



## Victorian Certificate of Education 2002

# FURTHER MATHEMATICS

## Written examination 1 (Facts, skills and applications)

Monday 4 November 2002

Reading time: 11.45 am to 12.00 noon (15 minutes)

Writing time: 12.00 noon to 1.30 pm (1 hour 30 minutes)

### MULTIPLE-CHOICE QUESTION BOOK

#### Structure of book

<i>Section</i>	<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of modules</i>	<i>Number of modules to be answered</i>	<i>Number of marks</i>
A	13	13			13
B	45	27	5	3	27
					Total 40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, up to four pages (two A4 sheets) of pre-written notes (typed or handwritten) and an approved scientific and/or graphics calculator (memory may be retained).
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

#### Materials supplied

- Question book of 33 pages with a detachable sheet of miscellaneous formulas in the centrefold.
- Answer sheet for multiple-choice questions.
- Working space is provided throughout the book.

#### Instructions

- Detach the formula sheet from the centre of this book during reading time.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

#### At the end of the examination

- You may keep this question book.

**Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.**

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Working space

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**SECTION A****Instructions for Section A**

Answer all questions in pencil on the answer sheet for multiple-choice questions.  
A correct answer scores 1, an incorrect answer scores 0. Marks will **not** be deducted for incorrect answers. No mark will be given if more than one answer is completed for any question.

**Core**

*The following information relates to Questions 1 to 4.*

Researchers conducted a survey of 403 school leavers who had recently entered the workforce. The aim was to determine whether the type of work they undertook was gender related. Work type was classified as ‘trade’, ‘clerical’, ‘manual’ or ‘professional’.

The data in the table below comes from the survey.

Work type	Gender	
	male	female
trade	104	18
clerical	21	143
manual	72	31
professional	8	6
<b>Total</b>	205	198

**Question 1**

In this survey, the variables

**work type** (trade, clerical, manual or professional)

and

**gender** (male or female)

are

- A. both categorical variables.
- B. both numerical variables.
- C. categorical and numerical variables respectively.
- D. numerical and categorical variables respectively.
- E. neither categorical nor numerical variables.

**Question 2**

Of the females surveyed, the percentage who became clerical workers is closest to

- A. 10%
- B. 14%
- C. 35%
- D. 72%
- E. 87%

**Question 3**

Of the school leavers surveyed, the percentage who became clerical or manual workers is closest to

- A. 26%
- B. 31%
- C. 41%
- D. 50%
- E. 66%

**Question 4**

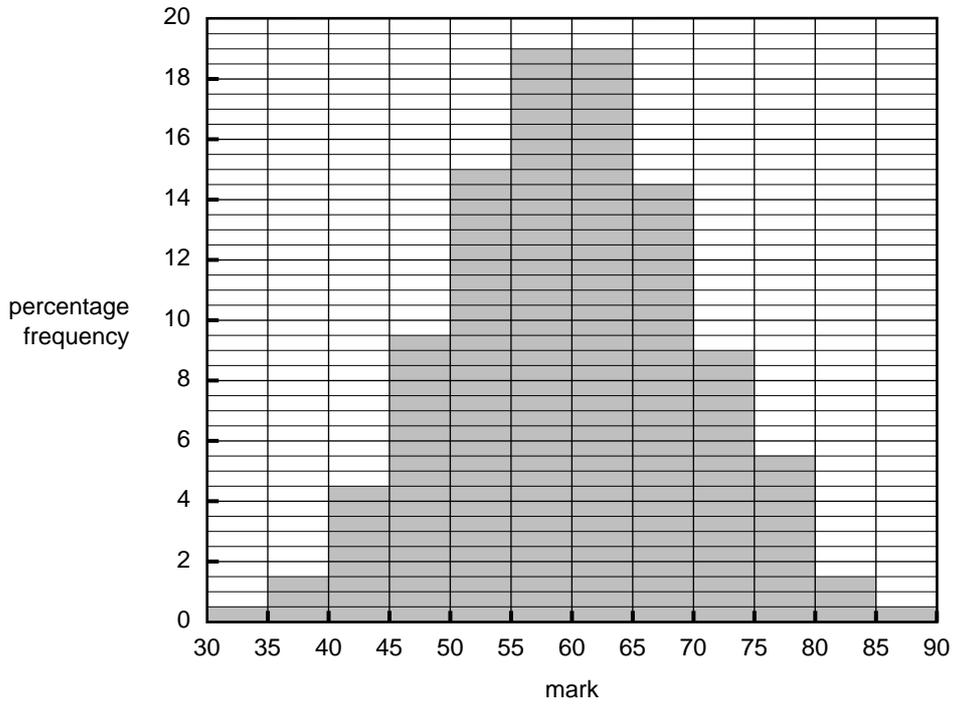
Which statement supports the suggestion that the type of work undertaken by school leavers is associated with gender?

- A. 50.9% of those surveyed are male compared to 49.1% female.
- B. 30.3% of those surveyed entered a trade while only 3.5% entered professions.
- C. 35.1% of male school leavers entered manual work compared to 15.7% of female school leavers.
- D. 403 school leavers participated in the survey.
- E. A similar but small percentage of male and female school leavers undertook professional work.

**CONTINUED OVER PAGE**

The following information relates to Questions 5 and 6.

1526 students sat for an examination. The histogram below shows the distribution of marks.



**Question 5**

The median examination mark of these students is closest to

- A. 50
- B. 55
- C. 60
- D. 65
- E. 70

**Question 6**

Using the 68-95-99.7% rule, the standard deviation of the marks is closest to

- A. 5
- B. 10
- C. 15
- D. 20
- E. 30

The following information relates to Questions 7 to 10.

The following data was recorded from measurements made on 12 men.

Age (years)	Weight (kg)	Waist (cm)
26	84	84
29	72	74
32	67	89
32	59	75
34	97	106
37	112	114
39	67	80
40	91	101
41	98	101
43	89	94
45	117	126
51	62	82

### Question 7

For these men, the median age ( $M$ ) and the interquartile range ( $IQR$ ), in years, are respectively

- A.  $M = 37$  and  $IQR = 10$
- B.  $M = 37.4$  and  $IQR = 7.2$
- C.  $M = 37.4$  and  $IQR = 6.9$
- D.  $M = 38$  and  $IQR = 10$
- E.  $M = 38$  and  $IQR = 25$

### Question 8

The sample of men has been drawn from a population in which the distribution of weights is bell-shaped, with a mean of 81.1 kg and a standard deviation of 17.9 kg.

The percentage of men in this population with a weight greater than that of the heaviest man in this sample is closest to

- A. 0.05%
- B. 2.5%
- C. 5%
- D. 50%
- E. 95%

### Question 9

A scatterplot of weight against waist measurement for this sample of men shows that there is a strong linear association between weight and waist measurement.

The least squares regression equation that would enable weight to be predicted from waist measurement is closest to

- A.  $\text{weight} = -25 + 1.11 \times \text{waist}$ .
- B.  $\text{weight} = -20 + 1.11 \times \text{waist}$ .
- C.  $\text{weight} = 1.11 - 20 \times \text{waist}$ .
- D.  $\text{weight} = 28 + 0.78 \times \text{waist}$ .
- E.  $\text{weight} = 0.78 + 28 \times \text{waist}$ .

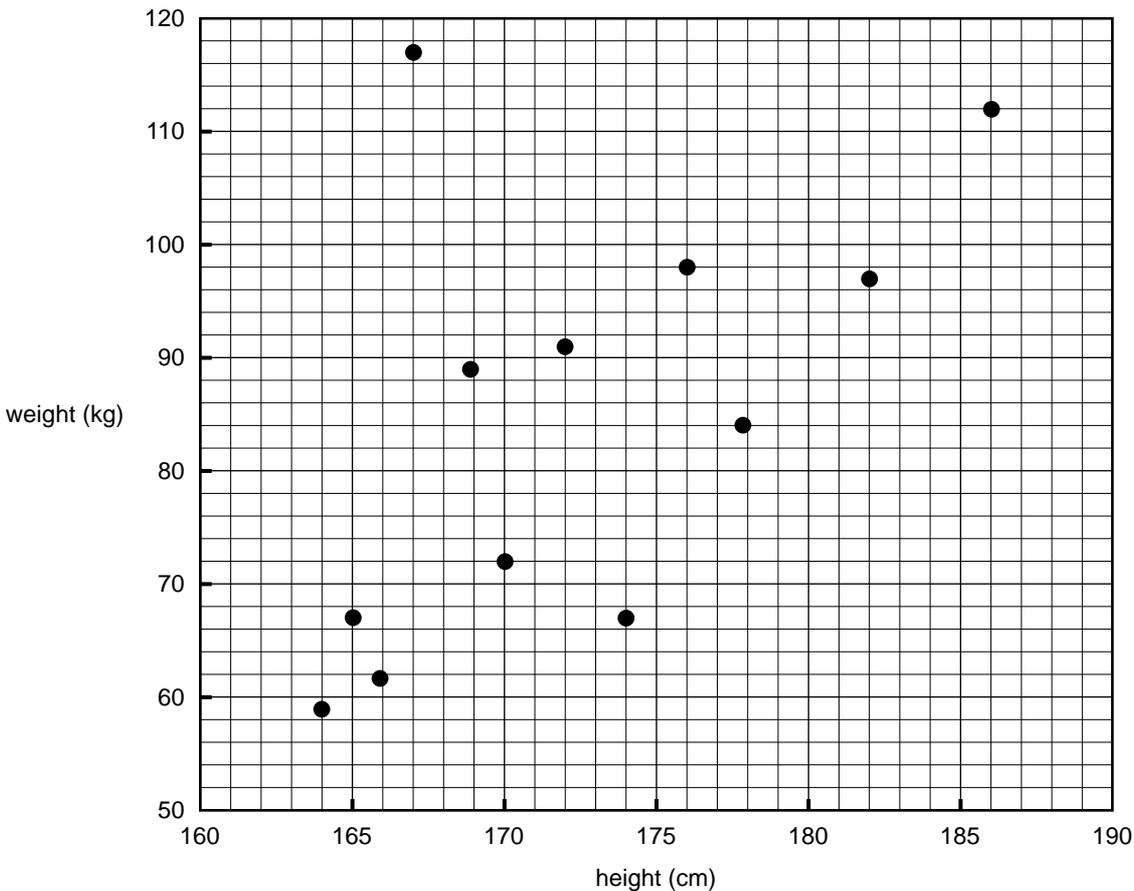
**Question 10**

The value of the product moment correlation coefficient,  $r$ , for weight against waist measurement, is closest to

- A. 0.6061
- B. -0.7785
- C. 0.8675
- D. 0.9314
- E. 0.9651

*The following information relates to Questions 11 and 12.*

A person’s weight is also known to be positively associated with their height. To investigate this association for 12 men, a scatterplot is constructed as shown below.



While there is a moderately strong positive linear relationship between weight and height, there is a clear outlier. Because of this, it is decided to model the relationship by fitting a 3-median line to the data displayed in the scatterplot.

**Question 11**

The slope of this 3-median line is closest to

- A. 1.1
- B. 1.5
- C. 1.6
- D. 2.3
- E. 2.8

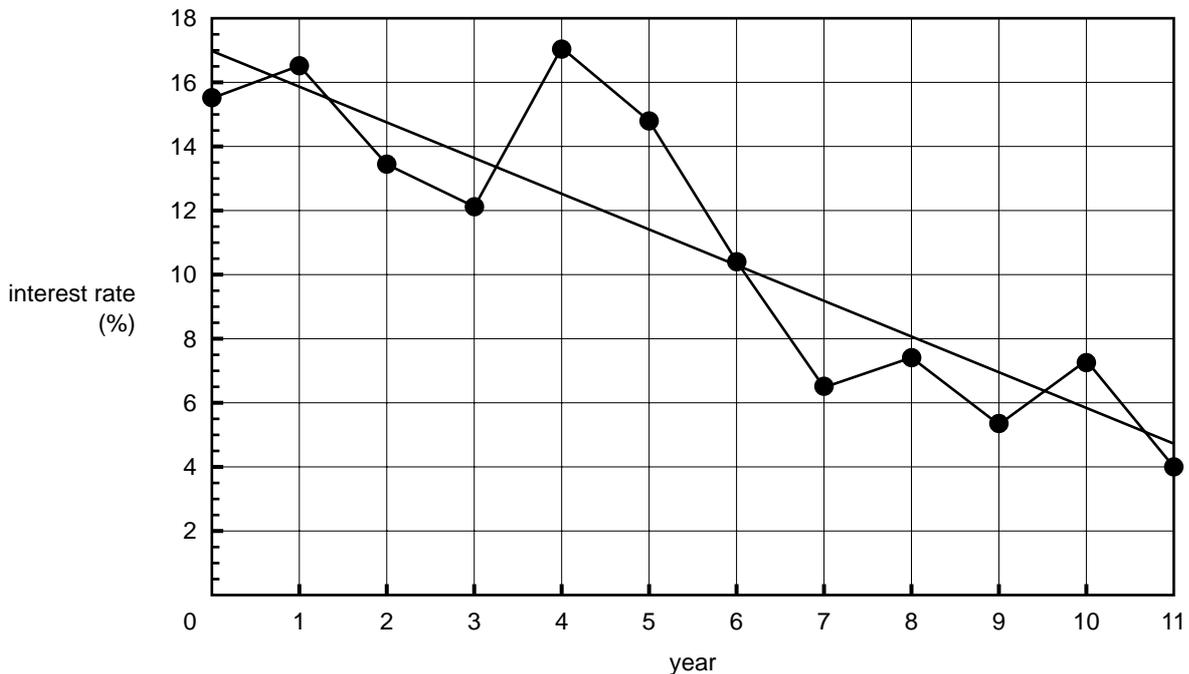
**Question 12**

When a least squares regression line is used to model this data, the coefficient of determination is found to be 0.3146. If the outlier is removed from the data, and a least squares regression line refitted to the data of the remaining 11 men, the value of the coefficient of determination will

- A. remain the same.
- B. increase.
- C. decrease.
- D. be halved.
- E. not be able to be determined.

**Question 13**

The time series shown below charts the change in interest rates over a period of years.



A trend line has been fitted to the data as shown.

The equation of the trend line is closest to

- A.  $\text{rate} = -1.1 \times \text{year}$ .
- B.  $\text{rate} = 15.6 - 1.5 \times \text{year}$ .
- C.  $\text{rate} = 17 + 1.1 \times \text{year}$ .
- D.  $\text{rate} = -1.1 + 17 \times \text{year}$ .
- E.  $\text{rate} = 17 - 1.1 \times \text{year}$ .

**SECTION B****Instructions for Section B**

Select three modules and answer **all** questions within the modules selected in pencil on the answer sheet provided for multiple-choice questions.

Show the modules you are answering by shading the matching boxes on your multiple-choice answer sheet.

A correct answer scores 1, an incorrect answer scores 0. Marks will **not** be deducted for incorrect answers. No mark will be given if more than one answer is completed for any question.

<b>Module</b>	<b>Page</b>
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Module 2: Geometry and trigonometry	15
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**Module 1: Number patterns and applications**

Before answering these questions you **must** shade the Number patterns and applications box on the answer sheet for multiple-choice questions.

**Question 1**

The following is an arithmetic sequence.

24, 20, 16, 12, ...

The sum of the first six terms is

- A. 28
- B. 36
- C. 80
- D. 84
- E. 204

**Question 2**

At Maggie's Music School, there are 512 students enrolled for lessons in Grade 1. Enrolments for music lessons decrease by 12.5% at each subsequent year level.

In Grade 4 the number of enrolments will be

- A. 1
- B. 64
- C. 256
- D. 300
- E. 343

**Question 3**

A hockey club began a fundraising campaign to raise money for a new clubroom. In the first year of fundraising, they raised \$15 000. After that, the amount of money they raised each year increased by 5% per year.

By the end of the fifth year, the total amount of money they had raised was closest to

- A. \$15 789
- B. \$18 750
- C. \$78 750
- D. \$82 884
- E. \$95 721

**Question 4**

Ethan's collection of CDs increases by four CDs each year. He has 13 CDs at the end of the third year.

At the end of the tenth year, the number of CDs he has is

- A. 17
- B. 36
- C. 37
- D. 41
- E. 53

**Question 5**

A large amount of oil was spilt from a ship at sea. When first noticed, it covered an area of 2 square kilometres. In the next hour, its area increased by 0.8 square kilometre. Thereafter, its area increased each hour by 20% of the previous hour's increase.

If this trend continues indefinitely, the eventual size of the oil spill, in square kilometres, will be closest to

- A. 1
- B. 2.4
- C. 2.8
- D. 3
- E. 12

**Question 6**

A sequence is defined by

$$t_{n+1} = t_n + 4 \quad \text{where} \quad t_1 = 3$$

The  $n$ th term in the sequence,  $t_n$ , is given by

- A.  $t_n = 4n + 3$
- B.  $t_n = 4n - 1$
- C.  $t_n = n + 2$
- D.  $t_n = n + 3$
- E.  $t_n = n + 4$

**Question 7**

Jan and her friend Hong are preparing for a trip to Thailand. They go to a currency exchange agency to buy some Thai baht (the Thai currency unit). Jan buys 5000 baht. This costs her \$223.21. Hong wants to buy 7000 baht.

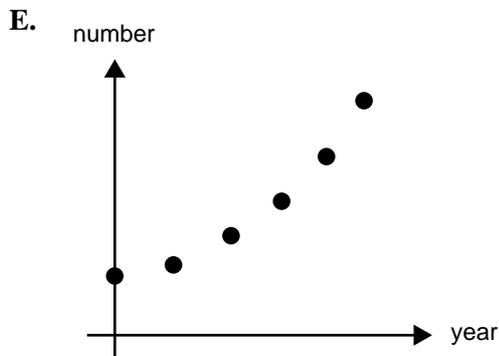
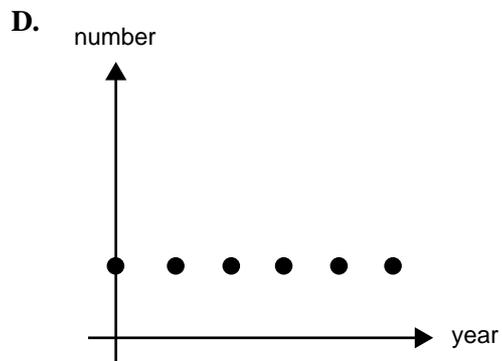
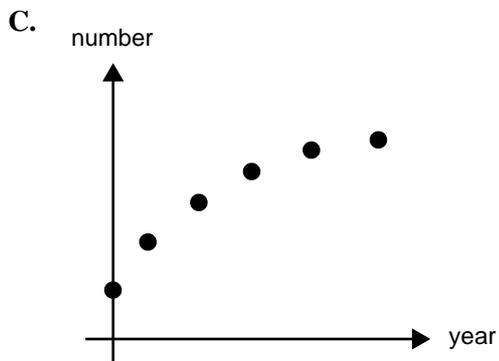
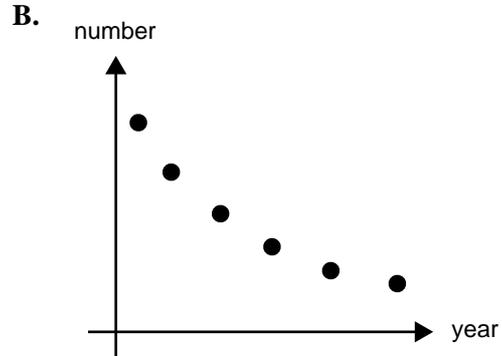
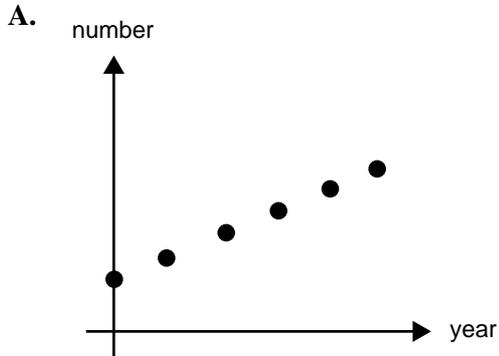
Ignoring any exchange fees, this will cost her

- A. \$159.44
- B. \$200.00
- C. \$312.49
- D. \$446.42
- E. \$2230.21

**Question 8**

At the start of each year, a cosmetics company increases the number of different eye shadow colours it manufactures by 10%.

The graph that most closely reflects the pattern of growth in the number of eye shadow colours manufactured by the company is



**Question 9**

A retiree invests \$250 000 in an income generating account that earns 4.54% per annum on the minimum yearly balance. Interest is credited to the account at the end of each year. Immediately after the interest is credited, a cheque for the amount of \$15 000 is paid out of the account to the retiree. The money remaining in the account is then reinvested. The same procedure is followed each year until the account runs out of money.

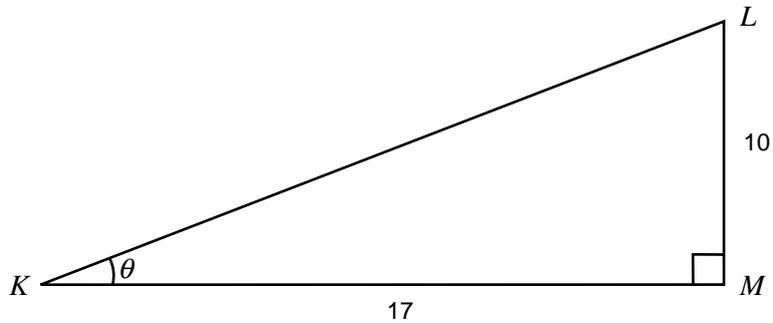
If  $A_n$  dollars is the amount of money invested at the start of the  $n$ th year, then a difference equation that describes the way in which the amount of money in the account decreases is

- A.  $A_{n+1} = 1.0454A_n - 15\,000$      where  $A_1 = 250\,000$
- B.  $A_{n+1} = 1.0454A_n - 250\,000$      where  $A_1 = 15\,000$
- C.  $A_{n+1} = 0.0454A_n - 15\,000$      where  $A_1 = 250\,000$
- D.  $A_{n+1} = 4.54A_n - 15\,000$      where  $A_1 = 250\,000$
- E.  $A_{n+1} = 0.0454A_n - 250\,000$      where  $A_1 = 15\,000$

## Module 2: Geometry and trigonometry

Before answering these questions you **must** shade the Geometry and trigonometry box on the answer sheet for multiple-choice questions.

### Question 1

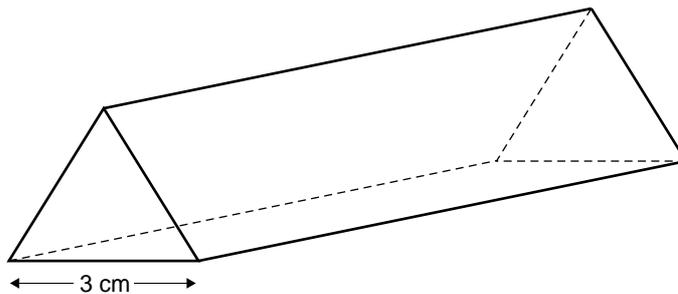


For triangle  $KLM$ , the size of angle  $\theta$  is closest to

- A.  $27^\circ$
- B.  $30^\circ$
- C.  $36^\circ$
- D.  $54^\circ$
- E.  $60^\circ$

### Question 2

A chocolate bar is made in the shape of a triangular prism. Each end is an equilateral triangle of side length 3 centimetres.

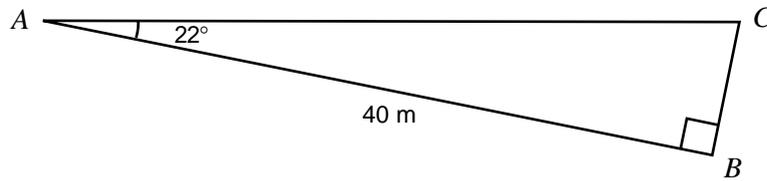


The area of a triangular end of this chocolate bar is closest to

- A.  $2.3 \text{ cm}^2$
- B.  $3.9 \text{ cm}^2$
- C.  $4.2 \text{ cm}^2$
- D.  $4.5 \text{ cm}^2$
- E.  $9.0 \text{ cm}^2$

**Question 3**

Tom and Matt are having a competition to see who can kick a football the longest distance. They both kick from point  $A$ , and the length of the kick is measured from point  $A$  to the point where the ball first makes contact with the ground. Tom's kick lands 40 metres away at point  $B$ . Matt kicks the ball at an angle of  $22^\circ$  to the direction of Tom's kick and it lands at point  $C$ , such that  $BC$  makes a right angle with  $AB$  as shown in the diagram below.

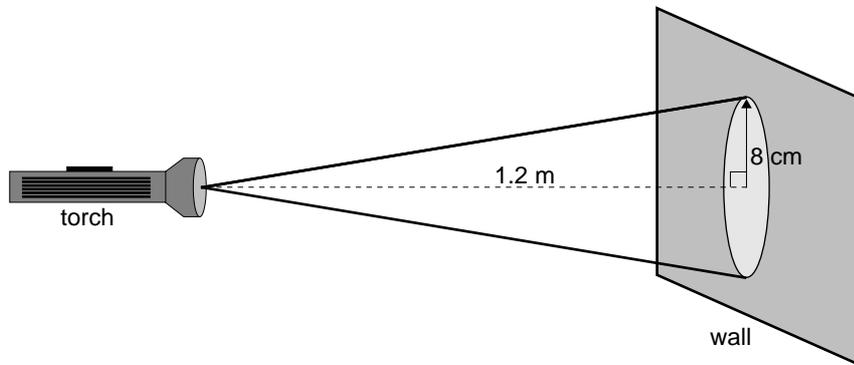


The length of Matt's kick,  $AC$ , is closest to

- A. 16 m
- B. 37 m
- C. 43 m
- D. 46 m
- E. 50 m

**Question 4**

A torch, which is held horizontally, is shone on to a wall from a distance of 1.2 metres as shown below. The circular area of light it creates on the wall has a radius of 8 centimetres.



The torch is now moved an additional 2 metres away from the wall. The radius of the circular area of light on the wall is now closest to

- A. 3 cm
- B. 10 cm
- C. 13 cm
- D. 16 cm
- E. 21 cm

**Question 5**

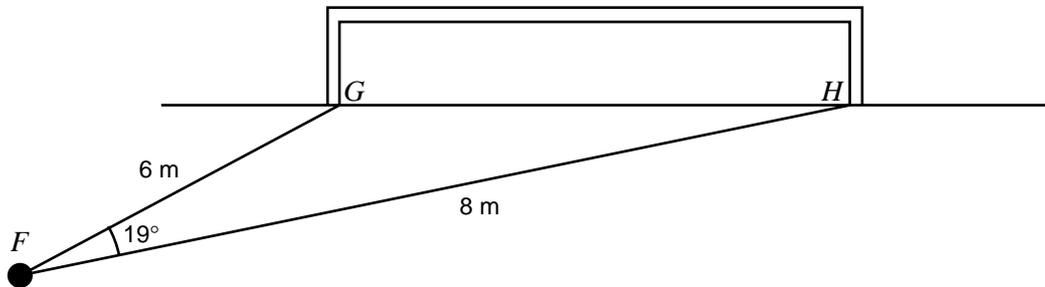
At the cinema, popcorn is sold in two different sized boxes. The boxes form similar rectangular prisms. The lengths of the sides of the smaller box are half the lengths of the corresponding sides of the larger box.

The ratio  $\text{Volume}_{\text{smaller box}} : \text{Volume}_{\text{larger box}}$  is

- A. 1:2
- B. 1:4
- C. 1:8
- D. 2:1
- E. 8:1

**Question 6**

In a game of beach soccer, a player at point  $F$  is attempting to kick a goal. She is 6 metres from one goal post at  $G$  and 8 metres from the other goal post at  $H$ . From where she is standing, the angle through which she can shoot at the goal and still score a goal is  $19^\circ$ , as shown below.

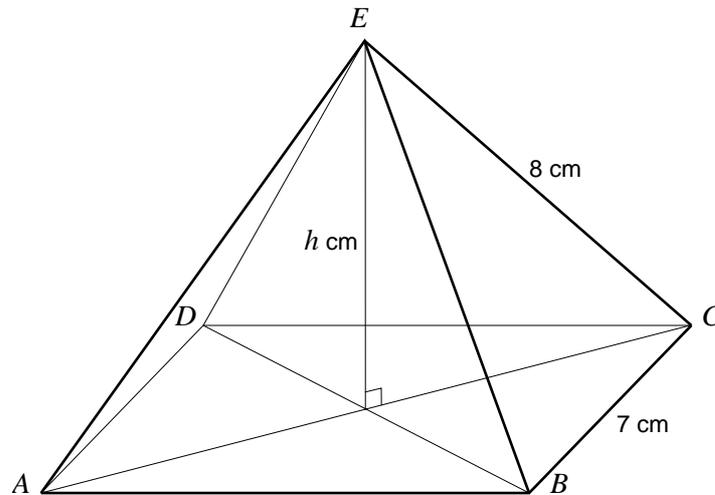


The distance between the goal posts,  $GH$ , is closest to

- A. 2.3 m
- B. 2.6 m
- C. 3.0 m
- D. 5.3 m
- E. 9.2 m

**Question 7**

For the square-based right pyramid,  $ABCDE$ , shown below, the sides of the base are 7 cm and the slant edges are 8 cm in length.



The vertical height,  $h$  cm, of this pyramid is closest to

- A. 3.9 cm
- B. 6.3 cm
- C. 7.2 cm
- D. 10.6 cm
- E. 12.7 cm

*The following information relates to Questions 8 and 9.*

In a race, a yacht rounds the first marker,  $M1$ , and then travels a distance of nine kilometres on a bearing of  $045^\circ$  true to a second marker,  $M2$ . From this marker, it sets off on a bearing of  $190^\circ$  true and travels a further ten kilometres to a third marker,  $M3$ .

**Question 8**

The bearing of marker  $M2$  from  $M3$  is

- A.  $10^\circ$  true
- B.  $35^\circ$  true
- C.  $55^\circ$  true
- D.  $145^\circ$  true
- E.  $190^\circ$  true

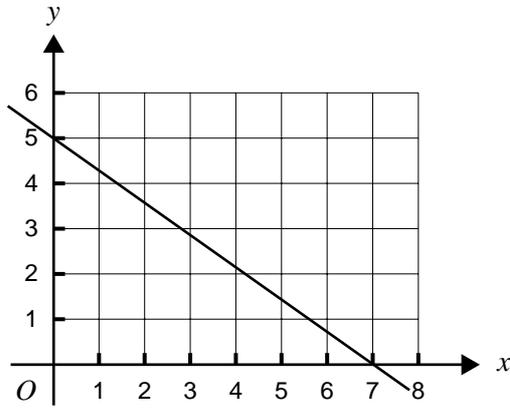
**Question 9**

The minimum distance the yacht will have to travel to get back from marker  $M3$  to the first marker,  $M1$ , is closest to

- A. 4.4 km
- B. 5.8 km
- C. 12.0 km
- D. 17.0 km
- E. 18.5 km

**Module 3: Graphs and relations**

Before answering these questions you **must** shade the Graphs and relations box on the answer sheet for multiple-choice questions.

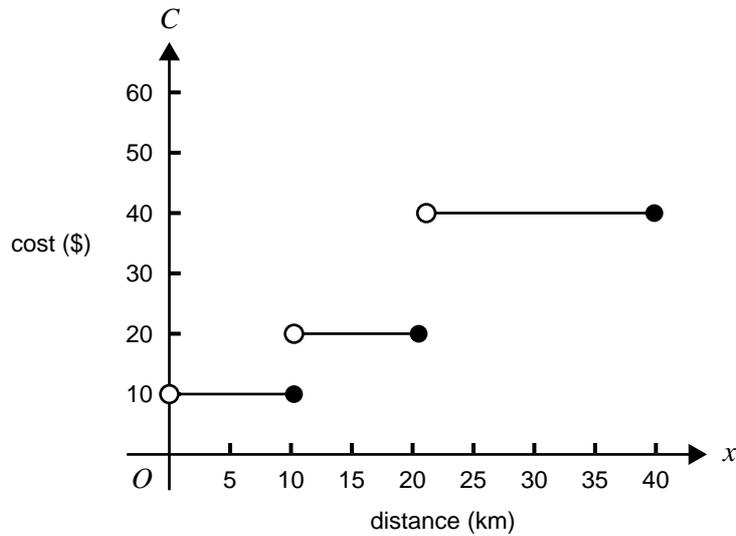
**Question 1**

The equation of this straight line is

- A.  $x = 7$
- B.  $y = 5$
- C.  $7x + 5y = 35$
- D.  $5x + 7y = 35$
- E.  $7x - 5y = 35$

**Question 2**

A courier company's charges are based on the distance required to deliver a parcel. Within a radius of 40 km, charges can be determined from a graph like that shown below.



Which one of the following rules could be used to describe this graph?

A. 
$$C = \begin{cases} 10 & \text{for } 0 < x < 10 \\ 20 & \text{for } 10 < x < 20 \\ 40 & \text{for } 20 < x < 40 \end{cases}$$

B. 
$$C = \begin{cases} 10 & \text{for } 0 \leq x \leq 10 \\ 20 & \text{for } 10 \leq x \leq 20 \\ 40 & \text{for } 20 \leq x \leq 40 \end{cases}$$

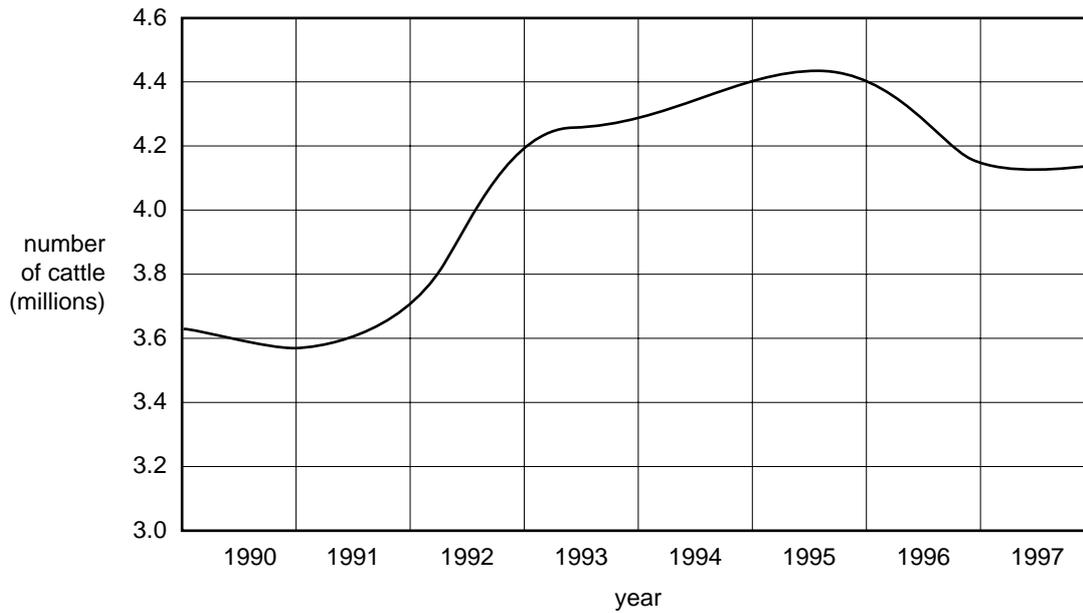
C. 
$$C = \begin{cases} 10 & \text{for } 0 < x \leq 10 \\ 20 & \text{for } 10 < x \leq 20 \\ 40 & \text{for } 20 < x \leq 40 \end{cases}$$

D. 
$$C = \begin{cases} 10 & \text{for } 0 \leq x < 10 \\ 20 & \text{for } 10 \leq x < 20 \\ 40 & \text{for } 20 \leq x < 40 \end{cases}$$

E. 
$$C = \begin{cases} 10 & \text{for } 0 < x < 10 \\ 20 & \text{for } 10 \leq x \leq 20 \\ 40 & \text{for } 20 < x < 40 \end{cases}$$

**Question 3**

The following graph shows the number of cattle in a particular state during the 1990s.



Of the years listed below, in which year did the number of cattle in the state remain relatively unchanged?

- A. 1991
- B. 1992
- C. 1994
- D. 1996
- E. 1997

**Question 4**

For the pair of simultaneous equations

$$4x = 7 - y$$

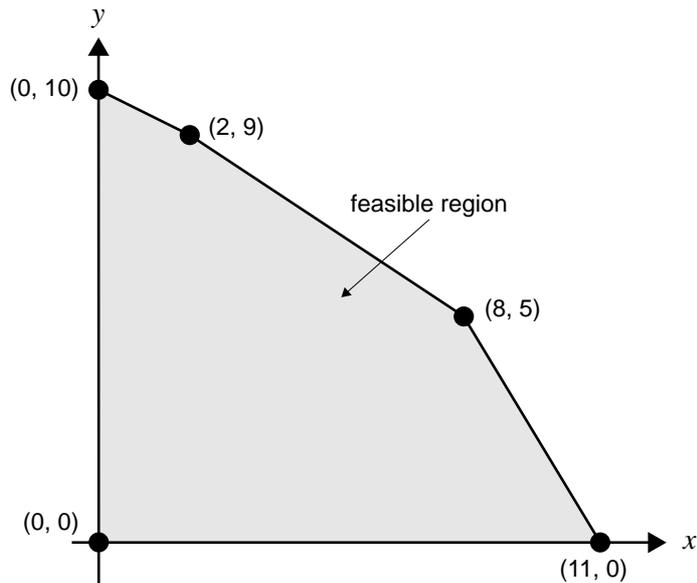
$$5x + 7y = 3$$

the solution is

- A.  $x = -2, y = -1$
- B.  $x = -2, y = 1$
- C.  $x = 1, y = 3$
- D.  $x = -1, y = 2$
- E.  $x = 2, y = -1$

**Question 5**

The feasible region for a particular linear programming problem is shown shaded on the following graph. All relevant vertices are labelled.



The **minimum** value of the expression  $4x - 2y$  for this feasible region is

- A. -20
- B. -10
- C. 0
- D. 22
- E. 44

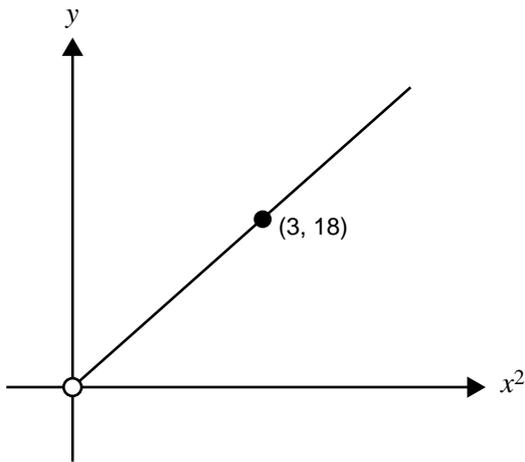
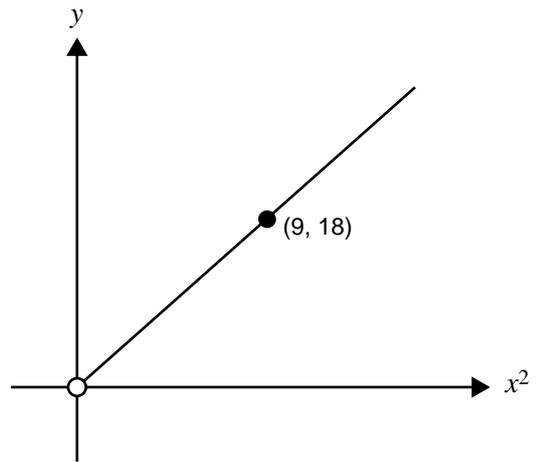
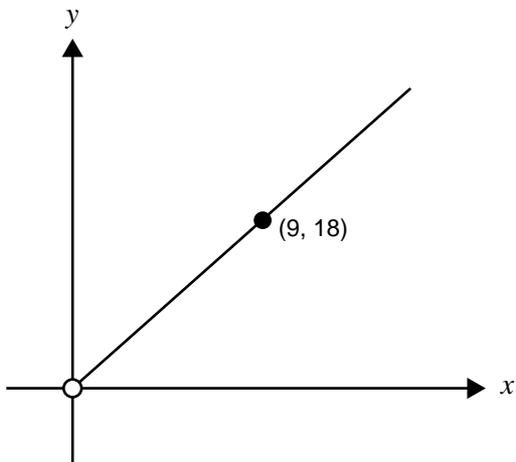
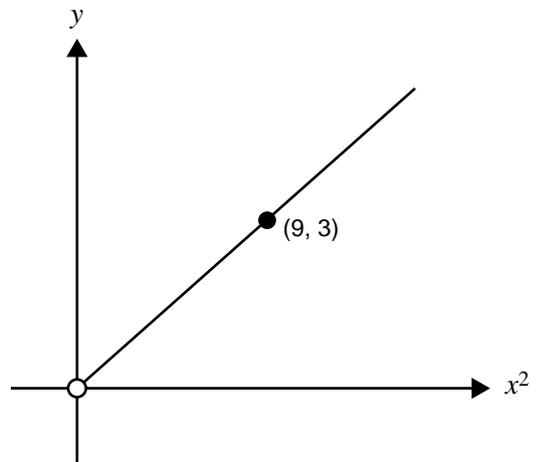
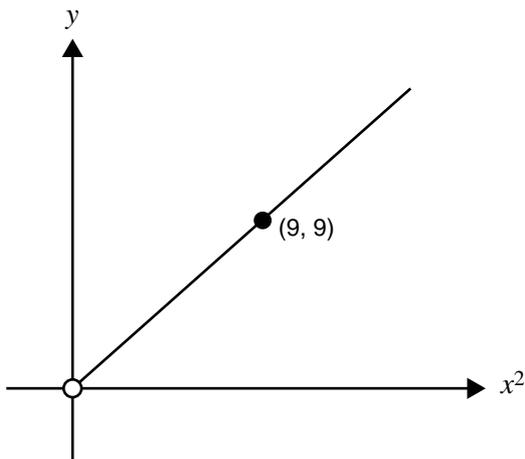
**Question 6**

For linear programming problems in general, which of the following statements is **false**?

- A. The objective function is a linear expression.
- B. The expressions defining the constraints are linear inequalities.
- C. There may be more than one optimal solution.
- D. The value of the objective function must be positive.
- E. The constraints are used to define the feasible region.

**Question 7**

The relation  $y = 2x^2$ , for  $x > 0$ , is represented by

**A.****B.****C.****D.****E.**

**Question 8**

The following inequalities define a region in the  $x$ - $y$  plane.

$$x \geq 0$$

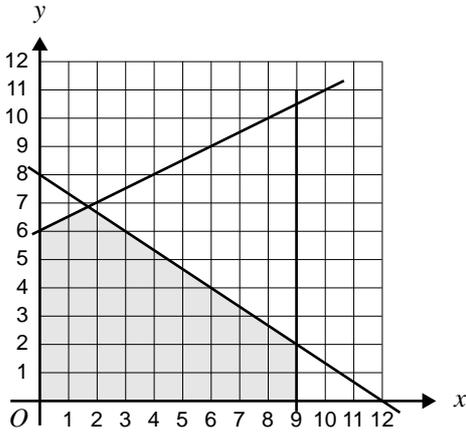
$$x \leq 9$$

$$2x + 3y \leq 24$$

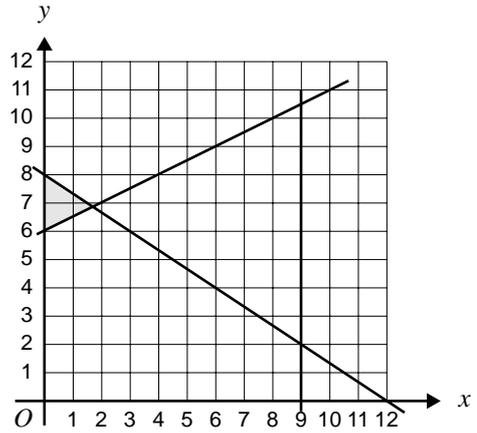
$$2y - x \geq 12$$

Which one of the following graphs represents this region?

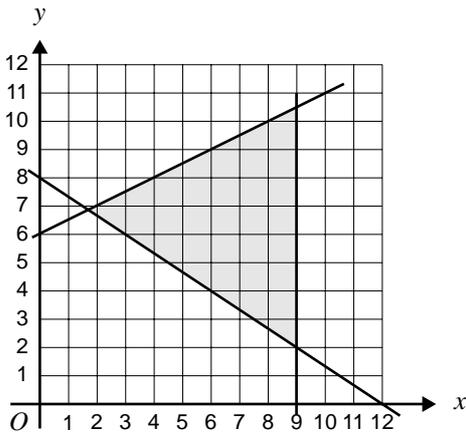
**A.**



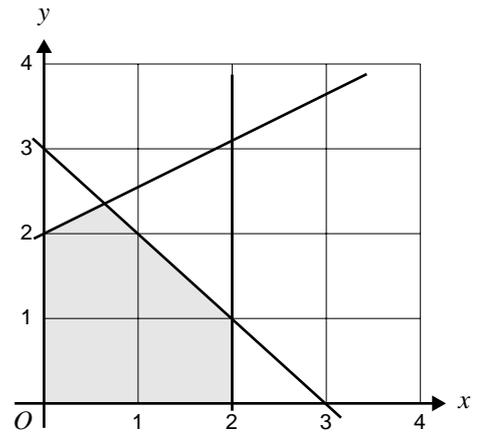
**B.**



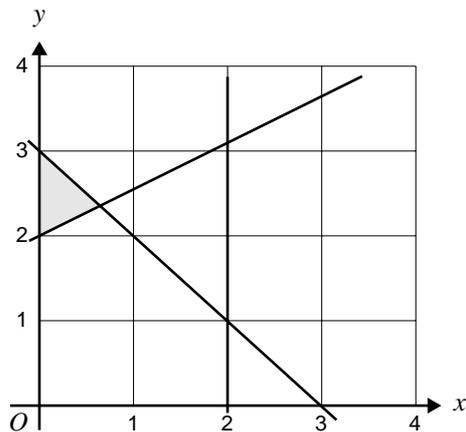
**C.**



**D.**



**E.**



**Question 9**

A company uses both trainee and qualified inspectors for quality control on an assembly line. The trainee inspectors can examine 15 items per hour, while the qualified inspectors can examine 25 items per hour.

The company needs at least 1800 items inspected in each eight-hour day.

Let  $x$  be the number of trainee inspectors employed by the company, and  $y$  be the number of qualified inspectors employed.

The constraint imposed by the number of items that need to be inspected in an eight-hour day is expressed by the inequality

- A.  $8x + 8y \geq 1800$
- B.  $15x + 25y \leq 1800$
- C.  $15x + 25y \geq 1800$
- D.  $120x + 200y \geq 1800$
- E.  $120x + 200y \leq 1800$

<b>CONTINUED OVER PAGE</b>
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**Module 4: Business-related mathematics**

Before answering these questions you **must** shade the Business-related mathematics box on the answer sheet for multiple-choice questions.

**Question 1**

Prani wants to earn \$1800 interest on a 3-year investment at 3% per annum simple interest.

The amount she needs to invest is closest to

- A. \$162
- B. \$1962
- C. \$1967
- D. \$20 000
- E. \$60 000

**Question 2**

An investment of \$16 000 is made at 4% interest per annum, compounding yearly.

The value of the investment at the end of the second year is

- A. \$17 280.71
- B. \$17 305.60
- C. \$17 325.71
- D. \$21 120.00
- E. \$21 897.10

**Question 3**

Kuong invests \$8500 at 6% interest per annum, compounding quarterly.

The amount of interest he earns during the fourth year of the investment is

- A. \$521.59
- B. \$623.62
- C. \$2231.05
- D. \$2286.38
- E. \$4489.32

**Question 4**

Rho takes a 20-year loan of \$172 000 at 6% per annum, compounding monthly and with monthly repayments.

To fully repay the loan in 20 years, the amount he must repay each month is

- A. \$716.67
- B. \$1216.54
- C. \$1232.26
- D. \$9058.63
- E. \$10 320.00

**Question 5**

A loan of \$18 000 is to be repaid by making 48 payments of \$480 per month.

For this loan, the effective rate of interest is closest to

- A. 7.0% per annum
- B. 13.7% per annum
- C. 26.7% per annum
- D. 28.0% per annum
- E. 54.9% per annum

**Question 6**

A machine is purchased for \$36 000. Its value depreciates at a rate of \$0.16 for each unit it produces. On average, the machine produces 24 000 units a year.

Using the unit cost method of depreciation, the value of the machine after six years of use is closest to

- A. \$3840
- B. \$7200
- C. \$12 960
- D. \$23 040
- E. \$32 160

**Question 7**

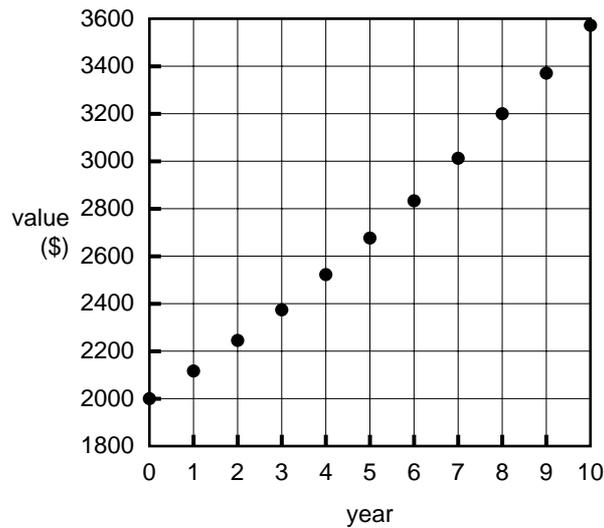
Ravi has a loan of \$135 000 at 7% per annum interest, compounding monthly. The loan is to be repaid monthly over 20 years. The scheduled repayments are \$1046.65 per month. However, he finds that he can afford to pay \$1200 per month and decides to do so for the duration of the loan.

The amount of time this will save in paying off the loan is closest to

- A. 6 months
- B. 1 year
- C. 5 years
- D. 10 years
- E. 15 years

**Question 8**

The following graph represents the growth of an investment over several years.



If  $A$  dollars is the value of the investment after  $n$  years, then a rule for describing the growth of this investment could be

- A.  $A = 2000 \times (1.06)^n$
- B.  $A = 2000 \times (0.06)^n$
- C.  $A = 2000 \times 1.06n$
- D.  $A = 2000 \times 0.06n$
- E.  $A = 2000 + (1.06)^n$

**Question 9**

Roland wants to increase the price of his products by 5%. By mistake, he increases the price by 8%. Realising the mistake, he tries to correct his mistake by discounting the **new** prices by 3%.

As a result, the products have had a price increase (from the original price) of

- A. 2.60%
- B. 3.00%
- C. 4.76%
- D. 5.00%
- E. 5.24%

## Module 5: Networks and decision mathematics

Before answering these questions you **must** shade the Networks and decision mathematics box on the answer sheet for multiple-choice questions.

### Question 1

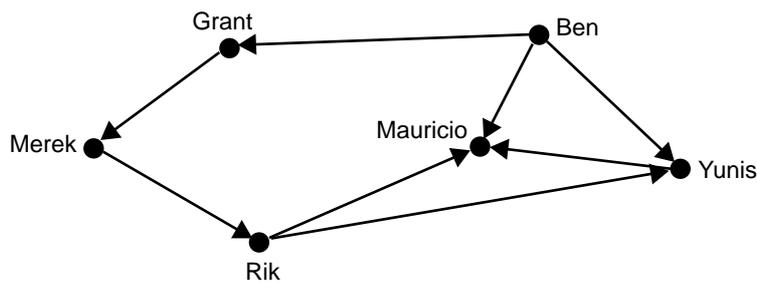
A connected planar graph of 15 edges divides the plane into eight distinct regions.

The number of vertices in this graph is

- A. 3
- B. 7
- C. 9
- D. 23
- E. 25

### Question 2

Six friends play a round-robin tennis tournament. There are no draws. The following directed graph summarises the results of the tournament to date. In this graph, an arrow going from Yunis to Mauricio, for example, indicates that Yunis defeated Mauricio.



On the basis of the information contained in the graph, which of the following statements is **false**?

- A. Rik defeated Yunis.
- B. Grant has only played two matches.
- C. Ben has won all his matches.
- D. Merek has not won a match yet.
- E. Eight matches have been played so far.

# **FURTHER MATHEMATICS**

## **Written examinations 1 and 2**

### **FORMULA SHEET**

#### **Directions to students**

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

## Further Mathematics Formulas

### Business-related mathematics

simple interest:  $I = \frac{PrT}{100}$

compound interest:  $A = PR^n$  where  $R = 1 + \frac{r}{100}$

hire purchase: effective rate of interest  $\approx \frac{2n}{n+1} \times \text{flat rate}$

annuities:  $A = PR^n - \frac{Q(R^n - 1)}{R - 1}$ , where  $R = 1 + \frac{r}{100}$

### Geometry and trigonometry

area of a triangle:  $\frac{1}{2}bh$

area of a triangle:  $\frac{1}{2}bc \sin A$

area of a circle:  $\pi r^2$

volume of a sphere:  $\frac{4}{3}\pi r^3$

volume of a cone:  $\frac{1}{3}\pi r^2h$

Pythagoras' theorem:  $c^2 = a^2 + b^2$

sine rule:  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule:  $c^2 = a^2 + b^2 - 2ab \cos C$

### Graphs and relations

#### Straight line graphs

gradient:  $m = \frac{y_2 - y_1}{x_2 - x_1}$

equation:  $y - y_1 = m(x - x_1)$  gradient-point form

$y = mx + c$  gradient-intercept form

$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$  two-point form

## Number patterns and applications

arithmetic series:  $a + (a + d) + \dots + (a + (n - 1)d) = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$

geometric series:  $a + ar + ar^2 + \dots + ar^{n-1} = \frac{a(1 - r^n)}{1 - r}, r \neq 1$

infinite geometric series:  $a + ar + ar^2 + ar^3 + \dots = \frac{a}{1 - r}, |r| < 1$

linear difference equations:  $t_n = at_{n-1} + b = a^{n-1}t_1 + b\frac{(a^{n-1} - 1)}{a - 1}, a \neq 1$   
 $= a^n t_0 + b\frac{(a^n - 1)}{a - 1}$

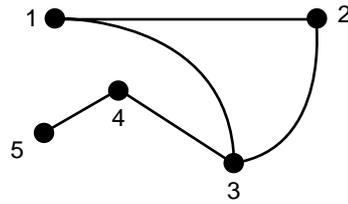
## Networks and decision mathematics

Euler's formula:  $v + f = e + 2$

## Statistics

seasonal index:  $\text{seasonal index} = \frac{\text{actual figure}}{\text{deseasonalised figure}}$

**Question 3**



Which one of the following adjacency matrices could be used to represent the graph above?

**A.**

$$\begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

**B.**

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$

**C.**

$$\begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

**D.**

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

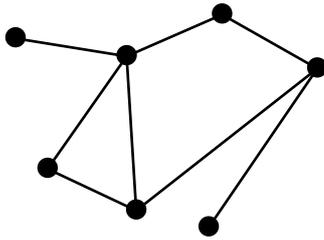
**E.**

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

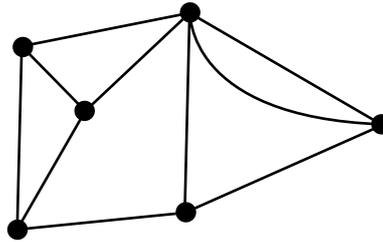
**Question 4**

Which one of the following graphs contains an Euler circuit?

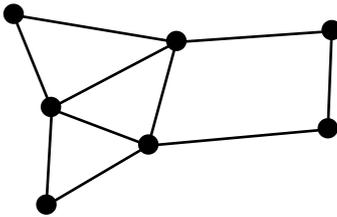
A.



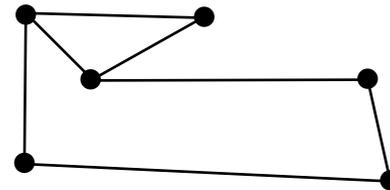
B.



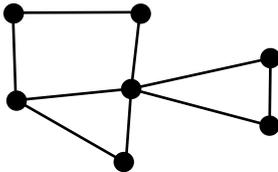
C.



D.



E.

**Question 5**

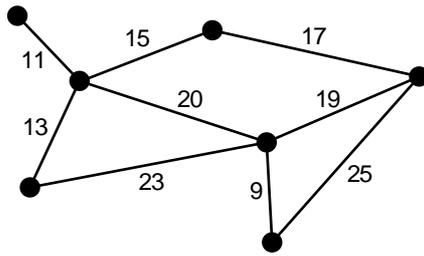
A factory has four types of machines, M1 to M4, to do a certain job and there are four tasks, T1 to T4, to be allocated to the machines. Each machine can do each task. The operating cost, in dollars, for doing a particular task on a given machine is shown in the following table.

Machine	T1	T2	T3	T4
M1	10	9	7	8
M2	6	8	7	7
M3	5	4	6	5
M4	2	3	4	5

If each task is to be assigned to only one machine, what is the optimal assignment which **minimises** the total cost?

- A. T1 to M1, T2 to M2, T3 to M3, T4 to M4  
 B. T1 to M4, T2 to M3, T3 to M2, T4 to M1  
 C. T1 to M3, T2 to M4, T3 to M1, T4 to M2  
 D. T1 to M2, T2 to M4, T3 to M1, T4 to M3  
 E. T1 to M4, T2 to M3, T3 to M1, T4 to M2

**Question 6**

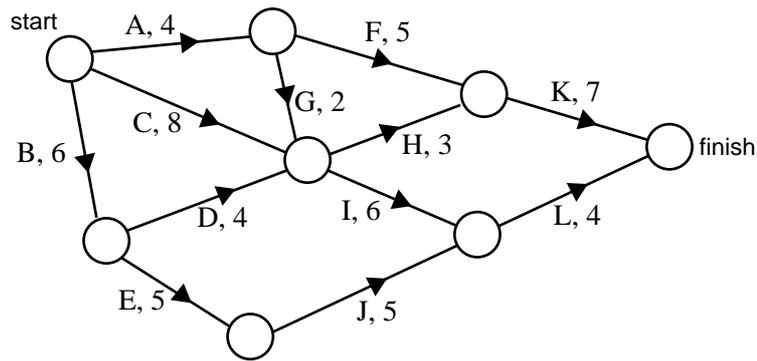


For the graph above, the total weight on the minimum weight spanning tree is

- A. 9
- B. 25
- C. 42
- D. 84
- E. 85

**Question 7**

The following network gives the time in days to complete the 12 activities for a building project.



The minimum time in which this building project can be completed is

- A. 8 days
- B. 16 days
- C. 18 days
- D. 20 days
- E. 59 days

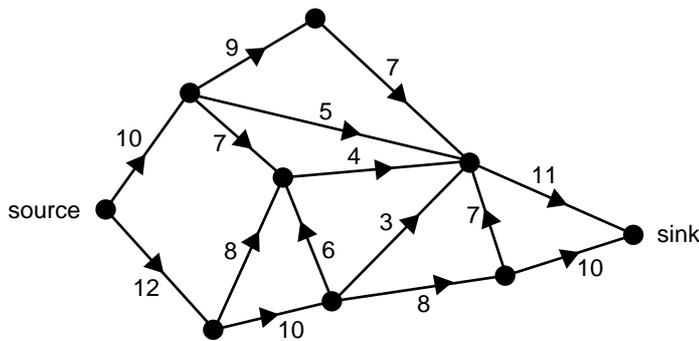
**Question 8**

The earliest and latest start times (in days) for seven activities in a design project are given in the following table.

activity	earliest start time	latest start time
<i>A</i>	0	4
<i>B</i>	0	0
<i>C</i>	6	6
<i>D</i>	9	9
<i>E</i>	6	11
<i>F</i>	13	13
<i>G</i>	18	18

The activities that lie on a critical path for this project are

- A. *B, C, D, F, G*
- B. *A, D, E, G*
- C. *C, D, E, F, G*
- D. *A, B, C, D, E, F, G*
- E. *B, E, G*

**Question 9**

The maximum flow through the network above is

- A. 19
- B. 21
- C. 22
- D. 27
- E. 34