VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education 2009

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

									Letter
Figures									
Words									

MATHEMATICAL METHODS

Written examination 1

Friday 6 November 2009

Reading time: 9.00 am to 9.15 am (15 minutes)
Writing time: 9.15 am to 10.15 am (1 hour)

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied
- Question and answer book of 9 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

a. Differentiate $x \log_e(x)$ with respect to x.

2 marks

b. For $f(x) = \dfrac{\cos(x)}{2x+2}$ find $f'(\pi)$.

3 marks

Question 2

a. Find an anti-derivative of $\frac{1}{1-2x}$ with respect to x.

2 marks

b. Evaluate $\int_1^4 \left(\sqrt{x}+1\right) dx$.

3 marks

Question 3

Let $f: R \setminus \{0\} \to R$ where $f(x) = \frac{3}{x} - 4$. Find f^{-1}, the inverse function of f.

3 marks

Question 4

Solve the equation $\tan(2x) = \sqrt{3}$ for $x \in \left(-\frac{\pi}{4}, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$.

3 marks

Question 5

Four identical balls are numbered 1, 2, 3 and 4 and put into a box. A ball is randomly drawn from the box, and not returned to the box. A second ball is then randomly drawn from the box.

a. What is the probability that the first ball drawn is numbered 4 and the second ball drawn is numbered 1?

1 mark

b. What is the probability that the sum of the numbers on the two balls is 5?

1 mark

c. Given that the sum of the numbers on the two balls is 5, what is the probability that the second ball drawn is numbered 1?

2 marks

Question 6

Oil is leaking at a constant rate to form a circular puddle on the floor. The oil is being added to the puddle at the rate of 10 mm^3 per minute causing the puddle to spread out evenly, with constant depth of 2 mm.

When the radius of the puddle is r mm, the volume, V mm^3, of oil in the puddle is given by $V = 2\pi r^2$.

Find the rate of change of the radius of the puddle when the radius is 30 mm. Give an exact answer, with units of mm per minute.

3 marks

Question 7

The random variable X has this probability distribution.

X	0	1	2	3	4
$\Pr(X = x)$	0.1	0.2	0.4	0.2	0.1

Find

a. $\Pr(X > 1 \mid X \leq 3)$

2 marks

b. $\text{Var}(X)$, the variance of X.

3 marks

Question 8

Let $f: R \rightarrow R, f(x) = e^x + k$, where k is a real number. The tangent to the graph of f at the point where $x = a$ passes through the point (0, 0). Find the value of k in terms of a.

3 marks

Question 9

Solve the equation $2\log_e(x) - \log_e(x+3) = \log_e\left(\frac{1}{2}\right)$ for x.

4 marks

Question 10

a. Use the relationship $f(x + h) \approx f(x) + hf'(x)$ for a small positive value of h, to find an approximate value for $\sqrt[3]{8.06}$.

4 marks

b. Explain why this approximate value is greater than the exact value for $\sqrt[3]{8.06}$.

1 mark

END OF QUESTION AND ANSWER BOOK

Victoria
The Place To Be

MATHEMATICAL METHODS AND MATHEMATICAL METHODS (CAS)

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

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Mathematical Methods and Mathematical Methods (CAS) Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

curved surface area of a cylinder: $2\pi rh$

volume of a cylinder: $\pi r^2 h$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a pyramid: $\frac{1}{3}Ah$

volume of a sphere: $\frac{4}{3}\pi r^3$

area of a triangle: $\frac{1}{2}bc\sin A$

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1} \qquad \int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax} \qquad \int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x} \qquad \int \frac{1}{x} dx = \log_e|x| + c$$

$$\frac{d}{dx}\left(\sin(ax)\right) = a\ \cos(ax) \qquad \int \sin(ax) dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}\left(\cos(ax)\right) = -a\ \sin(ax) \qquad \int \cos(ax) dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\ \sec^2(ax)$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + hf'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

mean: $\mu = \mathrm{E}(X)$

variance: $\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x-\mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$

END OF FORMULA SHEET