**2011 Assessment Report**

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

# 2011 Mathematical Methods (CAS) GA 3: Examination 2

## GENERAL COMMENTS

There were 15 983 students who sat the Mathematical Methods (CAS) Examination 2 in 2011. Students achieved scores across the whole spectrum of available marks. Responses showed that the paper was accessible and that it provided an opportunity for students to demonstrate what they knew.

In the multiple-choice section, Question 1 was answered correctly by the majority of students. Less than 50% of the students obtained the correct answers for Questions 13, 14, 16, 17, 19, 21 and 22. When finding the gradient of the normal, a number of students incorrectly gave the reciprocal of the gradient of the tangent, not the negative reciprocal. This occurred in multiple-choice Questions 2 and 17. Another common error occurred in Questions 12 and 13, where the incorrect area was typed into the technology when working with the normal distribution; for example, $X \sim N(u, \sigma^2)$, Pr $(X > a) = b$, Pr $(X \leq a) = 1 - b$. Then $a$ is found by using the inverse normal and using $1 - b$ as the area, not $b$.

In Section 2 it was pleasing to see a number of students using technology efficiently in Question 2 by defining the hybrid function at the start of the question. Technology enables students to approach questions in different ways and students should be familiar with a variety of methods. Many students, however, transcribed their equations incorrectly from their technology. This occurred in Questions 1dii., 3ci., 4bi. and 4di. Others chose not to use technology and made careless mistakes with their algebra, especially in Questions 3ci. and 4bi. For questions worth more than one mark, often only the equation to be solved followed by the answer is required. Students are not required to show all the steps. However, if it is a 'show that' question, such as Questions 1b., 4a. and 4c., the relevant steps must be clearly shown.

Correct mathematical notation must be used. Several students left out the variable they were integrating with respect to, such as *dt* in Question 1a., *dx* in Question 1d. and *dy* in Questions 2aii.–2cii. Variable names were also a problem this year, with *x* often being used when *t*, *m* or *y* were required. Another problem was the omission of brackets, especially in Questions 2d, 4bi. and 4c.

Answers must be given to the required accuracy. For numerical responses, an exact answer is required unless otherwise stated. A number of students gave the exact answer when it was required, but then wrote down an approximate answer as their final answer. This occurred in Questions 1di., 4b. and 4d. in particular. There appeared to be a common rounding error in Question 1di.; some students wrote their answers to the incorrect number of decimal places, others gave exact answers when an approximate answer was required.

Students should always carefully read and check questions to make sure they have:

- answered all parts of a question
- given coordinates when required
- given the answer to the required accuracy.

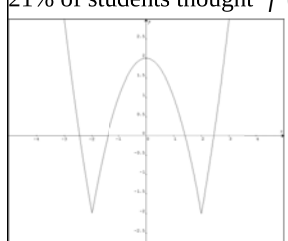
## SPECIFIC INFORMATION

### Section 1

**The table below indicates the percentage of students who chose each option. The correct answer is indicated by shading.**

| Question | % A | % B | % C | % D | % E | % No Answer | Comments |
|---|---|---|---|---|---|---|---|
| **1** | 84 | 2 | 7 | 4 | 3 | 0 | |
| **2** | 56 | 12 | 20 | 1 | 11 | 0 | $m_{tangent} = \frac{-4-0}{0+2} = -2$, $m_{normal} = \frac{-1}{-2} = \frac{1}{2}$ |
| **3** | 5 | 64 | 16 | 9 | 5 | 1 | |
| **4** | 52 | 23 | 20 | 2 | 3 | 0 | $\frac{d}{dx}\left(\log_e\left(2f(x)\right)\right) = \frac{2f'(x)}{2f(x)} = \frac{f'(x)}{f(x)}$ |
| **5** | 11 | 2 | 3 | 71 | 13 | 0 | |
| **6** | 3 | 4 | 16 | 8 | 68 | 0 | |
| **7** | 3 | 9 | 68 | 18 | 2 | 0 | |

| Question | % A | % B | % C | % D | % E | % No Answer | Comments |
|---|---|---|---|---|---|---|---|
| **8** | 3 | 8 | 11 | 58 | 20 | 1 | $h(x)=x^2-3x+3$. The turning point is at $x=\frac{3}{2}$.<br>$h\left(\frac{3}{2}\right)=\frac{3}{4}$, $h(5)=13$. The range of $h$ is $\left[\frac{3}{4},13\right)$.<br>Hence, the domain of $h^{-1}$ is $\left[\frac{3}{4},13\right)$. |
| **9** | 7 | 77 | 6 | 7 | 2 | 0 | |
| **10** | 7 | 5 | 75 | 5 | 8 | 0 | |
| **11** | 2 | 6 | 7 | 10 | 74 | 0 | |
| **12** | 5 | 4 | 28 | 59 | 3 | 0 | $X \sim N(30,25)$, $\Pr(X>a)=0.2$, $\Pr(X<a)=0.8$, $a=34.21$ correct to two decimal places. Option C (25.79) is obtained if 0.2 instead of 0.8 is used for the area. A similar error was made in Question 13. The answer needed to be greater than the mean. |
| **13** | 45 | 28 | 15 | 8 | 3 | 1 | $\Pr(Z>a)=\frac{1735}{2000}$, $\Pr(Z<a)=1-\frac{1735}{2000}$,<br>$a=-1.11465...$, $\frac{2.8-\mu}{0.2}\approx -1.11465$, $\mu=3.023$ correct to three decimal places. Option B (2.577) is obtained if $\frac{1735}{2000}$ is used for the area instead of $1-\frac{1735}{2000}$. A similar error was made in Question 12.<br>The mean needed to be greater than 2.8 because $\frac{1735}{2000}$ is greater than 0.5. No calculations were necessary. |
| **14** | 30 | 15 | 2 | 44 | 8 | 0 | Area = Area of the rectangle – Area under the curve = $e^2-\int_0^1\left(e^{2x}\right)dx$<br>Area = Area under the inverse function = $\int_1^{e^2}\left(\frac{\log_e(x)}{2}\right)dx$<br>Option A is obtained if only the first case is considered. |
| **15** | 14 | 67 | 7 | 5 | 7 | 0 | |
| **16** | 19 | 7 | 11 | 41 | 21 | 0 | $f(x)=\left\|x^2-4\right\|-2$ for all $x>0$, $f'(x)\neq 2x$<br>19% of students thought the graph of $f$ was not continuous everywhere.<br>21% of students thought $f'(x)\neq 2x$ for $x<-2$.<br> |

| Question | % A | % B | % C | % D | % E | % No Answer | Comments |
|---|---|---|---|---|---|---|---|
| **17** | 14 | 48 | 17 | 12 | 9 | 1 | $y = x^{\frac{3}{2}} + x$, $\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} + 1$, at $x = 4$, $\frac{dy}{dx} = 4$,<br>$m_{normal} = -\frac{1}{4}$, $4y + x = 7$, $y = -\frac{1}{4}x + \frac{7}{4}$<br>A similar error was made by students in Question 2. |
| **18** | 13 | 11 | 17 | 52 | 7 | 1 | $x^3 - 9x^2 + 15x + w = 0$, $\frac{d}{dx}\left(x^3 - 9x^2 + 15x + w\right) = 0$,<br>$x = 1$ or $x = 5$, $f(x) = x^3 - 9x^2 + 15x$, $f(1) = 7$,<br>$f(5) = -25$, $w < -7$ or $w > 25$ for one solution.<br>This question could be done by considering graphs and local maximum and local minimum turning points. Option D is the only one that specifies a set of the required form. |
| **19** | 7 | 15 | 27 | 43 | 8 | 0 | Area of the rectangles<br>$= f(1) + f(2) + f(3) + f(4) + f(5) + f(6)$<br>$= 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 = 91$<br>Actual area = $\int_0^6 \left(x^2\right) dx = 72$,<br>$p\% = \frac{91-72}{72} \times 100\% = 26\%$ correct to the nearest percentage. $p$ is closest to 25%.<br>Option C would be obtained if $91-72$ was considered or $p\% = \frac{91-72}{91} \times 100\% = 21\%$ correct to the nearest percentage. |
| **20** | 2 | 10 | 12 | 5 | 71 | 1 | |
| **21** | 17 | 25 | 23 | 19 | 15 | 1 | $\Pr(P \cap Q) = \Pr(P) \times \Pr(Q)$,<br>$\Pr(P' \cap Q) = \Pr(Q) - \Pr(P) \times \Pr(Q)$,<br>$\Pr(P) \times \Pr(Q) = \Pr(Q) - \Pr(P) \times \Pr(Q)$,<br>$2\Pr(P)\Pr(Q) = \Pr(Q)$, $\Pr(P) = \frac{1}{2}$<br>Given the stated condition, $P$ and $Q$ are independent if and only if $\Pr(P) = \frac{1}{2}$. |
| **22** | 15 | 45 | 17 | 8 | 15 | 1 | $\log_c(a) + \log_a(b) + \log_b(c)$<br>$= \frac{\log_a(a)}{\log_a(c)} + \frac{\log_b(b)}{\log_b(a)} + \frac{\log_c(c)}{\log_c(b)}$<br>$= \frac{1}{\log_a(c)} + \frac{1}{\log_b(a)} + \frac{1}{\log_c(b)}$<br>Alternatively, let $\log_e(a) = x$ then<br>$a = c^x$ and $1 = x\log_a(c)$.<br>So $x = \frac{1}{\log_{a(c)}}$ and similarly for the other terms.<br>Each of the options could also have been tested using technology. |

# Section 2

## Question 1

### 1a.

| Marks | 0 | 1 | 2 | 3 | Average |
|---|---|---|---|---|---|
| % | 74 | 3 | 3 | 20 | 0.7 |

$\frac{dV}{dt} = -\frac{t^2}{5}$, $V = \int_0^a \frac{t^2}{5} dt = 6075$ or $V = \int \left(-\frac{t^2}{5}\right) dt = -\frac{t^3}{15} + 6075 = 0$

or $\int \left(\frac{t^2}{5}\right) dt = \frac{t^3}{15} = 6075$, 45 minutes

This type of question appeared to be unfamiliar to many students and was not answered well. Many students did not attempt to integrate and solved $\frac{t^2}{5} = 6075$, getting $t = 45\sqrt{15}$.

### 1b.

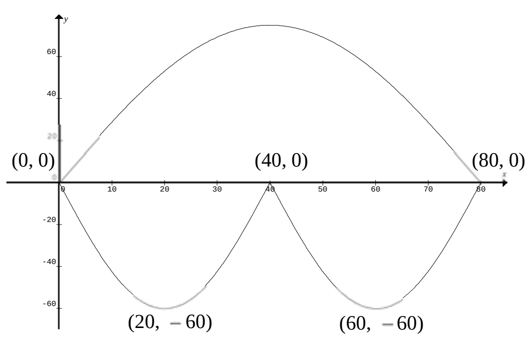
| Marks | 0 | 1 | 2 | 3 | Average |
|---|---|---|---|---|---|
| % | 31 | 8 | 4 | 56 | 1.9 |

$A = \pi r^2$, $\frac{dA}{dr} = 2\pi r$, $\frac{dA}{dt} = 20$, $\frac{dr}{dt} = \frac{dA}{dt} \times \frac{dr}{dA} = 20 \times \frac{1}{2\pi r} = \frac{10}{3\pi}$, when $r = 3$

It is pleasing to report that many students were able to write down a related rates equation. A number of students did not know the formula for the area of a circle. The steps must be shown clearly in a 'show that' question. Some students showed $r = 3$. There were some unusual choices of variable names, such as $V$ for area.

### 1c.

| Marks | 0 | 1 | 2 | Average |
|---|---|---|---|---|
| % | 23 | 22 | 54 | 1.3 |




Students seem to have taken more care when sketching graphs this year than in previous years. A sharp point was required at $x = 40$. Some students wrote their coordinates as (0, 40) and (0, 80) or left out (80, 0). Others had the graph above the $x$-axis or drew straight lines.

### 1di.

| Marks | 0 | 1 | Average |
|---|---|---|---|
| % | 20 | 80 | 0.8 |

$\int_0^{40} \sin\left(\frac{\pi x}{40}\right) dx = \frac{80}{\pi}$

This question was answered well. An exact answer was required. Many students wrote 25.46. Some wrote down the antiderivative but did not evaluate it.

**1dii.**

| Marks | 0 | 1 | 2 | Average |
|---|---|---|---|---|
| % | 55 | 16 | 29 | 0.8 |

$\int_0^{80} 75\sin(\frac{\pi x}{80})dx + 120\int_0^{40}\sin\left(\frac{\pi x}{40}\right)dx = 6875\text{ m}^2$ or $\int_0^{80} 75\sin(\frac{\pi x}{80})dx + 120\times\frac{80}{\pi} = 6875$ to the nearest square metre

Other methods were acceptable. Many students rounded incorrectly to 6876, rounding 6875.49 to 6875.5 then 6875.5 to 6876, or left their answer in exact form. Many wrote $+\ 120\int_0^{40}\sin\left(\frac{\pi x}{40}\right)dx$ or its equivalent form incorrectly, using incorrect terminals or $-\ 120\int_0^{40}\sin\left(\frac{\pi x}{40}\right)dx$ or forgetting to multiply their answer to Question 1di. by 60. Some students did not transcribe their equation correctly from the technology output.

**Question 2**

**2ai.**

| Marks | 0 | 1 | Average |
|---|---|---|---|
| % | 23 | 77 | 0.8 |

$X \sim N(3, 0.64)$, $\Pr(3 \le X \le 5) = 0.4938$ correct to four decimal places

This question was answered well.

**2aii.**

| Marks | 0 | 1 | 2 | 3 | Average |
|---|---|---|---|---|---|
| % | 27 | 7 | 8 | 58 | 2 |

$\Pr(3 \le Y \le 5) = \Pr(3 \le Y \le 4) + \Pr(4 < Y \le 5) = \int_3^4 \left(\frac{y}{16}\right)dy + \int_4^5 \left(0.25e^{-0.5(y-4)}\right)dy = 0.4155$ correct to four decimal places

or $\Pr(3 \le Y \le 5) = \int_3^5 \left(f(y)\right)dy = 0.4155$

The hybrid function could be defined using technology, giving the second method. As the rest of the question applied to the hybrid function this was a sensible approach. Some students incorrectly set up their integrals as:

$\int_3^4 \left(\frac{y}{16}\right)dy + \int_{4.001}^5 \left(0.25e^{-0.5(y-4)}\right)dy$ or $\int_3^4 \left(\frac{y}{16}\right)dy + \int_5^5 \left(0.25e^{-0.5(y-4)}\right)dy$ or $\int_3^5 \left(\frac{y}{16}\right)dy$ .

**2b.**

| Marks | 0 | 1 | 2 | 3 | Average |
|---|---|---|---|---|---|
| % | 32 | 18 | 14 | 37 | 1.6 |

$\int_0^4 \left(\frac{y^2}{16}\right)dy + \int_4^\infty \left(0.25ye^{-0.5(y-4)}\right)dy = 4.333$ correct to three decimal places or $\int_{-\infty}^{\infty}\left(y(f(y))dy\right) = 4.333$

Some students gave the exact answer, $\frac{13}{3}$. Others worked out each integral separately and did not add the two answers $\int_0^4 \left(\frac{y^2}{16}\right)dy = \frac{4}{3}$ and $\int_4^\infty \left(0.25ye^{-0.5(y-4)}\right)dy = 3$ or only considered $\int_0^4 \left(\frac{y^2}{16}\right)dy = 1.333$. Some found the mean of the two values, $\frac{1}{2}\left(\frac{4}{3}+3\right)=\frac{13}{6}=2.167$. Others used 3 from the previous question instead of 0,

$$\int_3^4\left(\frac{y^2}{16}\right)dy+\int_4^\infty\left(0.25ye^{-0.5(y-4)}\right)dy.$$

**2ci.**

| Marks | 0 | 1 | Average |
|---|---|---|---|
| % | 47 | 53 | **0.6** |

$\int_0^4\left(\frac{y}{16}\right)dy=\frac{1}{2}$ or $\int_{-\infty}^m(f(y))=\frac{1}{2}$, the median is 4

Some students left their answers as $-4$ and 4.

**2cii.**

| Marks | 0 | 1 | 2 | Average |
|---|---|---|---|---|
| % | 66 | 4 | 30 | **0.7** |

$\int_4^a 0.25e^{-0.5(y-4)}\,dy=0.2$ or $\int_a^\infty 0.25e^{-0.5(y-4)}\,dy=0.3$ or $\int_{-\infty}^a(f(y))\,dy=0.7$, $a$ = 5.02 correct to 2 decimal places

Some students attempted to use the normal distribution. Others solved $\int_4^a 0.25e^{-0.5(y-4)}\,dy=0.7$, for $a$.

**2d.**

| Marks | 0 | 1 | 2 | Average |
|---|---|---|---|---|
| % | 38 | 10 | 52 | **1.2** |

$W\sim Bi\left(10,\ \frac{9}{32}\right)$ or $\binom{10}{4}\left(\frac{9}{32}\right)^4\left(\frac{23}{32}\right)^6$, Pr($W$ = 4) = 0.1812 correct to four decimal places

Many students recognised that the distribution was binomial and gave the correct $n$ and $p$ values. Some wrote $\binom{10}{4}\frac{9^4}{32}\frac{23^6}{32}$. Others did not give the answer to the required number of decimal places.

**2e.**

| Marks | 0 | 1 | 2 | 3 | Average |
|---|---|---|---|---|---|
| % | 65 | 22 | 4 | 9 | **0.6** |

$\Pr(X>3)=0.5$, $\Pr(Y)>3=\frac{23}{32}$, Pr(longer than 3) = $0.5\times0.5+0.5\times\frac{23}{32}$ = 0.609375…,

Pr(Machine A|longer than 3) = $\frac{0.5\times0.5}{0.609375}$ = 0.4103 correct to four decimal places

This question was not answered well. Students who used a tree diagram were more successful. Some students gave probabilities greater than 1 in their working.

**Question 3**

**3ai.**

| Marks | 0 | 1 | Average |
|---|---|---|---|
| % | 5 | 95 | **1** |

$f'(x)=12x^2+5$

This question was answered very well; however, a small number of students tried to integrate.

**3aii.**

| Marks | 0 | 1 | Average |
|---|---|---|---|
| % | 53 | 47 | **0.5** |

$x^2 \geq 0$, for all of $x$, hence $12x^2 + 5 \geq 5$ for all of $x$

Many students found it difficult to explain their answer. 'Because it was translated 5 units up' was a common incomplete response.

**3bi.**

| Marks | 0 | 1 | Average |
|---|---|---|---|
| % | 91 | 9 | **0.1** |

0, 1, 2

**3bii.**

| Marks | 0 | 1 | Average |
|---|---|---|---|
| % | 80 | 20 | **0.2** |

0, 1

In Questions 3bi. and 3bii. many students did not know that a cubic function could have no stationary points if it is to be a one-to-one function even though earlier parts of the question were helpful in this regard. Some students thought the number of stationary points was required and others attempted to use the quadratic formula.

**3ci.**

| Marks | 0 | 1 | 2 | Average |
|---|---|---|---|---|
| % | 10 | 14 | 76 | **1.7** |

Let $y = 3 - 2x^3$, for the inverse rule interchange $x$ and $y$ and solve for $y$ to obtain $x = 3 - 2y^3$, $q^{-1}(x) = \sqrt[3]{\frac{3-x}{2}}$ or equivalent form.

Most students knew to interchange $x$ and $y$. Some did not transcribe the resulting expression from the technology. $q^{-1}(x) = \sqrt{\frac{3-x}{2}}$ and $q^{-1}(x) = \sqrt[3]{\frac{3+x}{2}}$ were common answers. Some students need to be careful with their mathematical notation as $q^{-1}(x) = 3\sqrt{\frac{3-x}{2}}$ and $q^{-1}(x) = \frac{\sqrt[3]{3-x}}{2}$ were cited. A few students tried to find the derivative or wrote $q'(x) = \sqrt[3]{\frac{3+x}{2}}$. A number did it by hand and had problems with the algebra and the negative sign. It was pleasing to see students changing their technology output $q^{-1}(x) = \frac{2^{\frac{2}{3}}\sqrt[3]{3-x}}{2}$ to $q^{-1}(x) = \frac{\sqrt[3]{3-x}}{2^{\frac{1}{3}}}$.

**3cii.**

| Marks | 0 | 1 | 2 | Average |
|---|---|---|---|---|
| % | 18 | 22 | 60 | **1.4** |

$3 - 2x^3 = x$, $x = 1$, $(1, 1)$

Many students were able to write down a correct equation. Some students did not find the coordinates.

**3di.**

| Marks | 0 | 1 | 2 | 3 | Average |
|---|---|---|---|---|---|
| % | 35 | 31 | 5 | 30 | **1.3** |

$g'(x) = 3x^2 + 4x + c = 0$, $\Delta = 16 - 12c = 0$ for one solution, $c = \frac{4}{3}$ or

$(x - A)^3 + B = x^3 - 3Ax^2 + 3A^2x - A^3 + B$, $-3A = 2$, $c = 3\left(-\frac{2}{3}\right)^2 = \frac{4}{3}$

It was pleasing to see a variety of methods being used. A number of students did not include + *c* in the derivative, $g'(x) = 3x^2 + 4x$. This result would occur if a multiplication sign was not put between *c* and *x* in $g(x) = x^3 + 2x^2 + cx + k$ when using technology. Others found $c = -x(3x+4)$.

**3dii.**

| Marks | 0 | 1 | 2 | 3 | Average |
|---|---|---|---|---|---|
| % | 79 | 9 | 2 | 9 | 0.4 |

$g'(x) = 3x^2 + 4x + \frac{4}{3} = 0$, $x = -\frac{2}{3}$, solve $g\left(-\frac{2}{3}\right) = -\frac{2}{3}$ for *k*, $k = -\frac{10}{27}$

This question was not answered well. Many students used *x* = 1 from Question 3cii. Others tried to solve $x^3 + 2x^2 + cx + k = 3x^2 + 4x + c$.

**Question 4**

**4a.**

| Marks | 0 | 1 | 2 | 3 | Average |
|---|---|---|---|---|---|
| % | 40 | 15 | 5 | 39 | 1.5 |

$n = m^2 - 1$, $L = \sqrt{(n-1)^2 + m^2} = \sqrt{(m^2-2)^2 + m^2} = \sqrt{m^2 + m^4 - 4m^2 + 4} = \sqrt{m^4 - 3m^2 + 4}$

Many students did not attempt this question. Some students knew the distance formula but were unsuccessful with the substitution. For 'show that' questions, students must show the steps involved and the final answer. Many students were able to do this; however, some had problems with the algebra.

**4bi.**

| Marks | 0 | 1 | 2 | 3 | Average |
|---|---|---|---|---|---|
| % | 25 | 27 | 12 | 36 | 1.6 |

$\frac{dL}{dm} = \frac{2m^3 - 3m}{\sqrt{m^4 - 3m^2 + 4}}$, $\frac{dL}{dm} = 0$, $m = \frac{\sqrt{6}}{2}$, $m \geq 0$, $n = m^2 - 1 = \frac{1}{2}$, $\left(\frac{\sqrt{6}}{2}, \frac{1}{2}\right)$ or $\left(\sqrt{\frac{3}{2}}, \frac{1}{2}\right)$

Most students were able to get the derivative. Some transcribed it incorrectly from their technology or wrote it as an incorrect expression $\frac{1}{2\sqrt{m^4 - 3m^2 + 4}} \times 4m^3 - 6m$. Some did it by hand but then did not answer the rest of the question. $\frac{dL}{dm} = m^2 - 1$ was often seen. There was some poor use of variable names with *x*s and *m*s in the same equation. Others forgot to find the coordinates or substituted their *m* value into *L* instead of $n = m^2 - 1$.

**4bii.**

| Marks | 0 | 1 | 2 | Average |
|---|---|---|---|---|
| % | 48 | 22 | 30 | 0.8 |

$L\left(\frac{\sqrt{6}}{2}\right) = \frac{\sqrt{7}}{2}$ km

This question was done well by students who were successful in Question 4bi. Others were able to gain a method mark by substituting their *m* value into *L*. However, some students gave an approximate answer when the exact answer was required.

**4c.**

| Marks | 0 | 1 | 2 | 3 | Average |
|---|---|---|---|---|---|
| % | 78 | 8 | 3 | 11 | 0.5 |

Distance from camp (0, 0) to river ($x$, $x^2 - 1$), $\sqrt{(x^2-1)^2 + x^2} = \sqrt{x^4 - x^2 + 1}$

Time from river to plant = $k\left(\frac{3}{4}-(x^2-1)\right)$, $T=\frac{\sqrt{x^4-x^2+1}}{2}+k\left(\frac{3}{4}-\left(x^2-1\right)\right)=\frac{\sqrt{x^4-x^2+1}}{2}+\frac{k}{4}\left(7-x^2\right)$

This question was not done well, and many students did not attempt it. As this was a 'show that' question, clear working was required to gain full marks. Some were able to use the distance formula but then struggled to find the equation for the time from the river to the plant.

**4di.**

| Marks | 0 | 1 | Average |
|---|---|---|---|
| % | 42 | 58 | **0.6** |

$\frac{dT}{dx}=\frac{x(2x^2-1)}{2\sqrt{x^4-x^2+1}}-\frac{\sqrt{13}x}{13}$ or $\frac{dT}{dx}=\frac{26x^3-2x\sqrt{13(x^4-x^2+1)}-13x}{26\sqrt{x^4-x^2+1}}$ or equivalent form

The question was done reasonably well. Some students transcribed the equation incorrectly from their technology, often writing 13 as 3 or not scrolling across to get all of the expression on the right-hand side of the equation. Others did it by hand and made errors when applying the chain rule. Some simplified technology output incorrectly.

**4dii.**

| Marks | 0 | 1 | 2 | Average |
|---|---|---|---|---|
| % | 56 | 22 | 22 | **0.7** |

$\frac{dT}{dx}=0$, $x=\frac{\sqrt{3}}{2}$, $\left(\frac{\sqrt{3}}{2},-\frac{1}{4}\right)$

Some students gave an approximate answer instead of the exact answer. Others gave two or more solutions, not considering the domain restriction. Some did not find the coordinates or substituted $x=\frac{\sqrt{3}}{2}$ into $T$ or $\frac{1}{4}k(7-4x^2)$, not $y=x^2-1$.

**4e.**

| Marks | 0 | 1 | 2 | Average |
|---|---|---|---|---|
| % | 79 | 5 | 17 | **0.4** |

$\frac{dT}{dx}=0$ with $x = 1$, $k=\frac{1}{4}$

This question was quite well done when attempted. Some substituted $x = 1$ into $T$.

**4f.**

| Marks | 0 | 1 | 2 | Average |
|---|---|---|---|---|
| % | 87 | 11 | 1 | **0.2** |

$\frac{dT}{dx}\le 0$ with $x=\frac{\sqrt{7}}{2}$, $k\ge\frac{5\sqrt{37}}{74}$. This can be observed from consideration of the family of graphs generated by $k$.

This question was not answered well. Of those students who did use the correct $x$ value, most left out the inequality sign, giving $k=\frac{5\sqrt{37}}{74}$ as their answer. Some students substituted $x=\frac{\sqrt{7}}{2}$ into $T$.