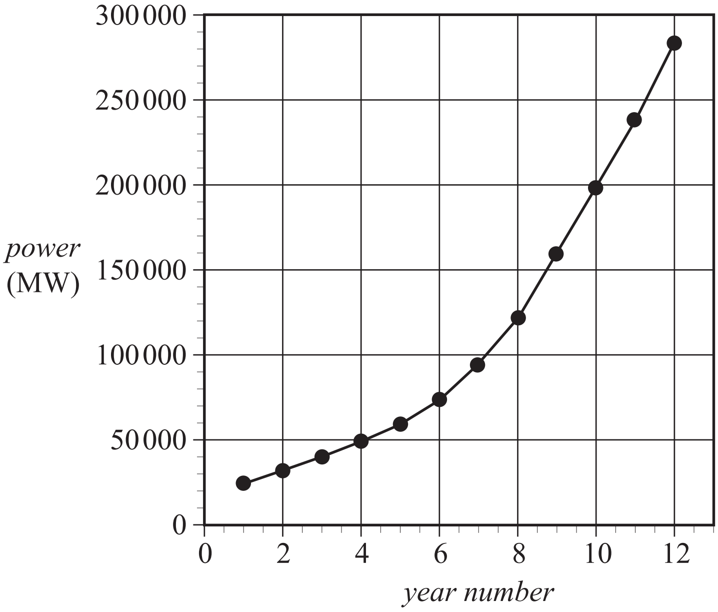
Question 6 (4 marks)

The time series data below shows the worldwide growth in electrical *power* generated by wind, in megawatts, for the period 2001–2012. The variable that represents *time*, in years, has been rescaled so that '1' represents 2001, '2' represents 2002, and so on.

This new variable is called *year number*.

A time series plot for the data is also shown.

Year number	***Power* (MW)**
1	23 900
2	31 100
3	39 431
4	47 620
5	59 091
6	73 957
7	93 924
8	120 696
9	159 052
10	197 956
11	238 110
12	282 850



Data: Global Wind Energy Council (GWEC), Global Statistics, 'Global Cumulative Installed Wind Capacity 2001–2016', <www.gwec.net/>

The relationship between *power* and *year number* is clearly non-linear.

A $\log_{10}$ transformation can be applied to the variable *power* to linearise the data.

a. Apply this transformation to the data to determine the equation of the least squares line that can be used to predict $\log_{10}$ (*power*) from *year number*.

Write the values of the intercept and slope of this least squares line in the appropriate boxes provided below.

Round your answers to four significant figures. 2 marks

$\log_{10}$ (*power*) = ☐ + ☐ × *year number*

b. Use the equation in **part a.** to predict the electrical *power*, in megawatts, expected to be generated by wind in 2020.

Round your answer to the nearest 1000 MW. 2 marks

Recursion and financial modelling

Question 7 (4 marks)

Roslyn invested some money in a savings account that earns interest compounding annually.

The interest is calculated and paid at the end of each year.

Let V_n be the amount of money in Roslyn's savings account, in dollars, after n years.

The recursive calculations below show the amount of money in Roslyn's savings account after one year and after two years.

$$V_0 = 5000$$
$$V_1 = 1.05 \times 5000 = 5250$$
$$V_2 = 1.05 \times 5250 = 5512.50$$

a. How much money did Roslyn initially invest? 1 mark

b. How much interest in total did she earn by the end of the second year? 1 mark

c. Let V_n be the amount of money in Roslyn's savings account, in dollars, after n years.

Write down a recurrence relation, in terms of V_0, V_{n+1} and V_n, that can be used to model the amount of money, in dollars, in Roslyn's savings account. 1 mark

d. Roslyn plans to use her savings to pay for a holiday.

The holiday will cost $6000.

What minimum annual percentage interest rate would have been required for Roslyn to have saved this $6000 after two years?

Round your answer to one decimal place. 1 mark

Question 8 (5 marks)

Richard will join Roslyn on the holiday.

He will sell his stereo system to help pay for his holiday.

The stereo system was originally purchased for \$8500.

Richard will sell the stereo system at a depreciated value.

a. Richard could use a flat rate depreciation method.

Let S_n be the value, in dollars, of Richard's stereo system *n* years after it was purchased.

The value of the stereo system, S_n, can be modelled by the recurrence relation below.

$$S_0 = 8500, \qquad S_{n+1} = S_n - 867$$

i. Using this depreciation method, what is the value of the stereo system four years after it was purchased? 1 mark

ii. Calculate the annual percentage flat rate of depreciation for this depreciation method. 1 mark

b. Richard could also use a reducing balance depreciation method, with an annual depreciation rate of 8%.

Using this depreciation method, what is the value of the stereo system four years after it was purchased?

Round your answer to the nearest cent. 1 mark

c. Four years after it was purchased, Richard sold his stereo system for \$4500.

Assuming a reducing balance depreciation method was used, what annual percentage rate of depreciation did this represent?

Round your answer to one decimal place. 2 marks

Question 9 (3 marks)

Andrew will also join Roslyn and Richard on the holiday.

Andrew borrowed $10 000 to pay for the holiday and for other expenses.

Interest on this loan will be charged at the rate of 12.9% per annum, compounding monthly.

Immediately after the interest has been calculated and charged each month, Andrew will make a repayment.

a. For the first year of this loan, Andrew will make interest-only repayments each month.

What is the value of each interest-only repayment? 1 mark

b. For the next three years of this loan, Andrew will make equal monthly repayments.

After these three years, the balance of Andrew's loan will be $3776.15

What amount, in dollars, will Andrew repay each month during these three years? 1 mark

c. Andrew will fully repay the outstanding balance of $3776.15 with a further 12 monthly repayments.

The first 11 repayments will each be $330.

The twelfth repayment will have a different value to ensure the loan is repaid exactly to the nearest cent.

What is the value of the twelfth repayment?

Round your answer to the nearest cent. 1 mark

END OF SECTION A
TURN OVER

SECTION B – Modules

Instructions for Section B

Select **two** modules and answer **all** questions within the selected modules.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Contents **Page**

Module 1 – Matrices

Question 1 (4 marks)

A region has four districts: North (N), South (S), East (E) and West (W).

Farmers from each district attended a conference in 2017.

Matrix F_{2017} below shows the number of farmers from each of these four districts who attended the 2017 conference.

$$F_{2017} = \begin{array}{c c} \begin{bmatrix} 36 \\ 20 \\ 28 \\ 16 \end{bmatrix} & \begin{array}{c} N \\ S \\ E \\ W \end{array} \end{array}$$

a. What is the order of matrix F_{2017}? 1 mark

b. How many of these farmers came from either the North or South district? 1 mark

The table below shows the cost per farmer, for each district, to attend the 2017 conference.

District	Cost per farmer ($)
North	25
South	20
East	45
West	35

c. Write down a matrix that could be multiplied by matrix F_{2017} to give the total cost for all farmers who attended the 2017 conference. 1 mark

d. The number of farmers who attended the 2018 conference increased by 25% for each district from the previous year.

Complete the product below with a scalar so that the product gives the number of farmers from each district who attended the 2018 conference. 1 mark

$$F_{2018} = \boxed{} \times F_{2017}$$

Question 2 (2 marks)

Five farmers, *A*, *B*, *C*, *D* and *E*, attended the 2018 conference.

Pairs of these farmers had previously attended one or more conferences together.

The number of conferences previously attended together is shown in matrix *M* below.

For example, the ‘1’ in the bottom row shows that *D* and *E* had attended one earlier conference together.

$$M = \begin{array}{c} \\ \\ \\ \\ \\ \\ \end{array}\begin{array}{ccccc|c} A & B & C & D & E & \\ \end{array}$$

$$\begin{array}{c|ccccc|c} & A & B & C & D & E & \\ \hline & 0 & 1 & 1 & 1 & 0 & A \\ & 1 & 0 & 2 & 1 & 3 & B \\ M = & 1 & 2 & 0 & 1 & 2 & C \\ & 1 & 1 & 1 & 0 & 1 & D \\ & 0 & 3 & 2 & 1 & 0 & E \end{array}$$

a. Which two farmers had not previously attended a conference together? 1 mark

b. What do the numbers in column *D* indicate? 1 mark

Question 3 (4 marks)

Three farmers, A, B and C, each placed orders for three types of fertilisers for their cornfields.

The types of fertilisers are Kalm (K), Nitro (N) and Phate (P).

The matrix below shows the amount of fertiliser, in tonnes, ordered by Farmer A and Farmer B.

$$\begin{array}{r c} & \begin{array}{ccc} K & N & P \end{array} \\ \begin{array}{r} \text{Farmer } A \\ \text{Farmer } B \end{array} & \begin{bmatrix} 2 & 4 & 2 \\ 2 & 5 & 1 \end{bmatrix} \end{array}$$

a. Farmer A and Farmer B each paid a total of \$16 000 for fertiliser.

What conclusion can be drawn about the prices of Nitro (N) and Phate (P)? 1 mark

Let x be the price per tonne of Kalm (K)
y be the price per tonne of Nitro (N)
z be the price per tonne of Phate (P).

The total cost of these two orders can be summarised by the matrix equation

$$\begin{bmatrix} 2 & 4 & 2 \\ 2 & 5 & 1 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 16000 \\ 16000 \end{bmatrix}$$

b. Explain why this equation cannot be solved using the matrix inverse method. 1 mark

c. The matrix equation below shows the fertiliser orders for all three farmers.

$$\begin{bmatrix} 2 & 4 & 2 \\ 2 & 5 & 1 \\ 1 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 16000 \\ 16000 \\ 6500 \end{bmatrix}$$

i. Complete the matrix equation below by filling in the missing elements. 1 mark

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ 0.5 & 0 & -1 \\ 1.5 & -1 & -1 \end{bmatrix} \begin{bmatrix} 16000 \\ 16000 \\ 6500 \end{bmatrix}$$

ii. Determine the cost, in dollars, of one tonne of Phate (P). 1 mark

Question 4 (2 marks)

Areas of farmland in the region are allocated to growing barley (B), corn (C) and wheat (W).

This allocation of farmland is to be changed each year, beginning in 2019.

The table below shows the areas of farmland, in hectares, allocated to each crop in 2018 ($n = 0$) and 2019 ($n = 1$).

Year	**2018**	**2019**
n	0	1
barley	2000	2100
corn	1000	1900
wheat	3000	2000

The planned annual change to the area allocated to each crop can be modelled by

$$H_{n+1} = RH_n + Q \qquad \text{where} \quad R = \begin{array}{c} \\ \\ \\ \\ \end{array}\begin{array}{c} \textit{this year} \\ \begin{array}{ccc} B & C & W \end{array} \\ \begin{bmatrix} 0.7 & 0.1 & 0.1 \\ 0.1 & 0.8 & 0.2 \\ 0.2 & 0.1 & 0.7 \end{bmatrix} \end{array} \begin{array}{l} \\ \\ B \\ C \ \ \textit{next year} \\ W \end{array}$$

H_n represents the state matrix that shows the area allocated to each crop n years after 2018.

Q is a matrix that contains additional fixed changes to the area that is allocated to each crop each year.

Complete H_2, the state matrix for 2020.

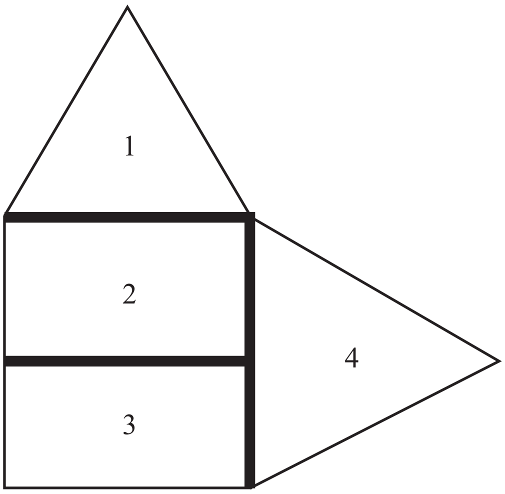
$$H_2 = \begin{bmatrix} \_\_\_\_\_\_ \\ \_\_\_\_\_\_ \\ \_\_\_\_\_\_ \end{bmatrix} \begin{array}{l} B \\ C \\ W \end{array}$$

Module 2 – Networks and decision mathematics

Question 1 (2 marks)

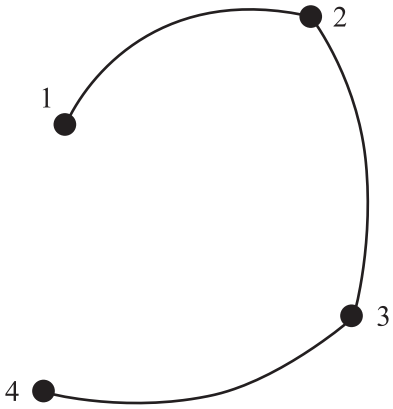
A farmer's property is divided into four areas labelled 1 to 4 on the diagram below.

The bold lines represent the boundary fences between two areas.



In the graph below, the four areas of the property are represented as vertices.

The edges of the graph represent the boundary fences between areas.



One of the edges is missing from this graph.

a. On the **graph above**, draw in the missing edge. 1 mark

(*Answer on the graph above.*)

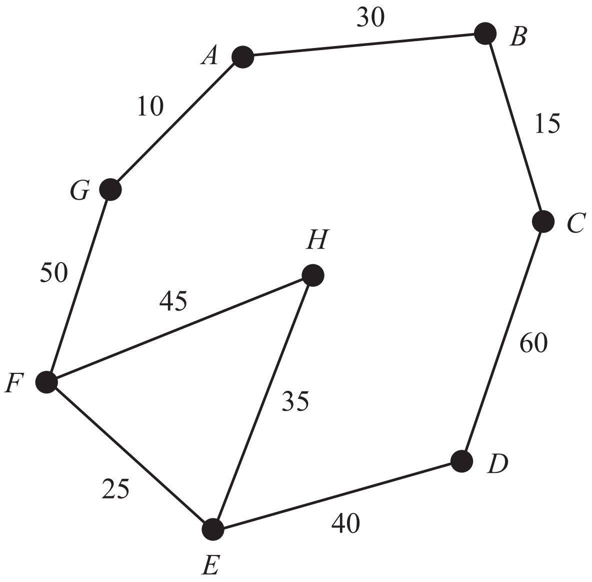
b. With this edge drawn in, what is the sum of the degrees of the vertices of the graph? 1 mark

Question 2 (3 marks)

Area 1 of the property contains eight large bushes that are labelled *A* to *H*, as shown on the graph below.

The farmer’s dog enjoys running around this area, stopping at each bush on the way.

The numbers on the edges joining the vertices give the shortest distance, in metres, between bushes.



a. Explain why the dog could not follow an Eulerian circuit through this network. 1 mark

b. If the dog follows the shortest Hamiltonian path, name a bush at which the dog could start and a bush at which the dog could finish. 1 mark

Start ☐ Finish ☐

c. The sum of all distances shown on the graph is 310 m.

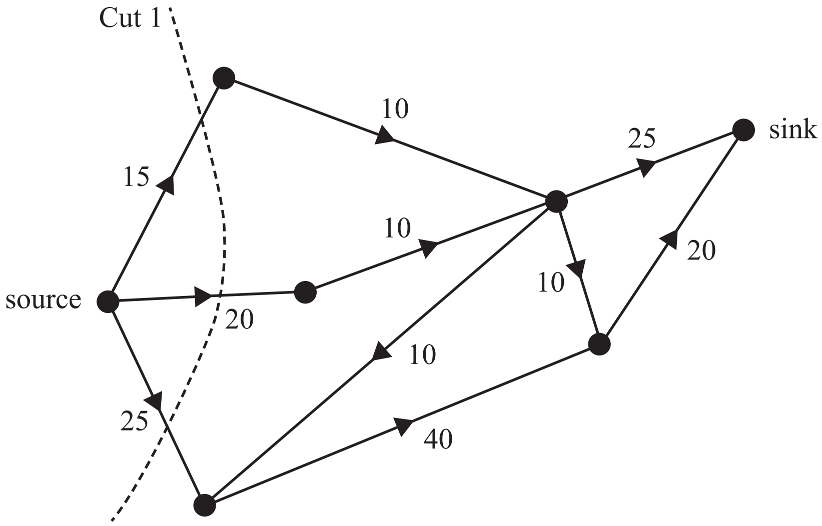
The dog starts and finishes at bush *F* and runs along every edge in the network.

What is the shortest distance, in metres, that the dog could have run? 1 mark

Question 3 (3 marks)

All areas of the property require a constant supply of water.

The following directed graph represents the capacity, in litres per minute, of a series of water pipes on the property connecting the source to the sink.



When considering the possible flow through this network, different cuts can be made.

Cut 1 is labelled on the graph above.

a. What is the capacity of Cut 1 in litres per minute? 1 mark

b. On the **graph above**, draw the cut (Cut 2) that has a capacity of 70 litres per minute. Label your answer clearly as Cut 2. 1 mark

(*Answer on the graph above.*)

c. Determine the maximum flow of water, in litres per minute, from the source to the sink. 1 mark

Question 4 (4 marks)

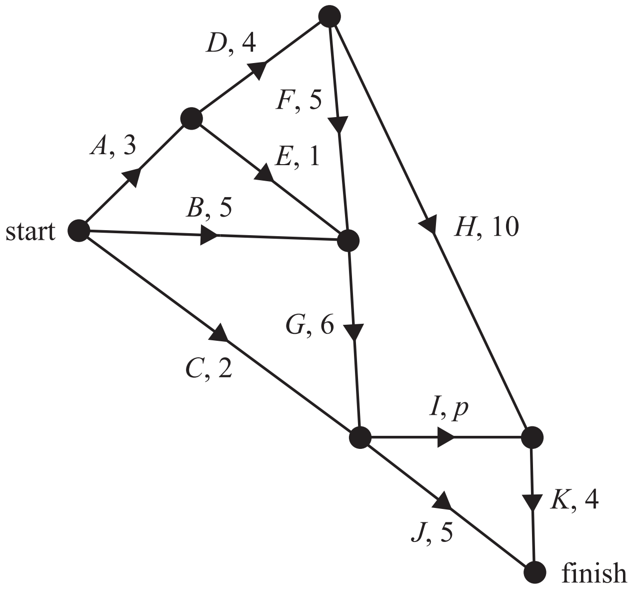
A barn will be built on the property.

This building project will involve 11 activities, *A* to *K*.

The directed network below shows these activities and their duration in days.

The duration of activity *I* is unknown at the start of the project.

Let the duration of activity *I* be *p* days.



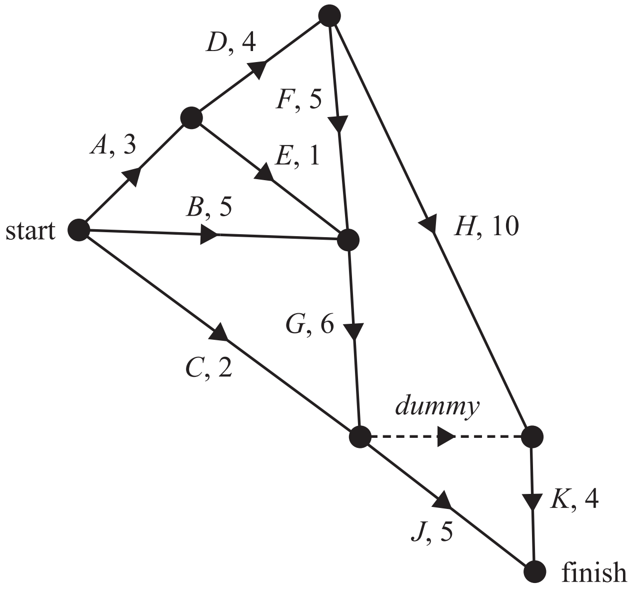
a. Determine the earliest starting time, in days, for activity *I*. 1 mark

b. Determine the value of *p*, in days, that would create more than one critical path. 1 mark

c. If the value of *p* is six days, what will be the float time, in days, of activity *H*? 1 mark

d. When a second barn is built later, activity *I* will not be needed.

A *dummy* activity is required, as shown on the revised directed network below.



Explain what this *dummy* activity indicates on the revised directed network. 1 mark

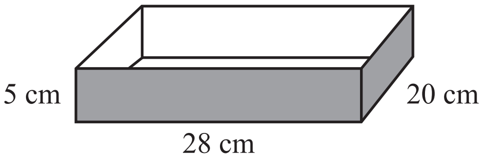
Module 3 – Geometry and measurement

Question 1 (5 marks)

Shannon is a baker.

One of her baking tins has a rectangular base of length 28 cm and width 20 cm.

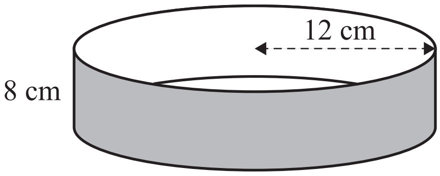
The height of this baking tin is 5 cm, as shown in the diagram below.



a. What is the volume of this tin, in cubic centimetres? 1 mark

Another baking tin has a circular base with a radius of 12 cm.

The height of this baking tin is 8 cm, as shown in the diagram below.

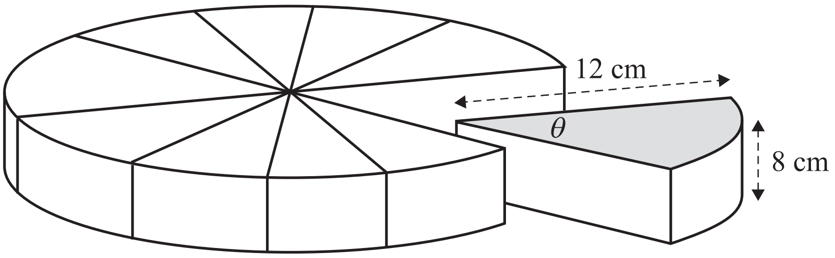


b. Shannon needs to cover the inside of both the base and side of this tin with baking paper.

What is the area of baking paper required, in square centimetres?

Round your answer to one decimal place. 2 marks

A cake cooked in the circular baking tin is cut into 10 pieces of equal size, as shown in the diagram below.



The angle θ is also shown on the diagram.

c. Show that the angle θ is equal to 36°. 1 mark

d. What is the volume, in cubic centimetres, of one piece of cake?
Round your answer to one decimal place. 1 mark

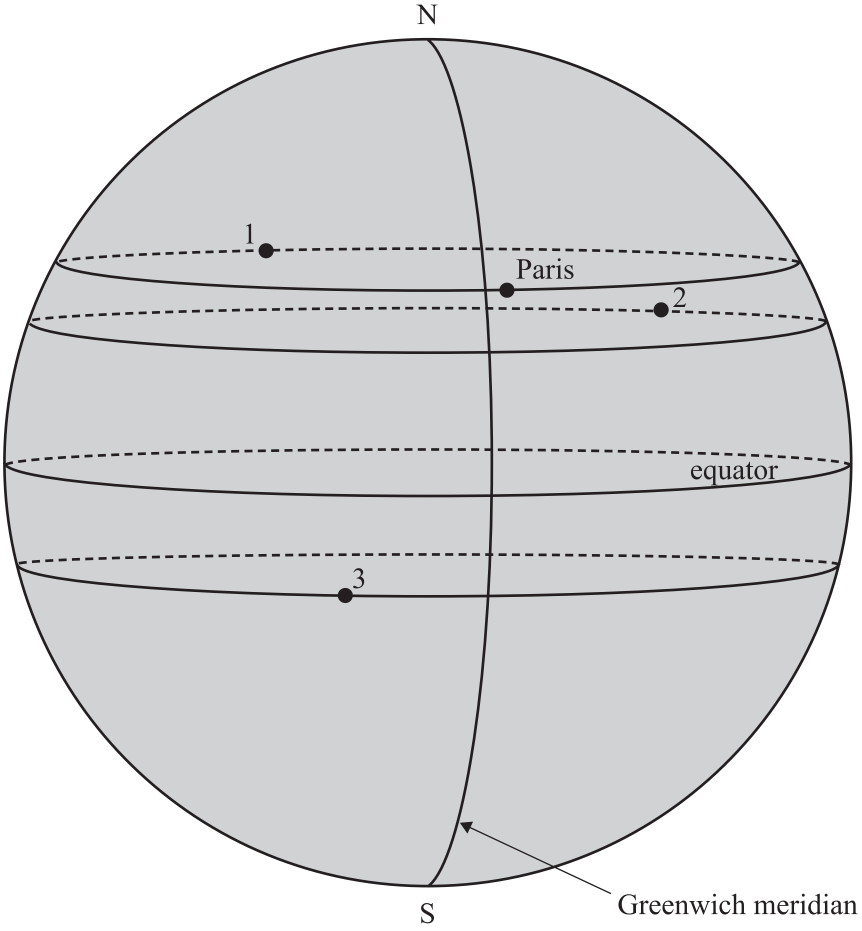
Question 2 (4 marks)

Shannon plans to travel to Paris, Beijing, Brasilia and Vancouver to try the local cake specialties:

- Paris (49° N, 2° E) in France
- Beijing (40° N, 116° E) in China
- Brasilia (16° S, 48° W) in Brazil
- Vancouver (49° N, 123° W) in Canada

The diagram below shows the position of Paris at latitude 49° N and longitude 2° E.

The three other cities are indicated on the diagram as 1, 2 and 3.



a. Complete the table below by matching the city with the corresponding city number (1, 2 and 3) given in the diagram above. 1 mark

City	City number
Beijing (40° N, 116° E)	
Brasilia (16° S, 48° W)	
Vancouver (49° N, 123° W)	

b. Shannon travelled from Sydney to Paris on Wednesday, 30 May. She left Sydney at 10.50 am.

The flight to Paris took 22 hours and 25 minutes.

The time difference between Sydney (34° S, 151° E) and Paris (49° N, 2° E) is eight hours.

On what day and at what time will Shannon arrive in Paris? 1 mark

c. On the day that Shannon arrives in Paris, the sun will rise at 5.54 am.

Assume that 15° of longitude equates to a one-hour time difference.

How long after the sun rises in Paris (49° N, 2° E) will the sun rise in Vancouver (49° N, 123° W)?

Write your answer in hours and minutes. 1 mark

d. Shannon travels to the French cities of Lyon (46° N, 5° E) and Marseille (43° N, 5° E).

Assume that the radius of Earth is 6400 km.

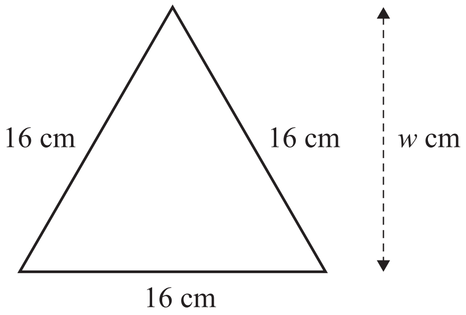
Find the shortest great circle distance between Lyon and Marseille.

Round your answer to the nearest kilometre. 1 mark

Question 3 (3 marks)

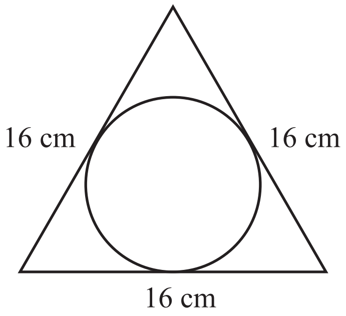
After returning from her travels, Shannon decides to design an interesting package for some of her smaller circular cakes.

She designs a triangular box with side lengths of 16 cm, as shown in the diagram below.



a. Show that the value of *w* on the diagram is 13.9, rounded to one decimal place. 1 mark

b. One circular cake is placed in the triangular box.



What is the diameter, in centimetres, of the largest cake that will fit in the triangular box?

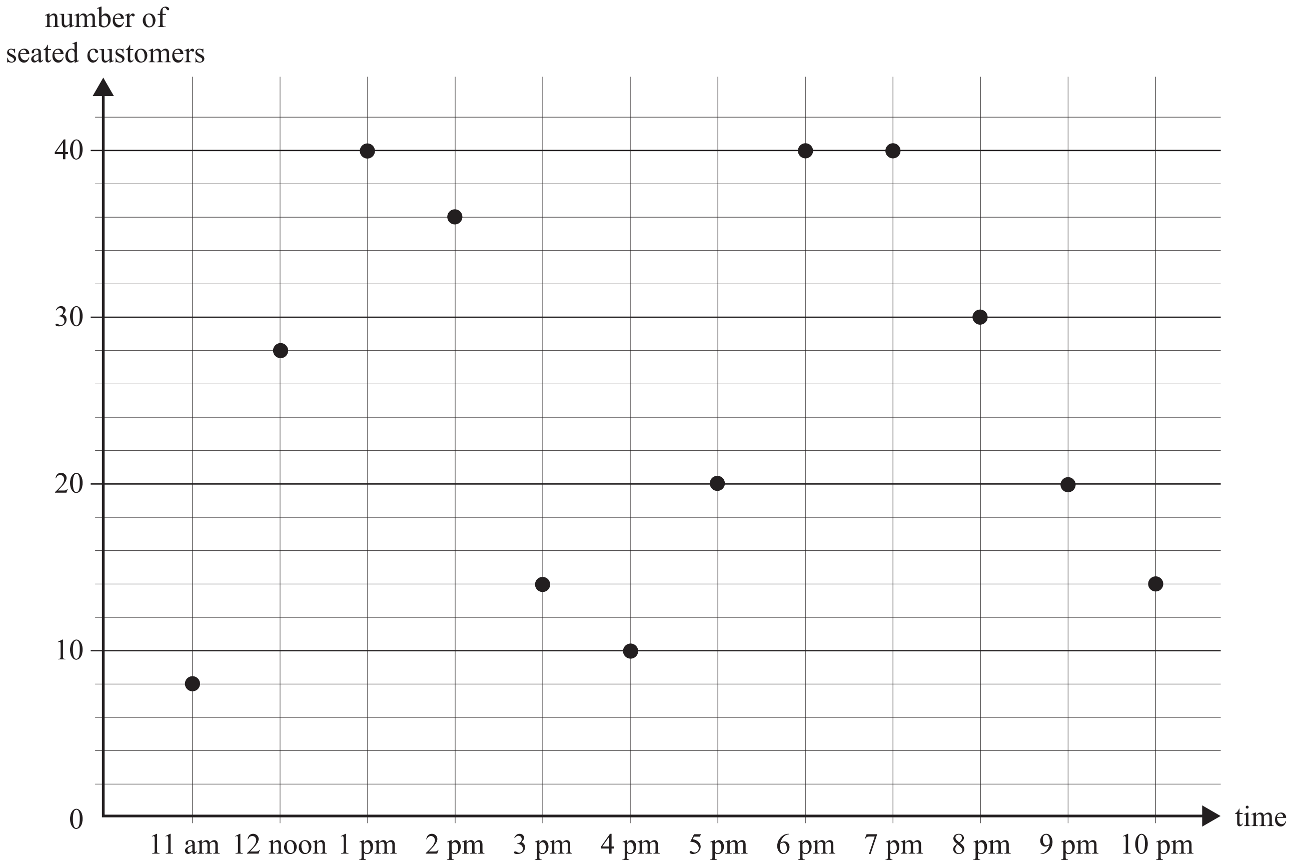
Round your answer to one decimal place. 2 marks

Module 4 – Graphs and relations

Question 1 (2 marks)

A hamburger restaurant recorded the number of seated customers each hour from 11 am to 10 pm.

The graph below shows the number of customers seated each hour on one particular day.



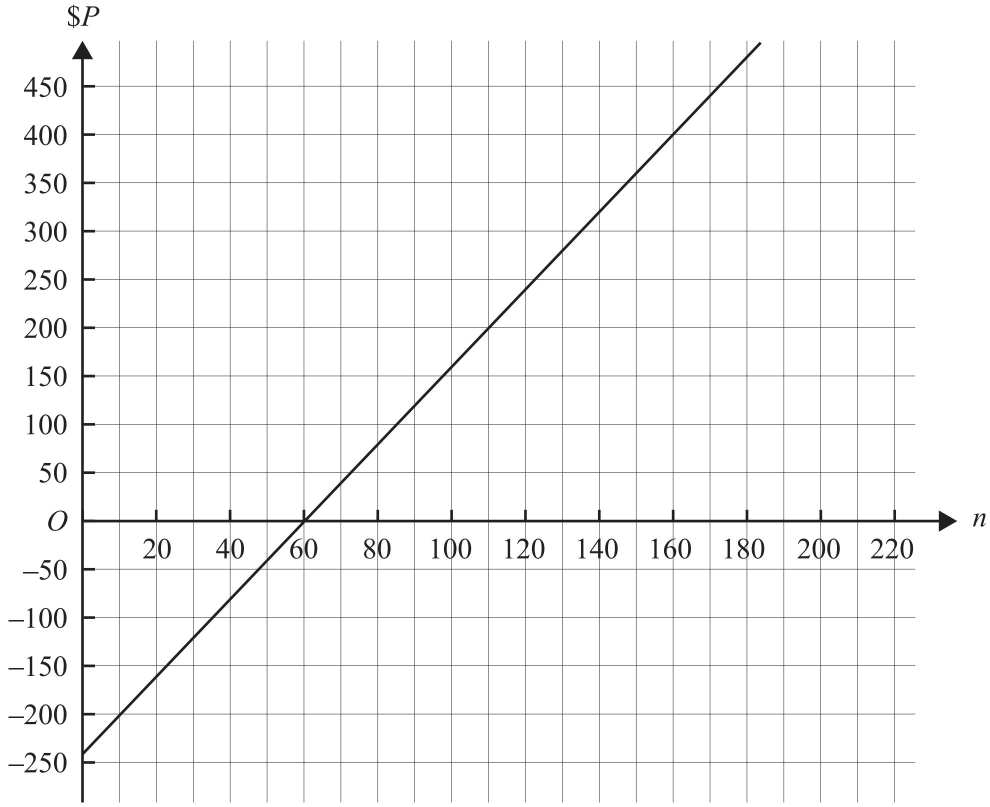
a. How many customers were seated at 4 pm? 1 mark

b. How many times did the restaurant record having 30 or more seated customers? 1 mark

Question 2 (4 marks)

The restaurant makes and sells bacon burgers.

The profit, P, in dollars, obtained from making and selling n bacon burgers is shown by the line in the graph below.



a. Determine the profit obtained from making and selling 110 bacon burgers. 1 mark

b. How many bacon burgers must be made and sold to break even? 1 mark

c. The profit obtained from selling 100 bacon burgers is \$160.

The cost, C, in dollars, of making n bacon burgers is given by the equation $C = 1.5n + 240$.

Calculate the selling price of each bacon burger. 2 marks

Question 3 (2 marks)

The restaurant also sells meal packs for large groups.

The restaurant charges \$10 per pack for the first 80 packs.

For every pack beyond the first 80 packs, the price reduces to \$8 per pack.

The revenue, R, in dollars, received from selling n meal packs can be determined as follows.

$$R = \begin{cases} 10n & 0 < n \leq 80 \\ 8n + c & n > 80 \end{cases}$$

A revenue of \$960 is received from selling 100 meal packs.

a. Show that $c = 160$. 1 mark

b. What revenue will the restaurant receive from selling a single order for 130 meal packs? 1 mark

Question 4 (4 marks)

The restaurant makes and sells two types of cheeseburgers: a single cheeseburger and a triple cheeseburger.

Let x be the number of single cheeseburgers made and sold in one day.

Let y be the number of triple cheeseburgers made and sold in one day.

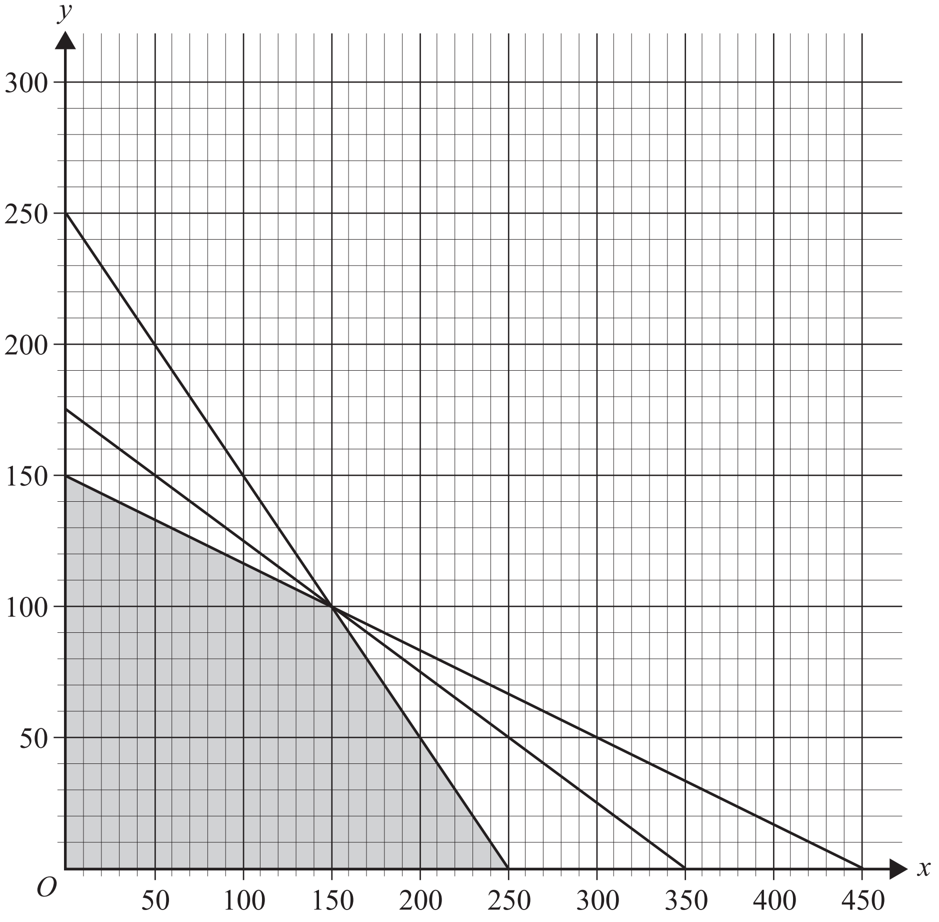
Each single cheeseburger contains one bun, one meat patty and one cheese slice.

Each triple cheeseburger contains one bun, three meat patties and two cheese slices.

The constraints on the production of cheeseburgers each day are given by Inequalities 1 to 5.

Inequality 1	$x \geq 0$
Inequality 2	$y \geq 0$
Inequality 3 (buns)	$x + y \leq 250$
Inequality 4 (meat patties)	$x + 3y \leq 450$
Inequality 5 (cheese slices)	$x + 2y \leq 350$

The graph below shows the lines that represent the boundaries of Inequalities 1 to 5. The feasible region has been shaded.



a. On Saturday, 100 single cheeseburgers were sold.

What is the maximum number of triple cheeseburgers that could have been sold on the same day? 1 mark

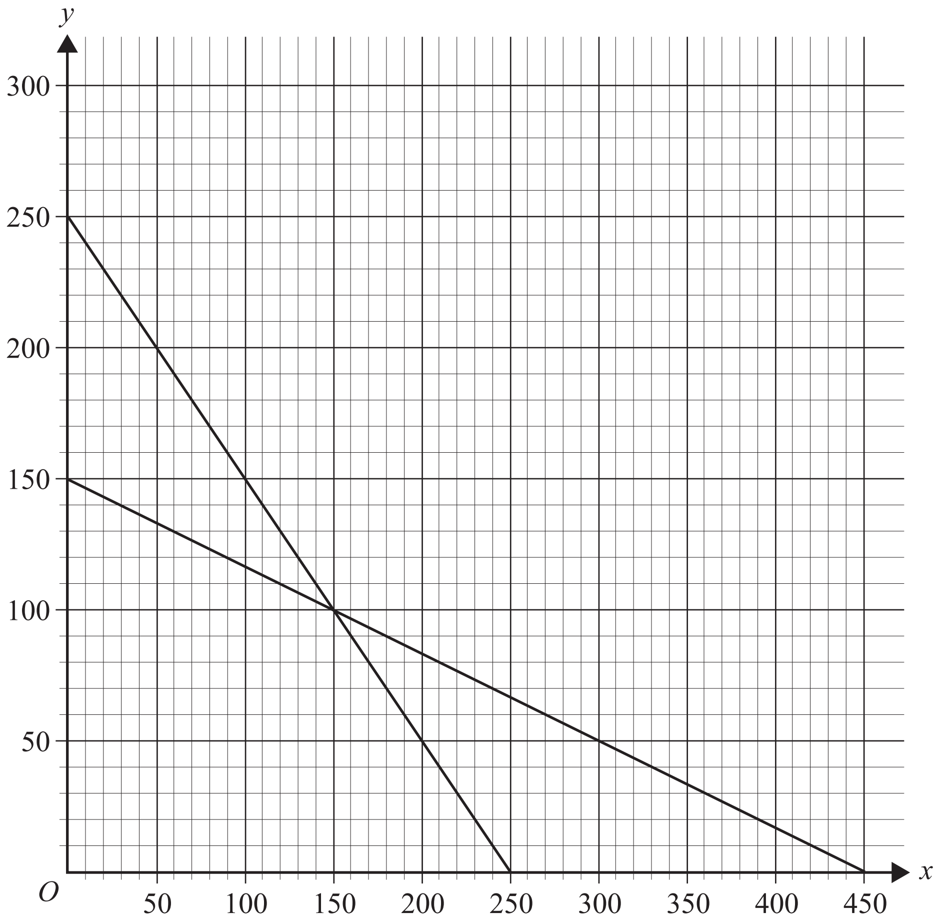
The profit for one single cheeseburger is \$1.50 and the profit for one triple cheeseburger is \$3.00

b. How many single cheeseburgers and how many triple cheeseburgers must the restaurant sell in a day in order to maximise profit? 1 mark

On Sunday, 30 cheese slices were found to be mouldy and could not be used.
This changed Inequality 5 to

$$x + 2y \leq 320$$

The graph below shows the lines that represent the boundaries of Inequalities 1 to 4.



c. Sketch the line $x + 2y = 320$ on the **graph above**. 1 mark

(*Answer on the graph above.*)

d. The maximum profit possible on this Sunday was \$480.

Calculate the **minimum** total number of cheeseburgers that need to be sold to make this profit. 1 mark

END OF QUESTION AND ANSWER BOOK

VICTORIAN CURRICULUM
AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2018

FURTHER MATHEMATICS

Written examination 2

FORMULA SHEET

Instructions

This formula sheet is provided for your reference.
A question and answer book is provided with this formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Further Mathematics formulas

Core – Data analysis

standardised score	$z = \dfrac{x - \bar{x}}{s_x}$
lower and upper fence in a boxplot	lower $Q_1 - 1.5 \times IQR$ upper $Q_3 + 1.5 \times IQR$
least squares line of best fit	$y = a + bx$, where $b = r\dfrac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$
residual value	residual value = actual value – predicted value
seasonal index	seasonal index = $\dfrac{\text{actual figure}}{\text{deseasonalised figure}}$

Core – Recursion and financial modelling

first-order linear recurrence relation	$u_0 = a, \quad u_{n+1} = bu_n + c$
effective rate of interest for a compound interest loan or investment	$r_{effective} = \left[\left(1 + \dfrac{r}{100n}\right)^n - 1\right] \times 100\%$

Module 1 – Matrices

determinant of a 2 × 2 matrix	$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \quad \det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$
inverse of a 2 × 2 matrix	$A^{-1} = \dfrac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, where $\det A \neq 0$
recurrence relation	$S_0 =$ initial state, $S_{n+1} = T\,S_n + B$

Module 2 – Networks and decision mathematics

Euler's formula	$v + f = e + 2$

Module 3 – Geometry and measurement

area of a triangle	$A = \frac{1}{2}bc\sin(\theta^\circ)$
Heron's formula	$A = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{1}{2}(a+b+c)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule	$a^2 = b^2 + c^2 - 2bc\cos(A)$
circumference of a circle	$2\pi r$
length of an arc	$r \times \frac{\pi}{180} \times \theta^\circ$
area of a circle	πr^2
area of a sector	$\pi r^2 \times \frac{\theta^\circ}{360}$
volume of a sphere	$\frac{4}{3}\pi r^3$
surface area of a sphere	$4\pi r^2$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a prism	area of base × height
volume of a pyramid	$\frac{1}{3}$ × area of base × height

Module 4 – Graphs and relations

gradient (slope) of a straight line	$m = \frac{y_2 - y_1}{x_2 - x_1}$
equation of a straight line	$y = mx + c$

END OF FORMULA SHEET