



**2009 Mathematical Methods (CAS) GA 2: Examination 1**

**GENERAL COMMENTS**

The number of students who sat for the 2009 examination was 7223. Almost 15% of students scored 90% or more of the available marks, compared with 16% in 2008, and just over 1% received full marks, compared with 3% in 2008.

It was encouraging to see that the use of the integrand, with 'dx', was not a problem for students this year. However, some students commenced finding the rule for an inverse function by writing  $f^{-1} = x =$  or similar, presumably in an attempt to indicate that they were finding  $f^{-1}(x)$  by solving  $x = f(y)$  for  $y$ .

Many students were unable to correctly evaluate simple arithmetic calculations involving decimals and fractions. In Questions 1a., 2b., 4, 5, 6, 7 and 10a., answers were often either left unsimplified or incorrectly evaluated when simplification was attempted.

The use of brackets in expressions where more than one term is multiplied or subtracted from another was a problem for a significant number of students. This was particularly evident in Questions 1b. and 2b.

Students are strongly advised to reread a question upon completion to ensure they have done all that is required. In Question 2b., many students applied the quotient rule but neglected to substitute  $x = \pi$  to complete their answer. In Question 3, most students knew what to do to find the rule of the inverse function  $f^{-1}(x)$  but neglected to state the domain of  $f^{-1}$ .

Students need to be aware that they must show working in questions worth more than one mark. Failure to show appropriate working will result in marks not being awarded. The number of marks allocated to a question is usually an indication of the number of steps/concepts required.

**Question 1a.**

Marks	0	1	2	Average
%	16	11	73	<b>1.6</b>

$$x \times \frac{1}{x} + \log_e(x)$$

$$= 1 + \log_e(x)$$

This was a straightforward differentiation question involving the use of the product rule. Many students 'cancelled'

$$x \times \frac{1}{x}$$

as zero rather than 1, to leave just the logarithm term.

**Question 1b.**

Marks	0	1	2	3	Average
%	12	42	10	37	<b>1.7</b>

$$f'(x) = \frac{(2x + 2) \times -\sin(x) - \cos(x) \times 2}{(2x + 2)^2}$$

$$f'(\pi) = \frac{1}{2(\pi + 1)^2}$$

The quotient rule was appropriately attempted by most students. However, too many students did not include the brackets on the first line and/or incorrectly cancelled out the  $2x + 2$  in the denominator with the  $2x + 2$  in the numerator.

A majority of students did not substitute  $x = \pi$  correctly.

# 2009 Assessment Report



## Question 2a.

Marks	0	1	2	Average
%	21	54	25	1.1

$$-\frac{1}{2} \log_e(|1 - 2x|)$$

This question was very poorly done. Most students realised that a logarithm would be involved in the answer; however, few obtained the correct result, commonly leaving off the absolute value operator.

## Question 2b.

Marks	0	1	2	3	Average
%	9	9	34	48	2.2

$$\int_1^4 (x^2 + 1) dx = \left[ \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + x \right]_1^4$$

$$= \left( \frac{2 \times 4^{\frac{3}{2}}}{3} + 4 \right) - \left( \frac{2}{3} + 1 \right)$$

$$= 7 \frac{2}{3} = \frac{23}{3}$$

Most students could anti-differentiate part of the expression and then set up the solution in appropriate limits. Many could not evaluate  $4^{\frac{3}{2}}$  as  $(\sqrt{4})^3 = 2^3 = 8$  or alternatively as  $\sqrt{4^3} = \sqrt{64} = 8$ .

## Question 3

Marks	0	1	2	3	Average
%	8	10	44	38	2.1

$$f^{-1} : R \setminus \{-4\} \rightarrow R \text{ where } f^{-1}(x) = \frac{3}{x+4}$$

Few students realised that the inverse function,  $f^{-1}$ , required the rule and the domain to be specified. The definition of  $f$  should have provided students with a clue.

## Question 4

Marks	0	1	2	3	Average
%	19	14	26	41	1.9

$$\tan(2x) = \sqrt{3} \Rightarrow 2x = \frac{\pi}{3}, \frac{4\pi}{3}, \frac{-2\pi}{3}$$

$$x = \frac{\pi}{6}, \frac{2\pi}{3}, \frac{-\pi}{3}$$

$$x = \frac{\pi}{6}, \frac{2\pi}{3} \text{ are within required domain.}$$

Most students correctly chose the initial angle but went on to have problems dividing by 2 and selecting the appropriate angles for the set domain.

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## Question 5a.

Marks	0	1	Average
%	22	78	0.8

$$\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$$

This was a straightforward question; however, some student responses showed poor arithmetic.

## Question 5b.

Marks	0	1	Average
%	37	63	0.7

$$4 \times \frac{1}{12} = \frac{1}{3}$$

Students encountered a similar problem to that experienced in Question 5a.

## Question 5c.

Marks	0	1	2	Average
%	49	11	40	0.9

$$\frac{1}{12} = \frac{3}{12} = \frac{1}{4}$$

Some students obtained the correct answer from first principles, using the restricted sample space  $\{(1,4), (2,3), (3,2), (4,1)\}$ . They obtained the correct answer of  $\frac{1}{4}$  despite sometimes having incorrect responses to Question 5a. and/or 5b.

## Question 6

Marks	0	1	2	3	Average
%	21	16	5	58	2

$$\frac{dV}{dr} = 4\pi r$$

$$\frac{dr}{dt} = \frac{dr}{dV} \frac{dV}{dt} = \frac{1}{4\pi r} \times 10$$

$$\frac{dr}{dt} = \frac{1}{12\pi} \text{ when } r = 30$$

Most students could differentiate  $V = 2\pi r^2$  and knew that the chain rule was required. However, many students could not put it together correctly.

## Question 7a.

Marks	0	1	2	Average
%	37	10	53	1.2

$$\Pr(X > 1 | X \leq 3) = \frac{\Pr(X = 2) + \Pr(X = 3)}{1 - \Pr(X = 4)} = \frac{0.6}{0.9} = \frac{6}{9} = \frac{2}{3}$$

The intersection was correctly identified by most students. What to do with the 'condition' eluded many. Some students did not write a proper fraction or stated a terminating decimal.



**Question 7b.**

Marks	0	1	2	3	Average
%	16	17	17	50	2

$$E(X) = 0 \times 0.1 + 1 \times 0.2 + 2 \times 0.4 + 3 \times 0.2 + 4 \times 0.1 = 0 + 0.2 + 0.8 + 0.6 + 0.4 = 2$$

$$E(X^2) = 0^2 \times 0.1 + 1^2 \times 0.2 + 2^2 \times 0.4 + 3^2 \times 0.2 + 4^2 \times 0.1 = 0 + 0.2 + 1.6 + 1.8 + 1.6 = 5.2$$

$$\text{Var}(X) = E(X^2) - E(X)^2 = 1.2 \quad \text{or use of } \text{Var}(X) = \sum (x - E(X))^2 \text{Pr}(X = x)$$

The majority of students started well and wrote down the correct rule for variance. However, many then neglected to square the mean to obtain the correct final answer.

**Question 8**

Marks	0	1	2	3	Average
%	42	17	4	37	1.4

$$f(a) = e^a + k$$

$$f'(x) = e^x, f'(a) = e^a \quad \text{so tangent is } y = e^a x$$

Then solve by using common  $y$ -values  $y_a = e^a a = f(a)$ , **or** common gradients  $\frac{e^a + k - 0}{a - 0} = e^a$

$$\text{both lead to } e^a a = e^a + k$$

$$\Rightarrow k = e^a a - e^a = e^a (a - 1)$$

(The equation of the *tangent* through  $(0,0)$  and  $(a, e^a + k)$  yields the same result.)

This question was poorly done and appeared to confuse many students. Common errors included incorrect differentiation, using  $x = 0$  instead of  $x = a$  to find the gradient of the tangent, and stating that the tangent was  $y = e^x x$ .

**Question 9**

Marks	0	1	2	3	4	Average
%	16	16	20	26	22	2.2

$$\log_e \left( \frac{x^2}{x+3} \right) = \log_e \left( \frac{1}{2} \right)$$

$$\frac{x^2}{x+3} = \frac{1}{2} \Rightarrow 2x^2 = x+3$$

$$2x^2 - x - 3 = 0$$

Solve (factorising or by formula) to get  $x = \frac{3}{2}$  or  $x = -1$ .

Since  $x$  must be positive, the answer is

$$x = \frac{3}{2}$$

Only a few students attained full marks on this question because many did not eliminate  $x = -1$  as a solution;  $2 \log_e(-1)$  is not defined as a real value.

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Some students incorrectly cancelled one  $x$  in the denominator with the  $x$  in the numerator to obtain  $x = \frac{3}{2}$ .

### Question 10a.

Marks	0	1	2	3	4	Average
%	47	8	9	9	27	1.6

$$f(x) = x^{\frac{1}{3}} \Rightarrow f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$$

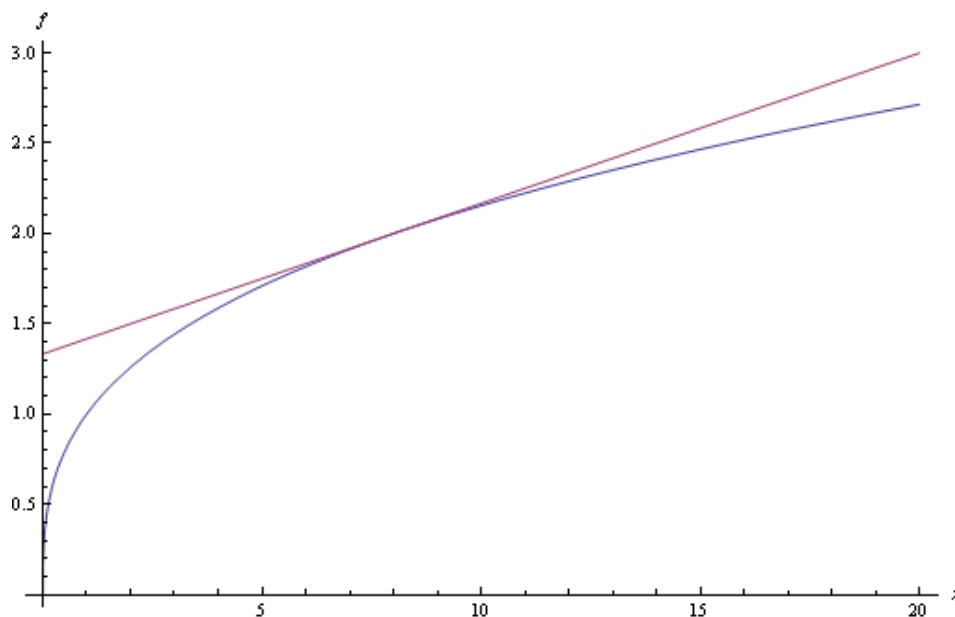
$$f(8) = 2 \text{ and } f'(8) = \frac{1}{12}$$

$$f(8 + 0.06) \approx f(8) + 0.06f'(8)$$

$$\sqrt[3]{8.06} \approx 2 + \frac{1}{200} = 2.005$$

### Question 10b.

Marks	0	1	Average
%	92	8	0.1



The gradient of the function is positive but decreasing for  $x \geq 8$ . The gradient of the tangent at  $x = 8$  is positive and constant. The approximation is determined by the  $y$ -value on the tangent which is greater than the actual value on the function for  $x > 8$ .