VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education 2022

FURTHER MATHEMATICS

Written examination 1

Friday 28 October 2022

Reading time: 2.00 pm to 2.15 pm (15 minutes)
Writing time: 2.15 pm to 3.45 pm (1 hour 30 minutes)

MULTIPLE-CHOICE QUESTION BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of modules*	*Number of modules to be answered*	*Number of marks*
A – Core	24	24			24
B – Modules	32	16	4	2	16
					Total 40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question book of 34 pages
- Formula sheet
- Answer sheet for multiple-choice questions
- Working space is provided throughout the book.

Instructions
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

At the end of the examination
- You may keep this question book and the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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SECTION A – Core

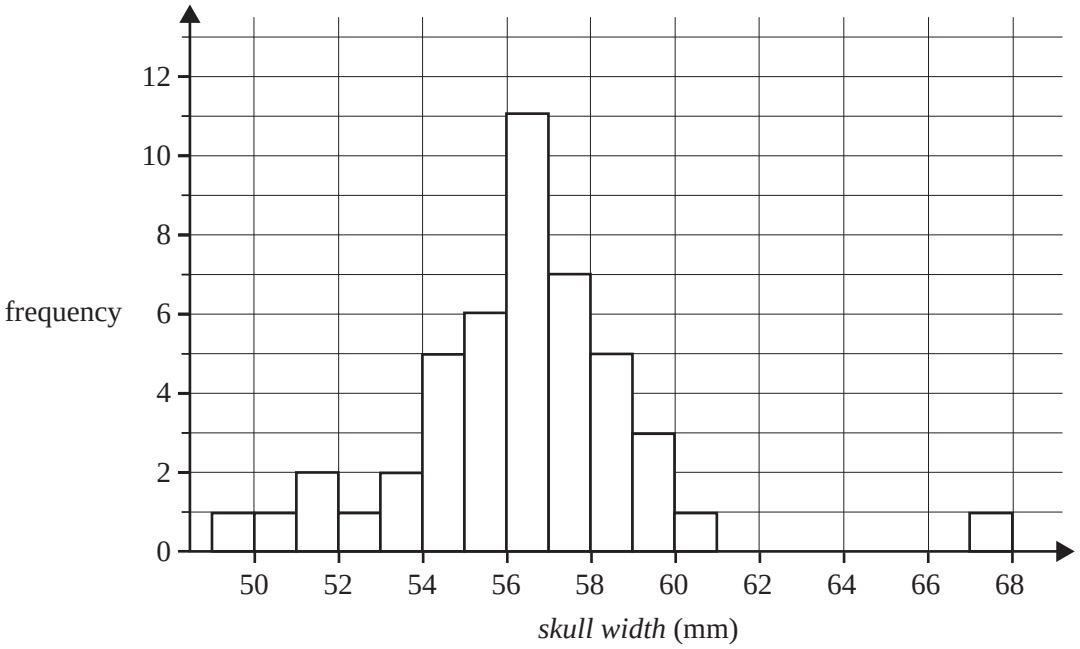
Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1; an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Data analysis

Use the following information to answer Questions 1–3.

The histogram below displays the distribution of *skull width*, in millimetres, for 46 female possums.



Data: adapted from DB Lindenmayer et al., 'Morphological variation among populations of the mountain brushtail possum, *Trichosurus caninus* Ogilby (Phalangeridae: Marsupialia)', *Australian Journal of Zoology*, 43(5), 1995, p. 453

Question 1

The shape of the distribution is best described as

A. negatively skewed.
B. approximately symmetric.
C. negatively skewed with a possible outlier.
D. positively skewed with a possible outlier.
E. approximately symmetric with a possible outlier.

Question 2

The percentage of the 46 possums with a *skull width* of less than 55 mm is closest to

A. 12%
B. 26%
C. 39%
D. 61%
E. 74%

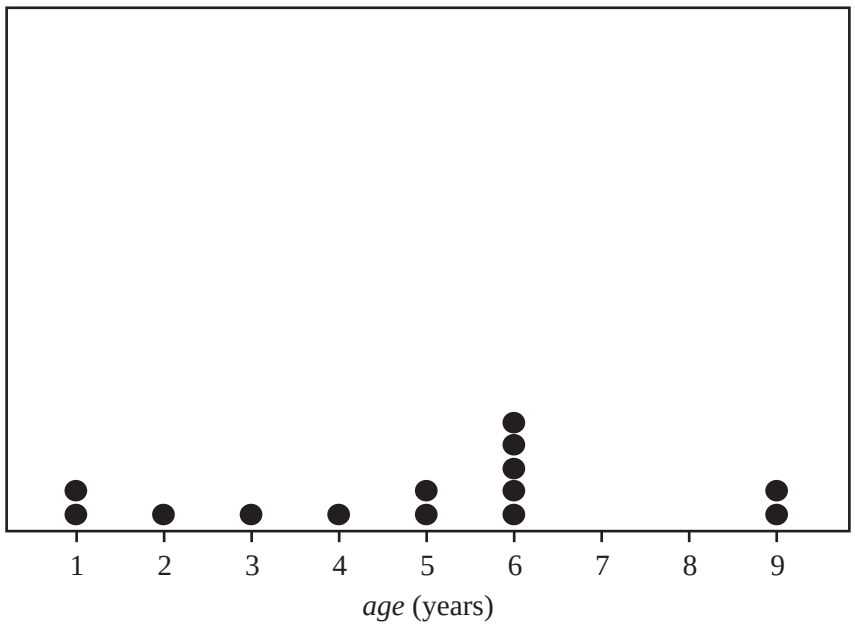
Question 3

The third quartile (Q_3) for this distribution, in millimetres, could be

A. 55.8
B. 56.2
C. 56.9
D. 57.7
E. 58.3

Question 4

The *age*, in years, of a sample of 14 possums is displayed in the dot plot below.



The mean and the standard deviation of *age* for this sample of possums are closest to

A. mean = 4.25 standard deviation = 2.6
B. mean = 4.8 standard deviation = 2.4
C. mean = 4.8 standard deviation = 2.5
D. mean = 4.9 standard deviation = 2.4
E. mean = 4.9 standard deviation = 2.5

Question 5

The possum population of a large city park is 2498.

The body lengths of this species of possum are known to be approximately normally distributed with a mean of 88 cm and a standard deviation of 4 cm.

Using the 68–95–99.7% rule, the number of possums in this park with a body length between 84 cm and 96 cm is closest to

A. 2036

B. 2043

C. 2047

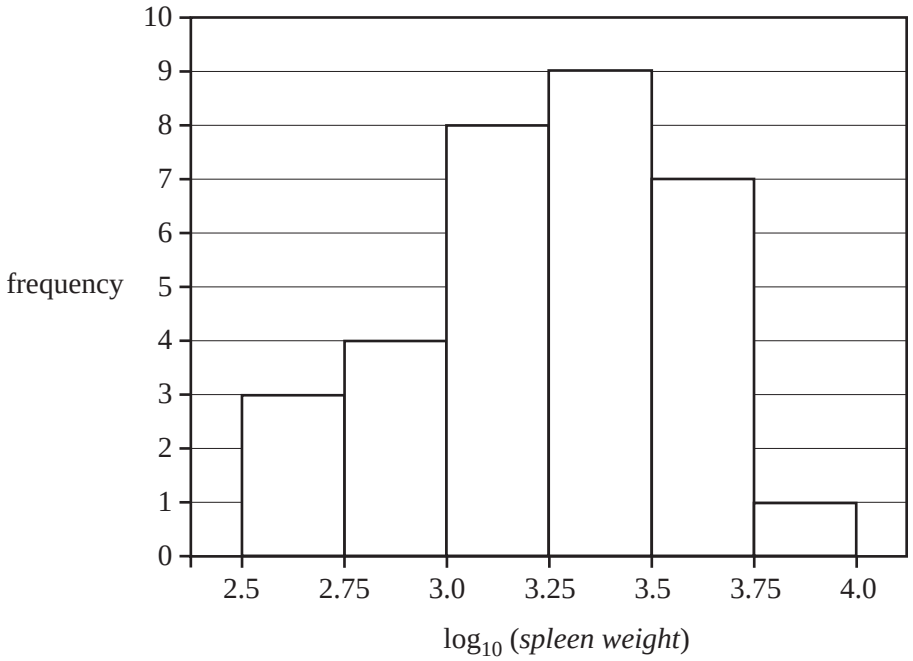
D. 2105

E. 2156

Question 6

The histogram below displays the distribution of *spleen weight* for a sample of 32 seals.

The histogram has a $\log_{10}$ scale.



Data: adapted from CL Stewardson et al., 'Gross and microscopic visceral anatomy of the male Cape fur seal, *Arctocephalus pusillus pusillus* (Pinnipedia: Otariidae), with reference to organ size and growth', *Journal of Anatomy*, 195, 1999, p. 240

The number of seals in this sample with a spleen weight of 1000 g or more is

A. 7

B. 8

C. 17

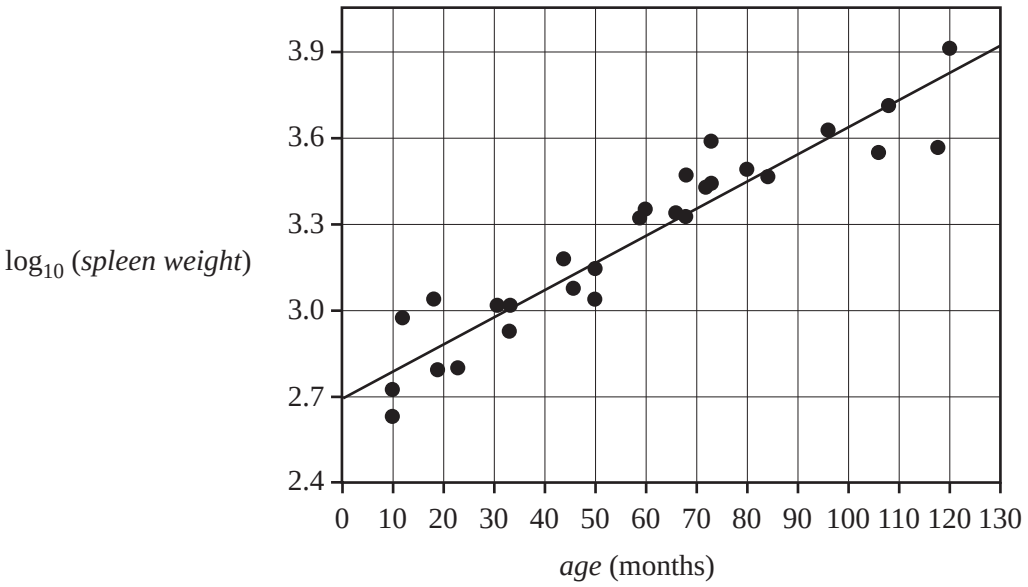
D. 25

E. 27

Use the following information to answer Questions 7 and 8.

The association between the weight of a seal's spleen, *spleen weight*, in grams, and its *age*, in months, for a sample of seals is non-linear.

This association can be linearised by applying a $\log_{10}$ transformation to the variable *spleen weight*.



Data: adapted from CL Stewardson et al., 'Gross and microscopic visceral anatomy of the male Cape fur seal, *Arctocephalus pusillus pusillus* (Pinnipedia: Otariidae), with reference to organ size and growth', *Journal of Anatomy*, 195, 1999, p. 240

The equation of the least squares line for this scatterplot is

$$\log_{10}(\textit{spleen weight}) = 2.698 + 0.009434 \times \textit{age}$$

Question 7

The equation of the least squares line predicts that, on average, for each one-month increase in the *age* of the seals, the increase in the value of $\log_{10}$ (*spleen weight*) is

A. 0.009434

B. 0.01000

C. 1.020

D. 2.698

E. 5.213

Question 8

Using the equation of the least squares line, the predicted *spleen weight* of a 30-month-old seal, in grams, is closest to

A. 3

B. 511

C. 772

D. 957

E. 1192

Use the following information to answer Questions 9–11.

Table 1 summarises the results of a study that compared the effectiveness of individual and group instruction (*instructional method*) when training future basketball referees.

Table 1

	Instructional method	
Test grade	**Individual**	**Group**
A (85% or above)	10	18
B (75–84%)	35	30
C (65–74%)	30	24
D (50–64%)	28	48
E (less than 50%)	12	6
Total	115	126

In this table, *test grade* is the response variable and *instructional method* is the explanatory variable.

Question 9

The variables *test grade* (A, B, C, D, E) and *instructional method* (individual, group) are

A. a numerical and a categorical variable respectively.

B. both nominal variables.

C. a nominal and an ordinal variable respectively.

D. both ordinal variables.

E. an ordinal and a nominal variable respectively.

Question 10

Of the students who received an A grade, the percentage who were instructed individually is closest to

A. 9%

B. 22%

C. 36%

D. 56%

E. 64%

Question 11

To become a qualified referee, a grade of A or B on the test is required. Those who receive a C, a D or an E will not qualify.

Using column percentages, a new two-way percentage frequency table is constructed from the data in Table 1.

In this new table, *qualified to be a referee* (yes, no) is the response variable and *instructional method* (individual, group) is the explanatory variable.

Which one of the following tables correctly displays the data from Table 1?

A.

	***Instructional method* (%)**	
Qualified to be a referee	**Individual**	**Group**
yes (A or B grade)	35	38
no (C, D or E grade)	65	62

B.

	***Instructional method* (%)**	
Qualified to be a referee	**Individual**	**Group**
yes (A or B grade)	39	38
no (C, D or E grade)	61	62

C.

	***Instructional method* (%)**	
Qualified to be a referee	**Individual**	**Group**
yes (A or B grade)	39	43
no (C, D or E grade)	61	57

D.

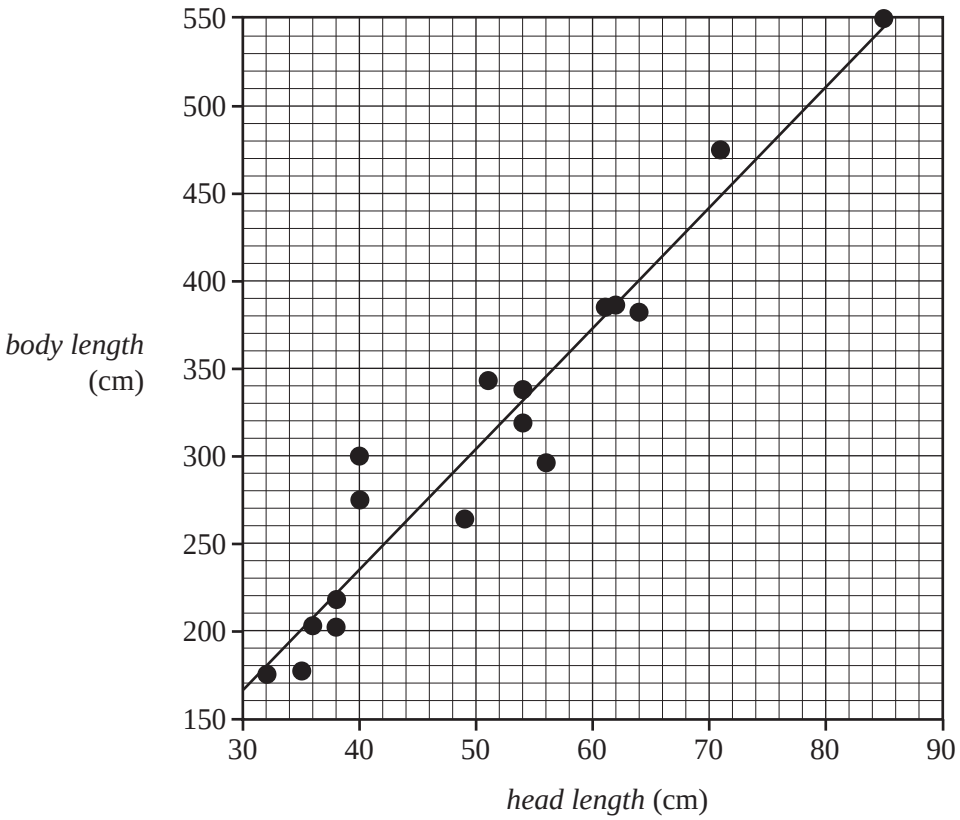
	***Instructional method* (%)**	
Qualified to be a referee	**Individual**	**Group**
yes (A or B grade)	61	62
no (C, D or E grade)	39	38

E.

	***Instructional method* (%)**	
Qualified to be a referee	**Individual**	**Group**
yes (A or B grade)	65	57
no (C, D or E grade)	35	43

Use the following information to answer Questions 12–14.

The scatterplot below displays the *body length*, in centimetres, of 17 crocodiles, plotted against their *head length*, in centimetres. A least squares line has been fitted to the scatterplot. The explanatory variable is *head length*.



Data: adapted from Data and Story Library (DASL), 'Crocodile_lengths', <https://dasl.datadescription.com/datafile/crocodile-lengths/>

Question 12

The equation of the least squares line is closest to

A. *head length* = −40 + 7 × *body length*

B. *body length* = −40 + 7 × *head length*

C. *head length* = 168 + 7 × *body length*

D. *body length* = 168 − 40 × *head length*

E. *body length* = 7 + 168 × *head length*

Question 13

The median *head length* of the 17 crocodiles, in centimetres, is closest to

A. 49

B. 51

C. 54

D. 300

E. 345

Question 14

The correlation coefficient r is equal to 0.963

The percentage of variation in *body length* that is **not** explained by the variation in *head length* is closest to

A. 0.9%

B. 3.7%

C. 7.3%

D. 92.7%

E. 96.3%

Question 15

The daily number of cups of coffee sold by a food truck over a three-week period is shown in the table below.

Week	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
1	123	91	120	176	145	99	187
2	98	104	145	163	134	128	206
3	125	134	128	187	156	102	179

The six-mean smoothed number of cups of coffee, with centring, sold on Thursday in Week 2 is closest to

A. 127

B. 138

C. 147

D. 155

E. 163

Question 16

The seasonal index for sales of sunscreen in summer is 1.25

To correct for seasonality, the actual sunscreen sales for summer should be

A. reduced by 20%

B. reduced by 25%

C. reduced by 80%

D. increased by 20%

E. increased by 25%

Recursion and financial modelling

Question 17

A sequence of numbers is generated by the recurrence relation shown below.

$$R_0 = 2, \quad R_{n+1} = 2 - R_n$$

The value of R_2 is

A. –4
B. –2
C. 0
D. 2
E. 4

Use the following information to answer Questions 18 and 19.

The balance of a loan, V_n, in dollars, after n months is modelled by the recurrence relation

$$V_0 = 400\,000, \quad V_{n+1} = 1.003V_n - 2024$$

Question 18

The balance of the loan first falls below $398 000 after how many months?

A. 1
B. 2
C. 3
D. 4
E. 5

Question 19

With a small change to the final payment, the loan is expected to be repaid in full in

A. 25 years.
B. 26 years.
C. 28 years.
D. 29 years.
E. 30 years.

Question 20

Nidhi owns equipment that is used for 10 hours per day for all 365 days of the year.

The value of the equipment is depreciated by Nidhi using the unit cost method.

The value of the equipment, E_n, in dollars, after n years can be modelled by the recurrence relation

$$E_0 = 100\,000, \quad E_{n+1} = E_n - 5475$$

The value of the equipment is depreciated by

A. $1.50 per hour.
B. $10 per hour.
C. $15 per hour.
D. $1.50 per day.
E. $10 per day.

Question 21

Consider the following four statements regarding nominal and effective interest rates as they apply to compound interest investments and loans:

- An effective interest rate is the same as a nominal interest rate if interest compounds annually.
- Effective interest rates increase as the number of compounding periods per year increases.
- A nominal rate of 12% per annum is equivalent to a nominal rate of 1% per month.
- An effective interest rate can be lower than a nominal interest rate.

How many of these four statements are true?

A. 0

B. 1

C. 2

D. 3

E. 4

Question 22

Tim deposited \$6000 into an investment account earning compound interest calculated monthly.

A rule for the balance, T_n, in dollars, after n years is given by $T_n = 6000 \times 1.003^{12n}$.

Let R_n be a new recurrence relation that models the balance of Tim's account after n months.

This recurrence relation is

A. $R_0 = 6000, \quad R_{n+1} = R_n + 18$

B. $R_0 = 6000, \quad R_{n+1} = R_n + 36$

C. $R_0 = 6000, \quad R_{n+1} = 1.003R_n$

D. $R_0 = 6000, \quad R_{n+1} = 1.0036R_n$

E. $R_0 = 6000, \quad R_{n+1} = 1.036R_n$

Question 23

Li invests $4000 for five years at 3.88% per annum, compounding annually.

Joseph invests a sum of money for five years, which earns simple interest paid annually.

Let J_n be the value, in dollars, of Joseph's investment after n years.

The two investments will finish at the same value, rounded to the nearest cent, if Joseph's investment is modelled by which one of the following recurrence relations?

A. $J_0 = 2000, \quad J_{n+1} = J_n + 467.72$

B. $J_0 = 2500, \quad J_{n+1} = J_n + 367.72$

C. $J_0 = 3000, \quad J_{n+1} = J_n + 317.72$

D. $J_0 = 3500, \quad J_{n+1} = J_n + 267.72$

E. $J_0 = 4000, \quad J_{n+1} = J_n + 67.72$

Question 24

On 1 January 2020, Dion invested $10 500 into an investment account paying compound interest of 0.52% quarterly.

At the end of each quarter, after the interest was credited, Dion added an additional amount of money.

Let D_n represent the additional amount, in dollars, added at the end of quarter n.

This additional amount per quarter is modelled by the recurrence relation

$$D_1 = C, \quad D_{n+1} = D_n$$

The balance of Dion's investment account on 1 January 2022 was $12 700.95

The value of C is

A. $71.69

B. $215.55

C. $260.22

D. $270.15

E. $275.12

END OF SECTION A

SECTION B – Modules

Instructions for Section B

Select **two** modules and answer **all** questions within the selected modules in pencil on the answer sheet provided for multiple-choice questions.

Show the modules you are answering by shading the matching boxes on your multiple-choice answer sheet **and** writing the name of the module in the box provided.

Choose the response that is **correct** for the question.

A correct answer scores 1; an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

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Contents **Page**

Module 1 – Matrices

Before answering these questions, you must **shade** the 'Matrices' box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Use the following information to answer Questions 1 and 2.

A bike rental business rents road bikes (R) and mountain bikes (M) in three sizes: child (C), junior (J) and adult (A).

Matrix B shows the daily rental cost, in dollars, for each type of bike.

$$B = \begin{array}{c} \\ C \\ J \\ A \end{array} \begin{array}{c} \begin{array}{cc} R & M \end{array} \\ \begin{bmatrix} 80 & 95 \\ 110 & 120 \\ 120 & 135 \end{bmatrix} \end{array}$$

The element in row i and column j in matrix B is b_{ij}.

Question 1

The daily cost of renting an adult mountain bike is shown in element

A. b_{12}
B. b_{21}
C. b_{23}
D. b_{31}
E. b_{32}

Question 2

On Sundays, the business increases the daily rental price for each type of bike by 10%.

To determine the rental cost for each type of bike on a Sunday, which one of the following matrix calculations needs to be completed?

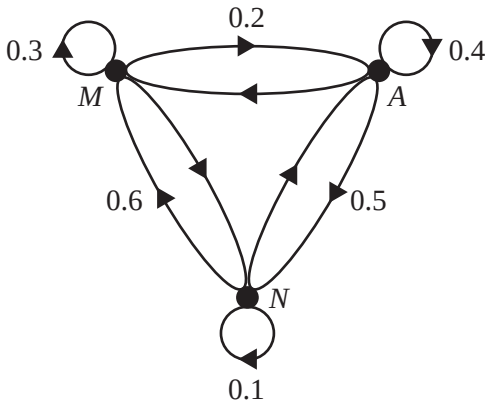
A. $0.01B$
B. $0.1B$
C. $1.01B$
D. $1.1B$
E. $11B$

Question 3

Each day, members of a swim centre can choose to attend a morning session (M), an afternoon session (A) or no session (N).

The transition diagram below shows the transition from day to day.

The transition diagram is incomplete.



Which one of the following transition matrices represents this transition diagram?

A.

$$\begin{array}{c} \\ M \\ A \\ N \end{array} \begin{array}{c} \begin{array}{ccc} M & A & N \end{array} \\ \begin{bmatrix} 0.2 & 0.1 & 0.1 \\ 0.3 & 0.5 & 0.3 \\ 0.5 & 0.4 & 0.6 \end{bmatrix} \end{array} \begin{array}{l} \\ M \\ A \\ N \end{array}$$

this day / *next day*

B.

$$\begin{array}{c} \\ M \\ A \\ N \end{array} \begin{array}{c} \begin{array}{ccc} M & A & N \end{array} \\ \begin{bmatrix} 0.3 & 0.1 & 0.6 \\ 0.5 & 0.4 & 0.3 \\ 0.2 & 0.5 & 0.1 \end{bmatrix} \end{array} \begin{array}{l} \\ M \\ A \\ N \end{array}$$

this day / *next day*

C.

$$\begin{array}{c} \\ M \\ A \\ N \end{array} \begin{array}{c} \begin{array}{ccc} M & A & N \end{array} \\ \begin{bmatrix} 0.3 & 0.1 & 0.6 \\ 0.2 & 0.4 & 0.3 \\ 0.5 & 0.5 & 0.1 \end{bmatrix} \end{array} \begin{array}{l} \\ M \\ A \\ N \end{array}$$

this day / *next day*

D.

$$\begin{array}{c} \\ M \\ A \\ N \end{array} \begin{array}{c} \begin{array}{ccc} M & A & N \end{array} \\ \begin{bmatrix} 0.3 & 0.5 & 0.2 \\ 0.1 & 0.4 & 0.7 \\ 0.6 & 0.1 & 0.1 \end{bmatrix} \end{array} \begin{array}{l} \\ M \\ A \\ N \end{array}$$

this day / *next day*

E.

$$\begin{array}{c} \\ M \\ A \\ N \end{array} \begin{array}{c} \begin{array}{ccc} M & A & N \end{array} \\ \begin{bmatrix} 0.3 & 0 & 0.6 \\ 0.2 & 0.4 & 0 \\ 0 & 0.5 & 0.1 \end{bmatrix} \end{array} \begin{array}{l} \\ M \\ A \\ N \end{array}$$

this day / *next day*

Question 4

The communication matrix below shows the communication links between five people: Steph (*S*), Tran (*T*), Ursula (*U*), Vinh (*V*) and Wanda (*W*).

$$\begin{array}{cc} & \begin{array}{ccccc} S & T & U & V & W \end{array} \\ \begin{array}{c} \\ S \\ T \\ \textit{sender} \quad U \\ V \\ W \end{array} & \begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

receiver

In this matrix:

- the '1' in row *S*, column *T* indicates that Steph can communicate directly with Tran
- the '0' in row *V*, column *W* indicates that Vinh cannot communicate directly with Wanda.

Ursula needs to communicate with Steph.

The sequence of communication links that will successfully allow Ursula to communicate with Steph is

A. *U-T-S*

B. *U-W-S*

C. *U-T-W-S*

D. *U-V-T-S*

E. *U-W-T-S*

Question 5

Matrix *E* is a 2 × 2 matrix.

Matrix *F* is a 2 × 3 matrix.

Matrix *G* is a 3 × 2 matrix.

Matrix *H* is a 3 × 3 matrix.

Which one of the following matrix products could have an inverse?

A. *EF*

B. *FH*

C. *GE*

D. *GF*

E. *HG*

Question 6

Consider the following system of simultaneous linear equations.

$$y + z = 4$$
$$x - y + z = 1$$
$$-x + y = 2$$

The solution to these simultaneous equations can be found by calculating

A. $\begin{bmatrix}4 & 1 & 2\end{bmatrix} \times \begin{bmatrix}0 & 1 & 1\\ 1 & -1 & 1\\ -1 & 1 & 0\end{bmatrix}$

B. $\begin{bmatrix}4\\ 1\\ 2\end{bmatrix} \times \begin{bmatrix}0 & 1 & 1\\ 1 & -1 & 1\\ -1 & 1 & 0\end{bmatrix}$

C. $\begin{bmatrix}4\\ 1\\ 2\end{bmatrix} \times \begin{bmatrix}1 & -1 & -2\\ 1 & -1 & -1\\ 0 & 1 & 1\end{bmatrix}$

D. $\begin{bmatrix}0 & 1 & 1\\ 1 & -1 & 1\\ -1 & 1 & 0\end{bmatrix} \times \begin{bmatrix}4\\ 1\\ 2\end{bmatrix}$

E. $\begin{bmatrix}1 & -1 & -2\\ 1 & -1 & -1\\ 0 & 1 & 1\end{bmatrix} \times \begin{bmatrix}4\\ 1\\ 2\end{bmatrix}$

Question 7

Matrix K is a permutation matrix.

$$K = \begin{bmatrix}0 & 0 & 1 & 0 & 0\\ 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 1\\ 1 & 0 & 0 & 0 & 0\end{bmatrix}$$

Matrix M is a column matrix that is multiplied once by matrix K to obtain matrix P.

When matrix M is multiplied by matrix K, the element m_{31} moves to element

A. p_{11}

B. p_{21}

C. p_{31}

D. p_{41}

E. p_{51}

Question 8

Two types of computers – laptops (L) and desktops (D) – can be serviced by Henry (H), Irvine (I) or Jean (J).

Matrix N shows the time, in minutes, it takes each person to service a laptop and a desktop.

$$N = \begin{matrix} \\ H \\ I \\ J \end{matrix} \begin{matrix} \begin{matrix} L & D \end{matrix} \\ \begin{bmatrix} 18 & 8 \\ 10 & 17 \\ 12 & 9 \end{bmatrix} \end{matrix}$$

Matrix Q shows the number of laptops and desktops in four different departments: marketing (M), advertising (A), publishing (P) and editing (E).

$$Q = \begin{matrix} \\ M \\ A \\ P \\ E \end{matrix} \begin{matrix} \begin{matrix} L & D \end{matrix} \\ \begin{bmatrix} 6 & 8 \\ 4 & 7 \\ 5 & 5 \\ 10 & 12 \end{bmatrix} \end{matrix}$$

A calculation that determines the total time that it would take each of Henry, Irvine or Jean, working alone, to service all the laptops and desktops in all four departments is

A. $\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \times \left(Q \times N^T \right)$

B. $\left(Q \times N^T \right) \times \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

C. $\left(N \times Q^T \right) \times Q$

D. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times N \times Q^T$

E. $\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \times Q \times N^T \times \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

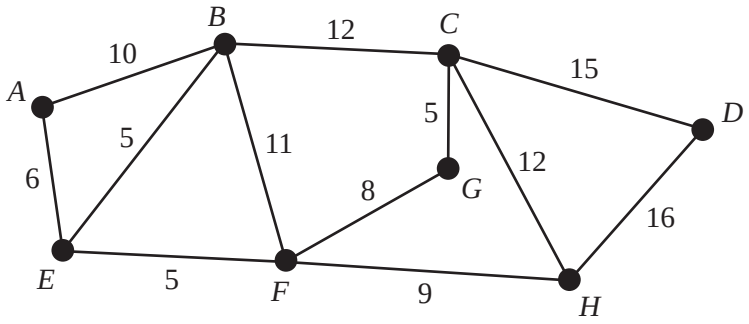
Module 2 – Networks and decision mathematics

Before answering these questions, you must **shade** the 'Networks and decision mathematics' box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

The network below shows the distances, in kilometres, along a series of roads.

The vertices *A*, *B*, *C*, *D*, *E*, *F*, *G* and *H* represent the intersections of these roads.



Prim's algorithm can be used to find the

A. critical path.

B. shortest path.

C. minimum cut.

D. minimum allocation.

E. minimum spanning tree.

Question 2

The map below shows seven countries within Central America.

A network diagram was drawn with seven vertices to represent each of the countries on the map of Central America. Edges were drawn to represent a border shared between two countries.

The number of edges that this network has is

A. 5

B. 6

C. 7

D. 8

E. 9

Question 3

An athletics club needs to select one team of four athletes.

The team is required to have one long jump, one high jump, one shot put and one javelin competitor.

The following table shows the best distances, in metres, for each athlete for each event.

Athlete	Long jump (m)	High jump (m)	Shot put (m)	Javelin (m)
Eve	4.8	1.7	13.1	40.9
Harsha	4.8	1.6	13.9	39.5
Shona	5.1	1.8	14.4	41.2
Taylor	4.8	1.7	12.8	39.8

The athletics club will allocate each athlete to one event in order to maximise the total distance that the team jumps and throws.

Which allocation of athlete to event must occur in order to maximise the total distance?

A.

long jump	high jump	shot put	javelin
Shona	Harsha	Eve	Taylor

B.

long jump	high jump	shot put	javelin
Shona	Taylor	Harsha	Eve

C.

long jump	high jump	shot put	javelin
Eve	Harsha	Taylor	Shona

D.

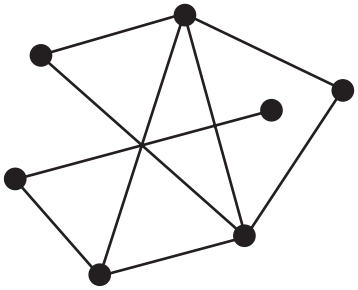
long jump	high jump	shot put	javelin
Harsha	Taylor	Shona	Eve

E.

long jump	high jump	shot put	javelin
Harsha	Taylor	Eve	Shona

Question 4

Consider the graph below.



The number of edges that need to be removed for this graph to be planar is

A. 0

B. 1

C. 2

D. 3

E. 4

Question 5

A connected graph consists of five vertices and four edges.

Which one of the following statements is **not** true?

A. The graph could be a tree.

B. The graph could be planar.

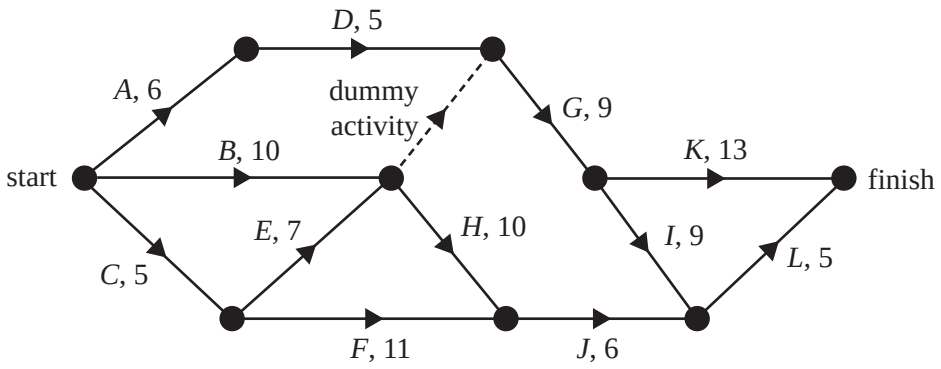
C. The graph could be bipartite.

D. The graph could contain a path.

E. The graph could contain a cycle.

Question 6

A landscaping project has 12 activities. The network below gives the time, in hours, that it takes to complete each activity.



The earliest start time, in hours, for activity *G* is

A. 10

B. 11

C. 12

D. 13

E. 14

Use the following information to answer Questions 7 and 8.

A project involves 11 activities, *A* to *K*.

The table below shows the earliest start time and duration, in days, for each activity.

The immediate predecessor(s) of each activity is also shown.

Activity	Earliest start time	Duration	Immediate predecessor(s)
A	0	6	–
B	0	7	–
C	6	10	*A*
D	6	7	*A*
E	7	8	*B*
F	15	2	*D*, *E*
G	15	2	*E*
H	17	3	*G*
I	20	6	*C*, *F*, *H*
J	17	5	*G*
K	26	2	*I*, *J*

Question 7

A directed network for this project will require a dummy activity.

The dummy activity will be drawn from the end of

A. activity *A* to the start of activity *D*.

B. activity *E* to the start of activity *F*.

C. activity *F* to the start of activity *I*.

D. activity *G* to the start of activity *H*.

E. activity *I* to the start of activity *J*.

Question 8

When this project is completed in the minimum time, the sum of all the float times, in days, will be

A. 0

B. 16

C. 18

D. 20

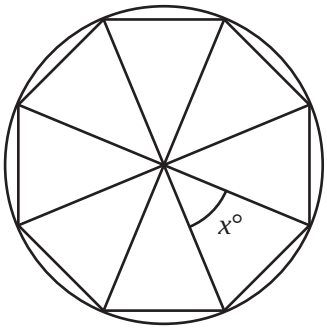
E. 28

Module 3 – Geometry and measurement

Before answering these questions, you must **shade** the 'Geometry and measurement' box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

A regular octagon is drawn inside a circle, as shown below.



The size of angle x is

A. 25°

B. 30°

C. 36°

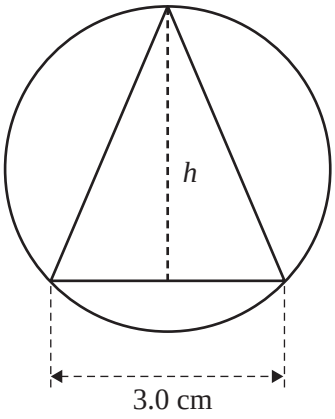
D. 45°

E. 60°

Use the following information to answer Questions 2 and 3.

An isosceles triangle is drawn within a circle, as shown in the diagram below.

The base length of the isosceles triangle is 3.0 cm and its vertical height is labelled *h*.



The area of the triangle is 5.25 cm^2.

Question 2

The value of *h*, in centimetres, is

A. 1.75

B. 2.5

C. 3.0

D. 3.5

E. 7.0

Question 3

The area of the triangle is 38.5% of the area of the circle.

The radius of the circle, in centimetres, is closest to

A. 1.04

B. 2.08

C. 3.12

D. 4.16

E. 5.20

Question 4

Three cities on the same parallel of latitude in the United States of America (USA) are Mountain Grove (37° N, 92° W), Santa Cruz (37° N, 122° W) and Williamsburg (37° N, 77° W).

Assume that a difference of 15° in longitude equates to a one-hour time difference.

Which one of the following statements is true?

A. When it is 3 pm in Mountain Grove, it is 5 pm in Williamsburg.

B. When it is 3 pm in Williamsburg, it is 5 pm in Santa Cruz.

C. When it is 3 pm in Santa Cruz, it is 6 pm in Williamsburg.

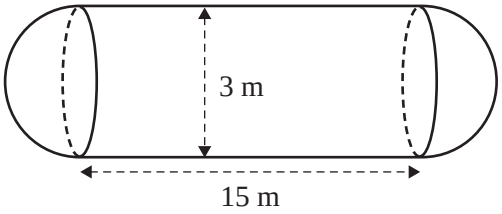
D. When it is 3 pm in Mountain Grove, it is 5 pm in Santa Cruz.

E. When it is 3 pm in Williamsburg, it is 5 pm in Mountain Grove.

Question 5

A large storage tank consists of a cylinder with a hemisphere at each end.

The cylinder has a length of 15 m and a diameter of 3 m, as shown in the diagram below.



A scale model of this storage tank is constructed.

The total length of the scale model is 300 mm.

The diameter of the scale model, in millimetres, is

A. 25

B. 30

C. 50

D. 60

E. 120

Question 6

An aeroplane completed a flight consisting of three stages.

Starting at Amberley, it flew 45 km directly due north to Beachwood.

From Beachwood, it flew 66 km on a bearing of 303° to Chalton.

From Chalton, it flew 98 km to return to Amberley.

The bearing, rounded to the nearest degree, of the trip from Chalton to Amberley, was closest to

A. 23°

B. 54°

C. 57°

D. 123°

E. 146°

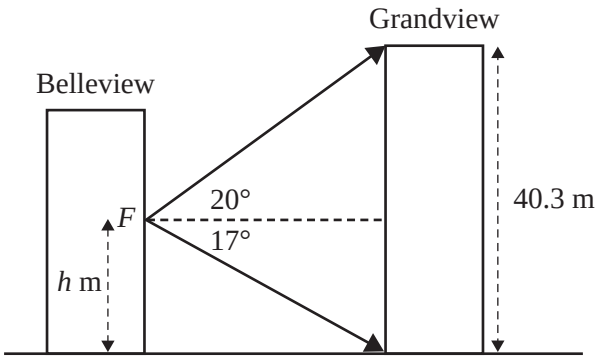
Question 7

Two residential towers, Belleview and Grandview, built on flat, level ground, face each other.

Grandview has a height of 40.3 m.

From Fran's window ledge in Belleview, *F*, she measured an angle of elevation of 20° to the top of Grandview and an angle of depression of 17° to the ground at Grandview.

This information is shown in the diagram below.



The height, *h*, from the ground to Fran's window ledge, in metres, is closest to

A. 8.8

B. 18.4

C. 21.9

D. 32.5

E. 211.6

Question 8

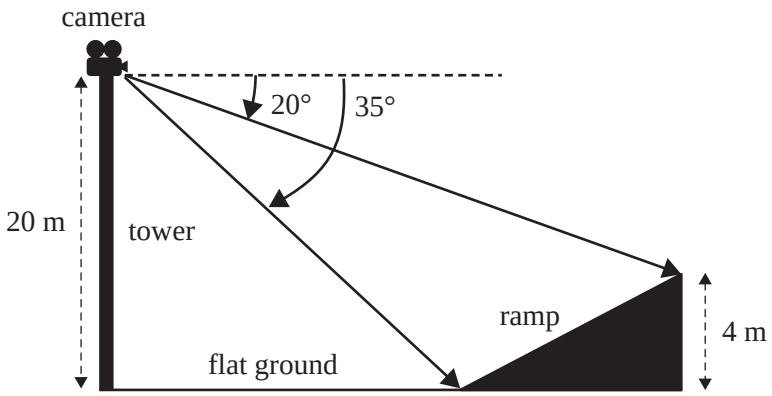
A movie camera is placed 20 m above ground level on a vertical tower.

Actors are filmed running away from the tower along a stretch of flat ground and up a ramp.

The end of the ramp is 4 m above ground level.

The angle of depression from the camera to the bottom of the ramp is 35° and the angle of depression from the camera to the top of the ramp is 20°.

This information is shown in the diagram below.



The length of the ramp, in metres, is closest to

A. 12

B. 14

C. 16

D. 35

E. 47

Module 4 – Graphs and relations

Before answering these questions, you must **shade** the 'Graphs and relations' box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

The expected height of a tree, H, in metres, n months after it was planted in a garden is modelled by the equation

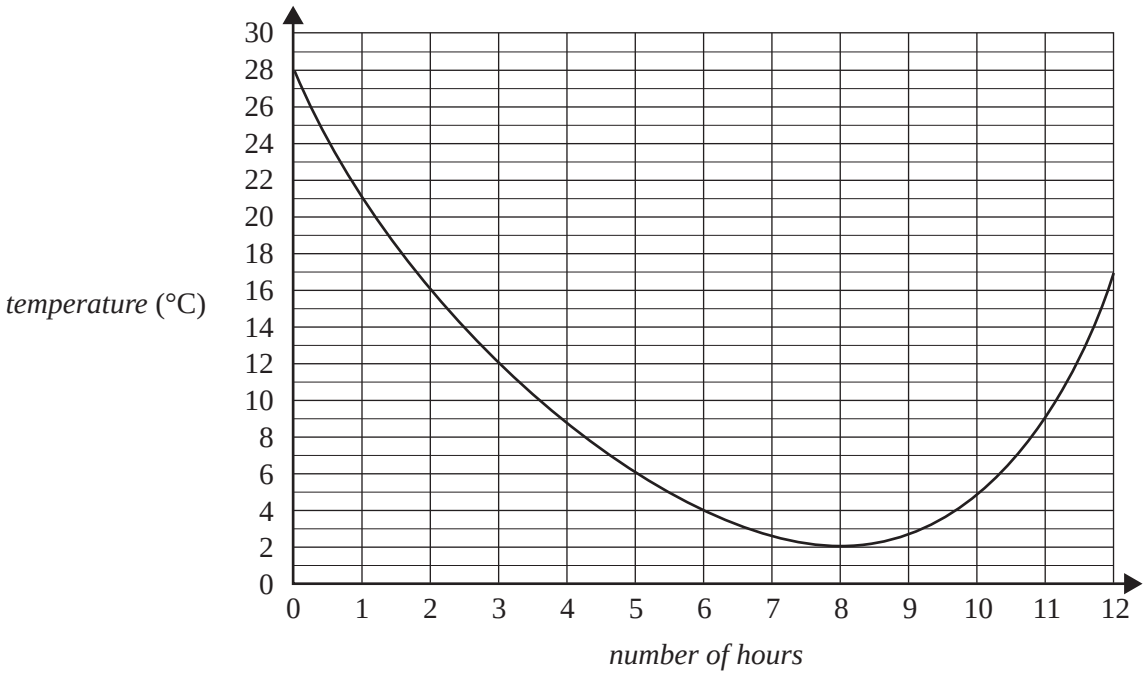
$$H = 1.5 + 0.12n$$

The expected height of the tree, in metres, eight months after it was planted is

A. 0.96
B. 1.44
C. 1.50
D. 1.628
E. 2.46

Question 2

The graph below shows the *temperature* inside a room, in degrees Celsius, over a 12-hour period.



Which one of the following statements is **not** true?

A. The minimum temperature during the 12-hour period was 2 °C.
B. The temperature decreased by 16 °C during the first three hours.
C. The temperature was 9 °C or lower for a total of seven hours.
D. The average increase in temperature from four hours to 12 hours was 1 °C per hour.
E. The average decrease in temperature during the first two hours was 5 °C per hour.

Question 3

Pedro makes and sells vases.

Each vase costs $40 to produce.

He also has one fixed cost of $1600 each month.

The selling price of each vase is $75.

Last month Pedro made and sold 150 vases.

The profit for this activity is

A. $3650

B. $5250

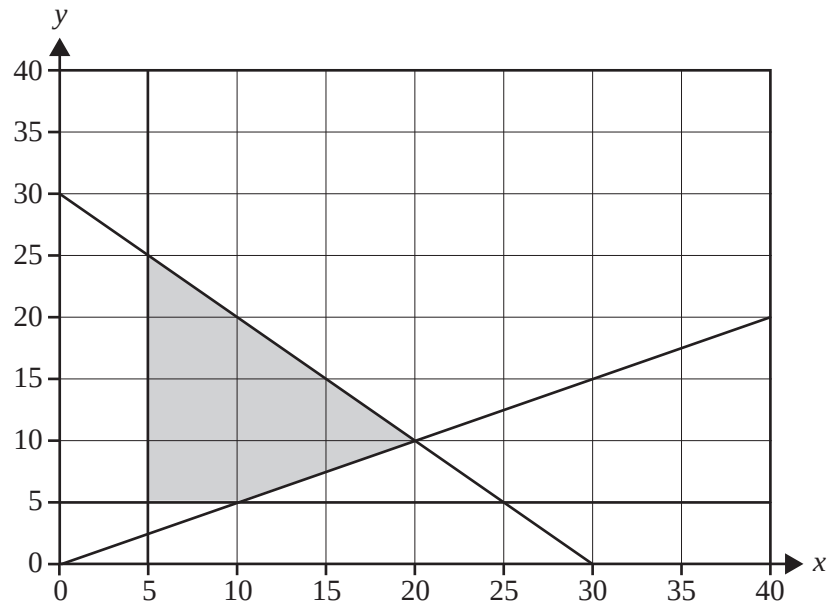
C. $6850

D. $7600

E. $11 250

Question 4

The shaded area in the graph below shows the feasible region for a linear programming problem.



The objective function is given by

$$Z = 3x - 5y$$

The maximum value of the objective function is

A. –10

B. 5

C. 10

D. 110

E. 125

Question 5

A car dealership purchased 11 sedans and seven station wagons for a total cost of $733 000.

Each sedan costs $7000 less than each station wagon.

The cost of each sedan that the car dealership purchased was

A. $31 000

B. $38 000

C. $41 000

D. $45 000

E. $52 000

Question 6

To make shades of pink paint, a painter mixes at least three drops of white paint with two drops of red paint.

Let x be the number of drops of white paint used.

Let y be the number of drops of red paint used.

An inequality representing this situation is

A. $x \geq \frac{2}{3}y$

B. $x \geq \frac{3}{2}y$

C. $y \geq \frac{2}{3}x$

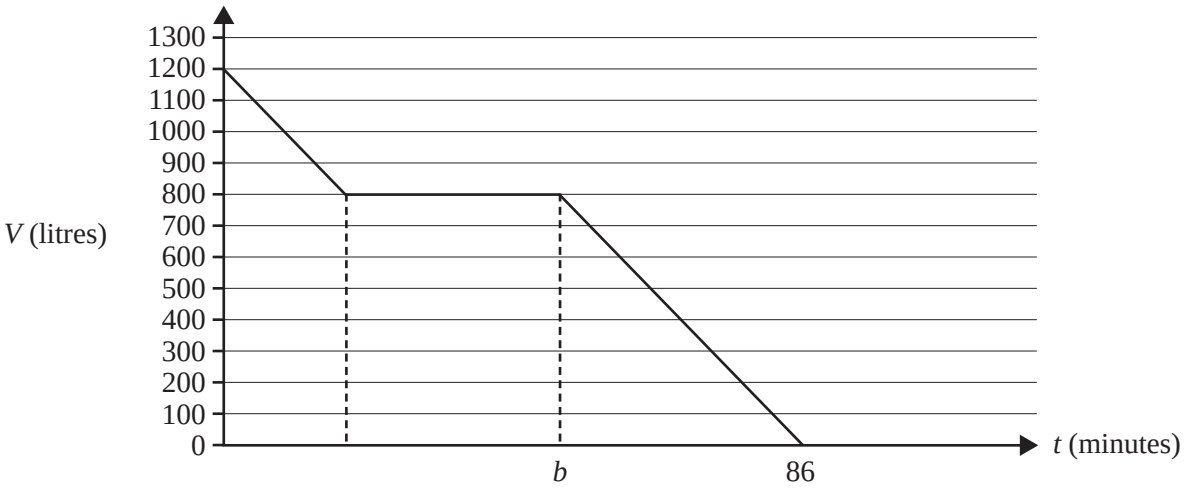
D. $y \geq \frac{3}{2}x$

E. $y \leq \frac{3}{2}x$

Question 7

A water tank with 1200 litres of water is being emptied.

The graph below shows the volume of water, V, in litres, in the tank at time t, in minutes.



The volume of water in the tank initially decreased by 400 litres at a rate of 25 litres per minute.

The volume of water did not change for the next 30 minutes.

Finally, the volume of water in the tank decreased to zero.

This whole process took 86 minutes.

The equation of the line between $t = b$ and $t = 86$ is

A. $V = -20t + 1720$

B. $V = -20t + 2150$

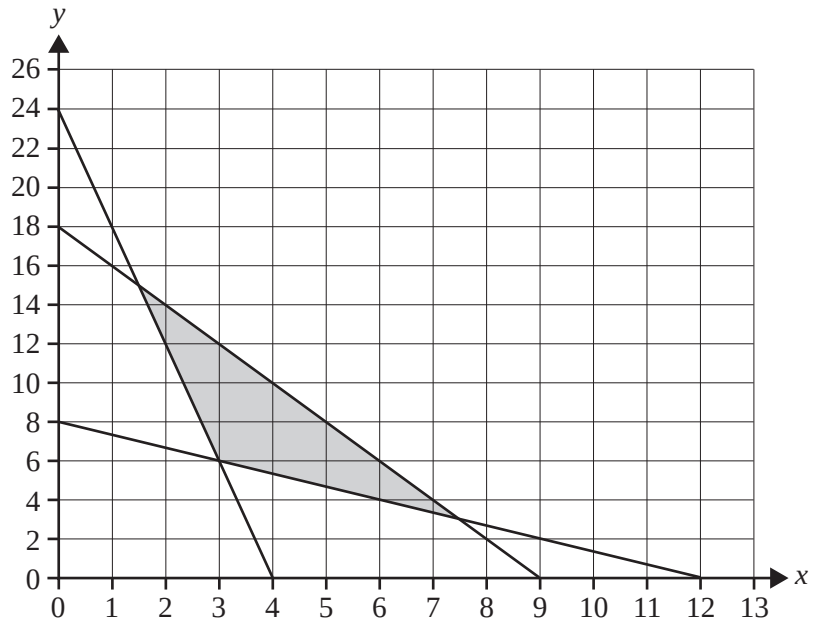
C. $V = -25t + 1720$

D. $V = -25t + 2150$

E. $V = -40t + 3440$

Question 8

The feasible region for a linear programming problem is shown shaded in the graph below.



The objective function for this problem is of the form

$$Z = ax + by \quad \text{where } a > 0 \text{ and } b > 0$$

x and y can only have integer values.

Which one of the following statements is true?

A. If $\frac{a}{b} = 2$, there are eight points that will maximise the value of Z.

B. There is a value for $\frac{a}{b}$ where the maximum value of Z will occur only at the point (3, 10).

C. If $\frac{a}{b} > 2$, then the maximum value of Z will occur at the point (2, 14).

D. If $\frac{a}{b} > 6$, then the minimum value of Z will occur at the point (2, 12).

E. For all values of $\frac{a}{b}$, the minimum value of Z will always occur at the point (3, 6).

END OF MULTIPLE-CHOICE QUESTION BOOK

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2022

FURTHER MATHEMATICS

Written examination 1

FORMULA SHEET

Instructions

This formula sheet is provided for your reference.
A multiple-choice question book is provided with this formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Further Mathematics formulas

Core – Data analysis

standardised score	$z = \dfrac{x - \bar{x}}{s_x}$
lower and upper fence in a boxplot	lower $Q_1 - 1.5 \times IQR$ upper $Q_3 + 1.5 \times IQR$
least squares line of best fit	$y = a + bx$, where $b = r\dfrac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$
residual value	residual value = actual value – predicted value
seasonal index	seasonal index = $\dfrac{\text{actual figure}}{\text{deseasonalised figure}}$

Core – Recursion and financial modelling

first-order linear recurrence relation	$u_0 = a$, $u_{n+1} = bu_n + c$
effective rate of interest for a compound interest loan or investment	$r_{effective} = \left[\left(1 + \dfrac{r}{100n}\right)^n - 1\right] \times 100\%$

Module 1 – Matrices

determinant of a 2 × 2 matrix	$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$
inverse of a 2 × 2 matrix	$A^{-1} = \dfrac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, where $\det A \neq 0$
recurrence relation	S_0 = initial state, $S_{n+1} = TS_n + B$

Module 2 – Networks and decision mathematics

Euler's formula	$v + f = e + 2$

Module 3 – Geometry and measurement

area of a triangle	$A = \frac{1}{2}bc\sin(\theta^\circ)$
Heron's formula	$A = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{1}{2}(a+b+c)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule	$a^2 = b^2 + c^2 - 2bc\cos(A)$
circumference of a circle	$2\pi r$
length of an arc	$r \times \frac{\pi}{180} \times \theta^\circ$
area of a circle	πr^2
area of a sector	$\pi r^2 \times \frac{\theta^\circ}{360}$
volume of a sphere	$\frac{4}{3}\pi r^3$
surface area of a sphere	$4\pi r^2$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a prism	area of base × height
volume of a pyramid	$\frac{1}{3}$ × area of base × height

Module 4 – Graphs and relations

gradient (slope) of a straight line	$m = \frac{y_2 - y_1}{x_2 - x_1}$
equation of a straight line	$y = mx + c$

END OF FORMULA SHEET