VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

2008 Further Mathematics GA 2: Written examination 1

GENERAL COMMENTS

As in 2007, the majority of students appeared to be well prepared for examination 1 in 2008, with the average marks for the Core – Data analysis and each of the modules exceeding 50 per cent. The number of students who sat for Further Mathematics examination 1 in 2008 was 25 769 compared to 25 644 in 2007.

SPECIFIC INFORMATION

The tables below indicate the percentage of students who chose each option. The correct answer is indicated by shading.

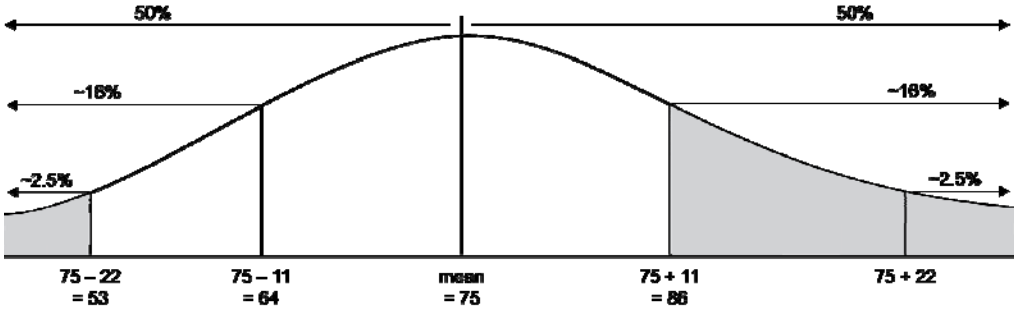
Section A
Core: Data analysis

Question	% A	% B	% C	% D	% E	% No Answer
1	1	2	14	82	2	0
2	0	1	10	6	83	0
3	13	42	37	6	2	0
4	53	6	18	15	8	0
5	15	15	60	5	5	0
6	10	71	11	5	2	0
7	22	13	15	44	5	0
8	25	10	62	2	1	0
9	2	10	55	11	21	0
10	18	13	6	6	58	0
11	5	2	4	50	39	0
12	6	16	56	14	7	0
13	23	7	5	63	2	0

Most questions on the Core material were answered correctly by more than 50 per cent of students. However, Questions 3, 7 and 11 were not well done.

From the box plot for Question 3 it could be seen that a time of 90 seconds roughly corresponded to the third quartile in the time distribution. Thus, the number of customers who spent more than 90 seconds moving along the aisle was around 25% of 79, or 20 customers (option B) and 42 per cent of students gave this correctly reasoned response. However, 37 per cent of students did not realise that the outliers were already accounted for in determining the top 25% of data values, and incorrectly chose option C.

Question 7 required two applications of the 68-95-99.7% rule to obtain the correct answer. One application to determine the percentage of students with pulse rates less than 53 beats/minutes (2.5%) and the second to determine the percentage of students with pulse rates greater than 86 beats/minutes (16%). The percentage of students with pulse rates less than 53 beats/minutes or greater than 86 beats/minutes is then the sum of these two values (18.5% – option D) and 44 per cent of students gave this correct response. In answering any question requiring the use of the 68-95-99.7% rule, a useful strategy is to first draw a normal curve and shade in the required area(s) as defined by the problem statement. This was an essential first step in answering Question 7.



standard deviation = 11

shaded region = less than 53 + greater than 86
≈ 2.5% + 16%
≈ 18.5%

Question 11 required students to choose the correct qualitative description of a time series plot from five alternatives. Only 39 per cent of students chose the correct alternative, 'an increasing trend only' (option E). Fifty per cent of students were apparently distracted by the two or three relatively large random fluctuations in monthly values, and incorrectly concluded that there was evidence of seasonality in addition to the increasing trend (option D).

Section B
Module 1: Number patterns and applications

Question	% A	% B	% C	% D	% E	% No Answer
1	2	2	3	5	87	0
2	2	2	2	93	1	0
3	4	77	12	5	1	0
4	3	12	10	12	61	0
5	3	60	18	12	5	1
6	38	1	2	57	1	0
7	75	6	4	11	4	0
8	6	8	24	45	15	1
9	16	16	13	41	13	1

This module was well answered with the notable exception of Question 9. Only 41 per cent of students obtained the correct answer to this question (48 060 litres – option D).

One solution strategy is to set up a difference equation to model this situation and use it to generate the sequence of terms representing the amount of water in the pool at the start of each week up to week 5.

Let V_n be the volume of water in the pool at the start of week n, then

$V_{n+1} = 0.98V_n + 500$

Given $V_1 = 50\,000$, then

$V_2 = 49\,500$

$V_3 = 49\,010$

$V_4 = 48\,529.8$

$V_5 = 48\,059.204$

Thus, the amount of water in the pool at the start of week 5 is 48 060 litres correct to the nearest litre (option D).

Module 2: Geometry and trigonometry

Question	% A	% B	% C	% D	% E	% No Answer
1	2	91	5	1	2	0
2	30	8	10	43	10	0
3	5	5	8	68	14	0
4	71	5	10	6	7	0
5	41	7	7	18	27	0
6	6	30	11	39	14	0
7	70	9	11	8	2	0
8	3	4	19	11	63	0
9	11	12	20	38	19	1

Questions in this module were either very well answered or poorly answered. Questions revealing significant student weaknesses were Questions 2, 5, 6 and 9.

Question 2 was a routine bearings question, yet only 43 per cent of students correctly chose 300° (option D). Thirty per cent of students chose 60° (option A). This suggests that a significant number of students do not to understand how bearings are determined. More work on the fundamentals is required.

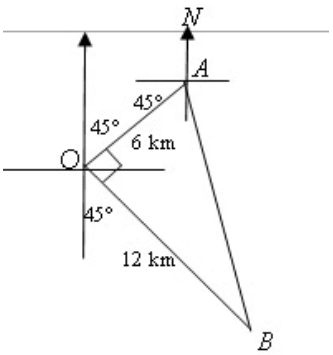
Question 5 assessed students' ability to recognise a situation that required the use of the sine rule and to then perform a rudimentary simplification. Only 41 per cent of students were able to successfully arrive at the correct answer to this question (option A). A further 27 per cent of students chose option E, suggesting that many students were able to recognise that an application of the sine rule was required but misapplied the rule or merely chose the response that appeared correct rather than working it out for themselves.

In Question 6, students were asked to determine the total surface area of a tent with semicircular ends. It was explicitly stated in the question that the base of the tent was to be included in the calculation. Despite this, 30 per cent of students apparently ignored this instruction and incorrectly chose the surface area of the tent excluding the base (option D). This highlights the need for students to carefully read a question before proceeding with their calculations.

Question 9 was correctly answered by only 38 per cent of students. The key to answering this question was for students to draw a clear diagram and include all of the relevant information.

The following is a possible solution strategy.

Draw a clear diagram. Note that ΔOAB is a right angled triangle.



To find the bearing of B from A, $\angle NAB$, first find $\angle OAB$

$\tan \angle OAB = \frac{12}{6} = 2$ so $\angle OAB = 63.43...°$

Then the bearing of B from A,

$\angle NAB = 270° - (45° + \angle OAB)$

$= 270° - (45° + 63.43...°)$

$= 161.56...°$ or 162° to the nearest degree (option D)

Module 3: Graphs and relations

Question	% A	% B	% C	% D	% E	% No Answer
1	2	90	5	1	2	0
2	93	3	2	1	1	0
3	6	5	70	7	11	0
4	1	5	29	47	17	0
5	14	48	12	18	7	1
6	7	10	10	65	7	1
7	11	23	50	5	10	1
8	14	17	15	46	7	1
9	17	6	17	19	41	1

This module was reasonably well answered, with the notable exception of Questions 4 and 9.

Question 4 related to the cost of parking over two days, given that the person parking had a choice of two different car parks with different fee structures. The correct answer, $15.50 (option C), was given by only 29 per cent of students. The most popular but incorrect choice, made by 47 per cent of students, was $16.00 (option C). Students giving this response failed to take into account that the person who was parking did not have to use the same car park on each occasion, although this was clearly stated in the problem statement.

Question 9 proved to be unexpectedly difficult with a success rate of only 17 per cent (option A). The key to answering this question was to recognise that not all objective functions have their maximum value at the same vertex. It is true that the objective function, $Z = x + 4y$ (option D), chosen by 47 per cent of students, had a larger value at the vertex M (20, 50) than the objective function $Z = x + y$ (option A). However, the maximum value of the objective function $Z = x + 4y$ occurs at the vertex (0, 60) not at M(20, 50). A check of the values of each objective function at each of the vertices will show that, of the objective functions given, only $Z = x + y$ (option A) has its maximum value at the vertex M.

Module 4: Business-related mathematics

Question	% A	% B	% C	% D	% E	% No Answer
1	1	7	2	2	87	0
2	10	7	9	68	5	1
3	25	47	13	5	9	1
4	15	8	62	8	6	1
5	12	52	19	10	5	1
6	51	10	18	8	11	1
7	11	25	13	24	27	1
8	8	53	17	12	9	1
9	20	9	33	17	21	1

This module was reasonably well done, with the exception of Questions 7 and 9.

Question 7 was poorly answered, with only 25 per cent of students choosing the correct option (graph B).
The key to answering this question was to realise that, for a reducing balance loan, the amount paid off the principal each year increases with time. It would appear that, by choosing either incorrect option D (24 per cent of students) or E (27 per cent of students), the majority of students confused the amount paid off the principal with the amount still owing on the loan.

In Question 9, students were asked to determine which of five given expressions, based on the compound interest formula, could be used to determine the interest earned in the fourth year by an $8000 investment. Only 33 per cent of students chose the correct option (C). Common errors included not recognising that interest was calculated quarterly (option B) or not converting the annual interest rate to a quarterly interest rate (option E).

Module 5: Networks and decision mathematics

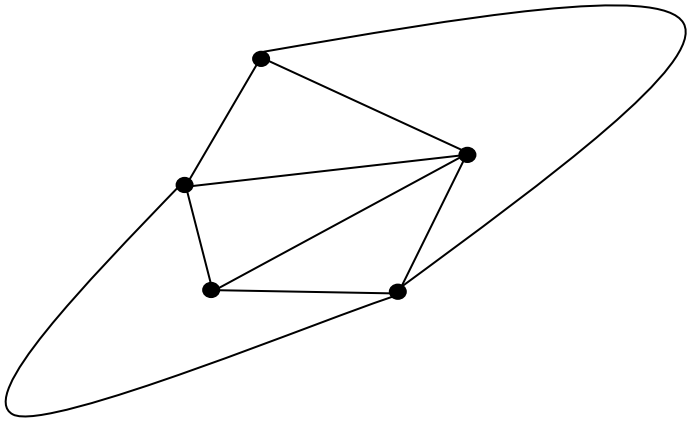
Question	% A	% B	% C	% D	% E	% No Answer
1	1	1	95	3	0	0
2	7	67	3	20	3	0
3	89	1	2	5	2	0
4	7	69	12	6	5	0
5	1	14	13	30	41	1
6	4	10	12	35	39	0
7	1	3	11	13	72	0
8	22	17	53	5	4	0
9	35	21	19	11	13	1

This module was well done with the exception of Questions 6, 7 and 9.

Question 6 was a routine question, but only 35 per cent of students correctly chose option D. The question required students to determine the capacity of a cut shown on a directed graph representing a 'flow' network. In such situations, the cut can be thought of as dividing the graph into a 'source region' and a 'sink region'. For an individual flow to contribute to the capacity of the cut the direction of the flow must be from the source region to the sink region. For the cut shown, one of the edges indicates a flow of 4 units in the reverse direction. This precludes this particular flow from contributing to the capacity of the cut. It was surprising that 39 per cent of students apparently did not understand this key idea and incorrectly chose option E.

For Question 7 only 11 per cent of students correctly chose option C. The question could have been approached in two ways. One approach was to recognise that the graph is planar (but not drawn in planar form), count the number of vertices (5) and edges (9), and then apply Euler's rule ($v + f = e + 2$) to determine that the graph had 6 faces (option C). The other approach was to realise that the regions delineated by the edges in a graph only represent faces if the graph is drawn in planar form. The graph given in the question was planar, but not drawn in planar form.

When redrawn in planar form, (one way of doing this is shown below) the graph is clearly seen to have six regions (option C).



By not redrawing the graph in planar form before trying to determine the number of faces, 72 per cent of students erroneously obtained the answer 9 (option E).

For Question 9 the original critical path is *ACFHJL*. A second critical path, *ABDJL*, emerges when the completion time of *D* is increased by 7 days (option A).

Module 6: Matrices

Question	% A	% B	% C	% D	% E	% No Answer
1	1	0	1	94	4	0
2	20	2	2	15	61	0
3	17	1	72	2	8	0
4	6	67	11	3	12	1
5	2	3	10	5	80	1
6	11	75	4	4	6	0
7	68	8	16	4	4	1
8	2	4	13	57	24	1
9	17	8	15	25	34	1

This module was well done with the exception of Question 9.

Question 9 was the third in a series of questions involving the use of a transition matrix to analyse the nesting behaviour of birds on an island. In this question, the number of birds nesting at site B in 2008 was given as 6000. The problem was to determine the total number of birds nesting on the island in the previous year, 2007, assuming that equal numbers of birds nested at each of the four sites in that year. Only 25 per cent of students chose the correct response 16 000 (option D).

The following is a possible solution strategy.

An equal but unknown number of birds nested at each of the four sites A, B, C and D in 2007.
Let x be this number.
6000 birds nested at site B in 2008
Then, using either the transition matrix or the diagram,
$6000 = x + 0.15x + 0.35x$
[Explanation: the number of birds nesting at site B in 2008 = 100% of the birds nesting at site B in 2007 + 35% of the birds nesting at site A in 2007 and 15% of the birds nesting at site C in 2007]
So $6000 = 1.5x$ or $x = 4000$ and the total number of birds nesting at all four sites in 2007 was $4 \times 4000 = 16\,000$ birds (option D).