VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

2007 Mathematical Methods & Mathematical Methods (CAS) GA 2: Exam 1

GENERAL COMMENTS

The number of students who sat for the 2007 examination was 15 770, which was 834 fewer than the 16 604 who sat in 2006. Around 8% scored 90% or more of the available marks, compared with 16% in 2006, and 2% received full marks, compared with 3% in 2006.

The overall quality of responses was not as strong as that of recent years; however, there were many very good responses and it was rewarding to see the quite substantial number who worked through the questions to obtain full marks. It was, however, disappointing to find that a significant number of students were unable to obtain any marks on the paper. There appeared to be a wider gap between those students who understood the material of the course and those who did not than in the previous year. Too many students were unable to correctly evaluate simple arithmetic calculations, including fractions, or simple algebraic expressions. Probability was an area of weakness and many students did not score well on these questions.

While there was no significant difference evident between the responses of Mathematical Methods students and Mathematical Methods (CAS) students, the mean performance on individual questions by Mathematical Methods (CAS) students was slightly better than their Mathematical Methods counterparts.

Students again need to be aware that the instruction to show appropriate working when more than one mark is available is applied rigorously when marking the papers. Failure to show appropriate working results in marks not being awarded where only an answer is given in response to the question. Similarly, incorrect, careless and sloppy notation is penalised: in particular, the dropping of '*dx*' from anti-differentiation and integration expressions. Graphs should be drawn showing correct features, such as smoothness and endpoints.

Students need also to be reminded that material from Units 1 and 2 of the study can be drawn on; Question 6, which related to basic probability, was an example of this. Question 11b. on conditional probability was another example and this area is often seen in either examination 1 or 2 questions. As noted in previous assessment reports, there continues to be difficulty with algebraic skills, setting out, graphing skills and the proper use of mathematical notation. This was evident again this year in almost every question on the paper.

SPECIFIC INFORMATION

Question 1

Marks	0	1	2	Average
%	16	21	62	1.5

$$f'(x)=\frac{3x^2\sin(x)-x^3\cos(x)}{\sin^2(x)}$$

Many students were able to obtain the correct answer; however, some students failed to gain both marks because they then incorrectly factorised the correct expression. Students should be advised against proceeding beyond what is explicitly asked for in a question. Students who attempted to use the product rule rather than the quotient rule often could not find the derivative of $(\sin(x))^{-1}$. Incorrect responses included simply differentiating the numerator and denominator separately to obtain $\frac{3x^2}{\cos(x)}$, cancelling sin(x) or adding numerator terms to give

$$f'(x)=\frac{3x^2\sin(x)+x^3\cos(x)}{\sin^2(x)}.$$

Question 2a.

Marks	0	1	2	Average
%	24	18	58	1.4

$\ln\left(2(3x+5)\right)=2$

$x=\dfrac{e^2-10}{6}$

This question was generally well answered, with many students knowing at least one logarithm law. Common mistakes included simply cancelling the logarithms, adding to obtain 3x + 7, or multiplying to get 6x + 5.

Question 2b.

Marks	0	1	2	Average
%	33	26	41	**1.1**

$g'(x)=\dfrac{\sec^2(x)}{\tan(x)}$

$g'\left(\dfrac{\pi}{4}\right)=\dfrac{\sec^2\left(\dfrac{\pi}{4}\right)}{\tan\left(\dfrac{\pi}{4}\right)}$

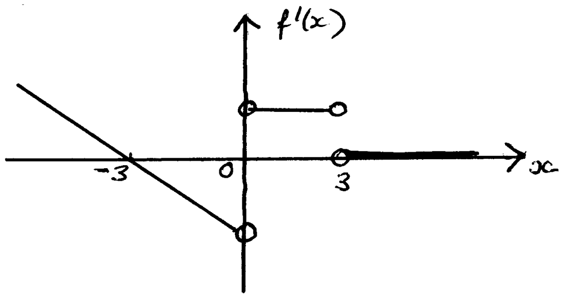
$=2$

Given that this was a standard application of the chain rule for differentiation, the question was not well answered. Some students attempted to use the product rule. Common responses were $\dfrac{1}{\tan(x)}$, $\dfrac{1}{\sec^2(x)}$ and $\dfrac{1}{\cos^2(x)}$. It was not uncommon to see students substitute $\dfrac{\pi}{4}$ into their expression but then fail to evaluate it, possibly due to not knowing the relevant exact values.

As seen in recent years, students' ability to deal effectively with both circular functions and logarithmic functions was an area of concern.

Question 3a.

Marks	0	1	2	3	Average
%	39	22	14	25	**1.2**



Many students had difficulty sketching the graph and including all the required information. Common errors were drawing the cusp point at x = 0 as a closed circle as well as the point at x= 3, not indicating endpoints or showing them as open circles at x = 0 or at x = 3, showing endpoints at extreme left and right, and excluding the section to the right of x = 3 altogether. Some students were happy to draw many-to-many relationships or inverse relations. Other students drew only the section (–3, ∞) correctly. This section could also have been drawn with a suitable curve.

Question 3b.

Marks	0	1	Average
%	55	45	**0.5**

$R\backslash\{0,3\}$

In this question, students often gave responses that were inconsistent with the graph they had drawn in part a. As always, *R* was a popular response regardless. Use of notation was slightly better than in previous years; however, union and intersection were sometimes confused as was the use of round, square or curly brackets.

Question 4

Marks	0	1	2	3	Average
%	28	20	10	42	**1.7**

$$\frac{dV}{dx} = 6x^{\frac{1}{2}}$$

$$\frac{dx}{dt} = \frac{dx}{dV} \cdot \frac{dV}{dt}$$

$$= \frac{8}{6x^{\frac{1}{2}}}$$

$$= \frac{2}{3}$$

Some students had difficulty deciding on how to label variables; *x* was sometimes interchangeable with *d* or *h* and little or no consistency was evident through the question. Most students were able to correctly differentiate *V* with respect to *x* but then either incorrectly used the chain rule or could not manage the arithmetic required. Common errors were to simply substitute 4 into *V* or to substitute into the derivative.

Question 5

Marks	0	1	2	Average
%	46	29	26	**0.8**

$$\Pr(X > 2) = \Pr(X = 3) + \Pr(X = 4)$$

$$= {}^4C_3 (0.5)^3 (0.5) + {}^4C_4 (0.5)^4 (0.5)^0$$

$$= \frac{5}{16}$$

Students generally either attempted to use the binomial distribution or a tree diagram to answer the question. Some were unable to evaluate $\left(\frac{1}{2}\right)^4 = \frac{1}{16}$ or the binomial coefficients. Students should know that ${}^nC_n = 1$ and ${}^nC_{n-1} = n$. Notation was poor and often made it difficult to ascertain if the student really understood what was required. Common responses were $0.5^2 + 0.5^3 + 0.5^4$, or a combinations of such, and incomplete tree diagrams. Probabilities greater than 1 were also seen.

Question 6

Students generally either did very well on this question or not well at all. For a basic probability question it was very poorly done. Probabilities greater than 1 were seen too often and arithmetic was again an issue for some.

Question 6a.

Marks	0	1	Average
%	69	31	**0.3**

$$\Pr(A' \cap B) = PR(B) - \Pr(A \cap B) = \frac{5}{24}$$

Students who used a Venn diagram or Karnaugh map usually obtained the correct answer. Many students either just added or multiplied $\Pr(\mathrm{A}')$ and $\Pr(\mathrm{B})$. Some students attempted unsuccessfully to develop a formula.

Question 6b.

Marks	0	1	Average
%	77	23	**0.2**

$\Pr(A' \cap B) = \Pr(B) = \frac{1}{3}$

A lot of incorrect attempts were seen for this question. A very popular incorrect response was $\frac{4}{15}$, obtained by confusing independent events with mutually exclusive events.

Question 7

Marks	0	1	2	3	Average
%	56	9	7	28	1.1

$$x\sin(3x) = \frac{\left(\cos(3x) - f'(x)\right)}{3}$$

$\int x\sin(3x)dx = \frac{\sin(3x)}{9} - \frac{x\cos(3x)}{3}$ is an antiderivative.

This question should have been a standard and familiar calculus problem, but was not well answered. Some students differentiated, while others ignored *x* and proceeded to anti-differentiate regardless. The instruction 'hence' was often ignored. Setting out and notation were very poor in this question and '*dx*' often did not appear. Many students were unable to rearrange the expression.

Question 8a.

Marks	0	1	2	Average
%	41	14	45	1.1

$$\frac{2\pi x}{3} = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$

$$\frac{2\pi x}{3} = \frac{4\pi}{3}, \frac{5\pi}{3}$$

$$x = 2, \frac{5}{2}$$

This question was handled reasonably well. The biggest hurdle for many students was not knowing the correct basic angle for the exact value and therefore being unable to obtain the correct result for *x*. Other common errors were giving answers outside the given domain, including negative values, or only finding $x = 2$.

Question 8b.

Marks	0	1	2	Average
%	62	18	20	0.6

The graph of $y = \sin\left(\frac{2\pi x}{3}\right)$ has first maximum turning point at $x = \frac{3}{2\pi} \times \frac{\pi}{2} = \frac{3}{4}$. The graph is translated 1 unit to the right, so $x = \frac{3}{4} + \frac{4}{4} = \frac{7}{4}$ is where the maximum first occurs.

or

The maximum first occurs when

$$\sin\frac{\left(2\pi(x-1)\right)}{3} = 1$$

$$\frac{2\pi(x-1)}{3} = \frac{\pi}{2}$$

$$x = \frac{7}{4}$$

or

$$g'(x) = 2\pi\cos\left(\frac{2\pi(x-1)}{3}\right) = 0$$

$$\frac{2\pi(x-1)}{3} = -\frac{\pi}{2}, \frac{\pi}{2}$$

$$x = \frac{1}{4} \text{ for minimum}, \frac{7}{4} \text{ for maximum}$$

Students found this question quite challenging, and a significant number did not know what was required. Many students encountered problems when attempting to set up an equation: $3f(x-1)+2$ became $3f'(x-1)=0$ and this led to $x = 1$. Many variations on this theme were seen. Some students attempted to find the derivative of an almost correct expression, which they then equated to zero. Other students attempted to find when $\sin\frac{(2\pi(x-1))}{3} = 1$; this was sometimes successful. Only a small number of students attempted to find the answer by transformation.

This question proved to be a good discriminator of overall student performance.

Question 9
This should have been a straightforward question.

Question 9a.

Marks	0	1	2	Average
%	35	20	44	1.1

$f'(x) = 0.5e^{\left(\frac{x}{2}\right)}$

$f'(0) = 0.5$

normal: $y = -2x + 2$

Students often had difficulty correctly finding the y intercept; (0, 1) was a common answer, leading to the normal being $y = -2x + 1$. Quite a few students differentiated but then did not find the value of the derivative at $x = 0$ and simply substituted the expression for the derivative into the equation of the normal, which led to incorrect attempts at algebraic manipulation and non-linear normals. Some students found a tangent instead.

Question 9b.

Marks	0	1	2	3	Average
%	38	18	17	27	1.3

$$\text{Area} = \int_0^1 e^{\left(\frac{x}{2}\right)} + 1 - (-2x+2)\,dx$$

$$= \left[2e^{\left(\frac{x}{2}\right)} - x + x^2\right]_0^1$$

$$= 2e^{\frac{1}{2}} - 2$$

Most students correctly set up an area expression but some were unable to proceed due to their ‘normal’ not being a linear expression. Many had the correct terminals, although 2 instead of 1 was a popular incorrect value. Arithmetic errors appeared frequently and subtraction mistakes in the integrand were also common. Some students also lost ‘1’ from the equation of the curve. The use of ‘*dx*’ was surprisingly good. A few students used the area under the curve minus the triangle with some success.

Question 10

Marks	0	1	2	3	Average
%	31	15	21	33	**1.6**

$$\int_0^9 kx^{\frac{1}{2}}\,dx = 27$$

$$\left[\frac{2kx^{\frac{3}{2}}}{3}\right]_0^9 = 27$$

$$k = \frac{3}{2}$$

This question was generally well answered. Many students were able to set up the correct expression and integrate correctly. However the $\frac{1}{\left(\frac{3}{2}\right)}$ sometimes became $\frac{3}{2}$ instead of $\frac{2}{3}$. Too many students were unable to evaluate the expression $9^{\left(\frac{3}{2}\right)}$ correctly. Some students who had little idea simply set the given expression equal to 27.
$\int_0^9 \left(kx^{\frac{1}{2}} - 9\right)dx = 27$ was also quite common.

Question 11

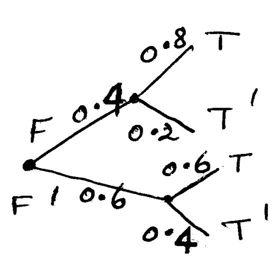
Some students who were unable to obtain any other marks on the paper correctly answered both parts of this question. However, overall this was not well answered.

Question 11a.

Marks	0	1	2	Average
%	37	14	49	**1.1**

$$\begin{aligned}\Pr(T) &= \Pr(T|F)\cdot\Pr(F) + \Pr(T|F').\Pr(F') \\ &= 0.8\times 0.4 + 0.6\times 0.6 \\ &= 0.68\end{aligned}$$

or, using a tree:



Common incorrect responses included incorrect values on the branches of the tree or just taking one branch to obtain $0.8\times 0.4 = 0.32$. Multiplication of decimals was not well done by some students.

Question 11b.

Marks	0	1	2	Average
%	62	19	19	**0.6**

$$\begin{aligned}\Pr(F|T) &= \frac{\Pr(F\cap T)}{\Pr(T)}\\ &= \frac{\Pr(T|F)\cdot\Pr(F)}{\Pr(T)}\\ &= \frac{0.8\times 0.4}{0.68}\\ &= \frac{8}{17}\end{aligned}$$

This was a poorly answered question, with many students obtaining no marks. Again, students struggled with conditional probability. Despite copying the formula from the formula sheet, students did not apply it to their values. Common responses included $\Pr(T|F)\cdot\Pr(F)$ or $\dfrac{\Pr(F)}{\text{answer to part a.}}$. Quite a few students didn't use their previous answers at all.

Question 12

Marks	0	1	2	3	4	Average
%	67	5	4	3	20	**1.0**

The equation of line *OP* is $y = \frac{1}{2}x$ (*OP* is perpendicular to $y = 10 - 2x$ for minimum length)

The point of intersection $\frac{1}{2}x = 10 - 2x$ gives $x = 4$. The coordinates of *P* are (4, 2), hence the minimum length is $2\sqrt{5}$

or

$$\begin{aligned}l &= \sqrt{\left(x^2 + (10-2x)^2\right)}\\ l^2 &= \left(5x^2 - 40x + 100\right)\\ \frac{d\left(l^2\right)}{dx} &= (10x - 40) = 0\\ \text{so } x &= 4\end{aligned}$$

or

$$\begin{aligned}l &= \sqrt{\left(x^2 + (10-2x)^2\right)}\\ &= \sqrt{5x^2 - 40x + 100}\end{aligned}$$

so applying the quadratic formula and symmetry, or similar consideration, to $5x^2 - 40x + 100$ identifies the minimum value as occurring at $x = 4$.

This proved to be a challenging question for most students. Many did not know what they were required to find, or how to go about it if they did. Many different methods of solution were attempted, the most common being to use the normal, or the distance expression. A common, incorrect response was to find intercepts (0, 10) and (5, 0) on the line then use the distance between two points to find the distance to be $5\sqrt{5}$. Other common, incorrect responses involved taking *P* as the midpoint of the line, guessing and checking solutions without justification, incorrectly differentiating the distance equation, or using an incorrect diagram. A small number of students used a vector approach to solve the problem. Some attempts at geometrical solutions were also seen with varying degrees of success.