The case of technology in senior secondary mathematics: Curriculum and assessment congruence?

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Abstract

This paper outlines how curriculum and assessment congruence considerations have been addressed in the context of the incorporation of computer algebra system (CAS) technology into Victorian senior *secondary* mathematics curriculum and assessment, in particular examinations, over the period 2000–2010. The role of some related research is discussed.

Introduction

The relationship between curriculum and assessment is central to discourse in mathematics education. It is a focus of close attention in the senior secondary years where there is a strong connection to matters of certification and pathways into post-secondary education, training and work. A key aspect of mathematics is the role of technology in working mathematically. How this is reflected in senior secondary mathematics curriculum and assessment is one of the big issues of our time, especially as various software and hand-held devices that support and integrate powerful numerical, statistical, graphical, geometric and symbolic functionality have become readily available for widespread use in school mathematics. The notion of *congruence* is used here as a metaphor for *effective alignment* between the use of technology as an enabling tool in the curriculum and its use in related assessment. The term *technology* will be understood to indicate a synergy between an *artefact* and the *knowledge and understanding* of how it can be used as a tool for a *purpose*. Relevant research includes philosophical studies or meta-analyses of beliefs and values (see, for example, Bishop, 2007; Ernest, 1991), rationales, policies, trials and pilot studies (see, for example, Stacey, McCrae, Chick, Asp & Leigh-Lancaster, 2000) and strategies and processes that lead to certain directions and approaches being taken within and across jurisdictions. The re-energising of discussions on the role of digital technologies in the school mathematics curriculum arising from the emerging Australian national curriculum initiative is a good example of a contemporary context for these considerations (ACARA, 2009).

It has been common to associate mathematical functionality with certain devices; for example, numerical with scientific calculators; statistical with spreadsheet based applications; geometry with dynamic geometry software; graphing with graphics calculators; and symbolic manipulation with computer algebra systems (CAS). These associations have been used as the basis of jurisdiction specifications for proscribed, permitted or prescribed technology access in formal assessment, especially examinations. Over the past half-decade they have become less distinctive with multiple functionalities available on a single platform, for example CASIO *Classpad* or Texas Instruments *Nspire* hand-held devices and general purpose CAS software such as *Maple* and *Mathematica*. These technologies can also be used for developing documents that integrate text with 'live' mathematical computations (calculations, tables, graphs, diagrams, symbolic expressions) and as presentation tools.

In their complementary relationship, curriculum and assessment are key indicators of educational beliefs, values and preferences; for example, what is, or is not to be done, and how it may be done, by and for whom, and in what contexts. If *curriculum* is to say what students should, as a consequence of their learning, know and be able to do (concepts, skills, processes and the like) and *assessment* is the means by which judgments are made about progress and achievement, then a curriculum that sets expectations for the active use of technology as an enabling tool for working mathematically requires congruent expectations and practices for assessment. This is typically informed by inter- jurisdiction benchmarking research of curriculum and/or assessment routinely carried out by education authorities as part of the development – evaluation – review cycle (see, for example, Coupland, 2007).

A brief historical background

Over the past few decades, various technologies have been used in senior secondary mathematics curricula and related Year 12 final examinations in Victoria. While different models have been used to design and develop these curricula, there have been essentially three main types of final year mathematics courses:

- a practically oriented statistics and discrete mathematics course (e.g. networks), often with a business/financial mathematics component/option
- a mainstream function, algebra, calculus and probability course
- an advanced mathematics functions and relations, algebra, calculus, vectors, complex numbers, differential equations and mechanics course (this course assumes concurrent or previous study of the mainstream calculus based course).

In Victoria, from 1993 these have been called Further Mathematics, Mathematical Methods/Mathematical Methods CAS and Specialist Mathematics respectively, and their corresponding assumed technologies for examinations are shown in Table 1.

Table 1: Assumed technology for end of year 12 final examinations in Victoria from 1970

Stage	Assumed technology for end of Year 12 examinations in Victoria
Pre-1978	Four-figure logarithm tables and/or an approved slide rule.
1978–1996	Scientific calculator. Until 1990 there was a single 3-hour examination. From 1991 there were two 1½-hour examinations.
1997	Scientific calculator – approved graphics calculator permitted but not assumed.
1998–1999	Approved graphics calculator assumed for Mathematical Methods and Specialist Mathematics (both examinations). Scientific calculator with bivariate statistical functionality or approved graphics calculator assumed for Further Mathematics (both examinations).
2000–2005	Approved graphics calculator for Further Mathematics, Mathematical Methods and Specialist Mathematics (both examinations). Approved CAS (calculator or software) for Mathematical Methods CAS pilot study, 2002–2005 (both examinations).
2006–2009	Approved graphics calculator or CAS for Further Mathematics (both examinations). Mathematical Methods and Mathematical Methods (CAS) were alternative but like studies with a common technology free Examination 1 (worth 40 marks) and a separate technology assumed Examination 2 (worth 80 marks), with around 70% – 80% common material, approved graphics calculator assumed for Mathematical Methods Examination 2, approved CAS assumed for Mathematical Methods (CAS) Examination 2. Specialist Mathematics – technology free Examination 1. Approved graphics calculator or CAS assumed for Examination 2 (technology active but graphics calculator/CAS neutral).
2010–2013	Approved CAS or graphics calculator assumed for Further Mathematics (both examinations). Mathematical Methods (CAS) and Specialist Mathematics each have a 1-hour technology free examination. Mathematical Methods (CAS) and Specialist Mathematics each have a 2-hour technology active examination. An approved CAS (calculator or software) is the assumed technology.
2014 and beyond	(Draft) Australian curriculum has four senior secondary mathematics studies: Essential mathematics (Course A); General mathematics (Course B); Mathematical methods (Course C) and Specialist mathematics (Course D), currently under consultation. If things proceed well, 2014 could be the first year of implementation in Victoria. Assessment remains the province of states and territory jurisdictions for the interim.

The extent to which a technology such as CAS is actively used in curriculum, pedagogy and assessment has much variation across jurisdictions (see, for example, Leigh-Lancaster, 2000). A curriculum may specify expected student use of CAS in working mathematically, while precluding, permitting or assuming its use in components of school-based or examination assessment. Decisions about possible or required use (or not) may rest with the class teacher, or be partly or wholly prescribed by the relevant authority. With respect to the use of CAS in examination assessment, it may be the case that the use of technology is precluded for some components (College Board AP Calculus, Denmark, Sweden, and Victoria, Western Australia, New Zealand) and permitted (College Board AP Calculus, Sweden) or assumed (Denmark, Victoria, Western Australia, New Zealand) for other components. Other jurisdictions permit but do not require CAS for all examination assessment (France, Tasmania). Some jurisdictions do not have externally set examinations, with only school-based assessment (Ontario Canada, Queensland), but have a curriculum that explicitly incorporates the use of CAS while teachers decide locally what technology is to be used in assessment (typically with at least graphics calculator functionality assumed). A summary of jurisdictions which permit or require student access to CAS for some components of their senior secondary curriculum and assessment can be found at Computer Algebra in Mathematics Education (see CAME, 2010). Thus there will be multiple assessment models, and their efficacy with respect to the aims of the corresponding curriculum is a rich area for research.

Mathematical Methods – Mathematical Methods (CAS) 2006–2009

The Victorian model for trialling, development and implementation of Mathematical Methods (CAS), has been substantially informed by experience and expertise from other jurisdictions – the College Board, Denmark, France, Austria and Switzerland. It is, however, quite unique. Victoria is the only jurisdiction to have moved from an established study, Mathematical Methods (1992–2009) to concurrent piloting of a related equivalent and alternative study, Mathematical Methods CAS (2001–2005); then concurrent implementation of both fully accredited studies as equivalent but alternative (2006–2009) with a transition to the CAS version replacing the 'parent' version of the study from 2009 (Units 1 and 2 – Year 11 level) and 2010 (Units 3 and 4 – Year 12 level). During the concurrent implementation phase, both studies had a common technology free examination; and each had its own technology assumed examination with 70 % – 80 % questions common to the two papers. The first phase of the VCAA Mathematical Methods (CAS) pilot study was founded in the work of the Computer Algebra System – Curriculum Assessment and Teaching (CAS-CAT) project (2000 – 2002) an Australian Research Council grant funded research project partnership between the VCAA, the University of Melbourne, and calculator companies. The expanded pilot (2001–2005) also incorporated the use of CAS software.

Questions of interest include consideration of matters such as potential and actual curriculum gains, the perceived and actual impact of regular student access to CAS on student facility with traditional 'by-hand' skills, changes in teacher pedagogy and student approaches to working mathematically, use of technology with respect to gender, and performance of the two cohorts with respect to assessment in concurrent advanced mathematics study – Specialist Mathematics. The performance of the two cohorts on common assessment items in examinations has been monitored closely by the VCAA and reported in Assessment Reports (see, for example, VCAA, 2010a, 2010b) and papers (see, for example, Evans, Jones, Leigh-Lancaster, Les, Norton & Wu, 2008).

Facility with traditional 'by-hand' skills is an area of some interest – mean score data on the technology free Examination 1 for 2006–2009 consistently indicate that, in general, the Mathematical Methods (CAS) cohort perform at least as well as the Mathematical Methods cohort on related questions. In particular for 2009 (where the size of the cohorts was around 7000–8000), the distribution of student scores for each cohort across the mark range from 0 to 40 shows that at the top end, the performance of the two cohorts is essentially the same; at the very bottom end, the performance of the Mathematical Methods (CAS) cohort tends to be better, while from the low to high mark range the Mathematical Methods (CAS) cohort consistently achieves a slightly higher score than the Mathematical Methods cohort. This pattern persists when the data is controlled for general mathematical ability using the Mathematics, Science and Technology component of the General Ability Test (which has moderate correlation with respect to study specific ability) conducted in the middle of the same year. When Examination 1 results are used to control for ability on common Examination 2 extended response questions (that is, technology independent or graphics calculator/ CAS functionality neutral) comprising 21 items for a score of 35 marks out of a total of 80 marks, a similar pattern is observed, as shown in Figure 1.

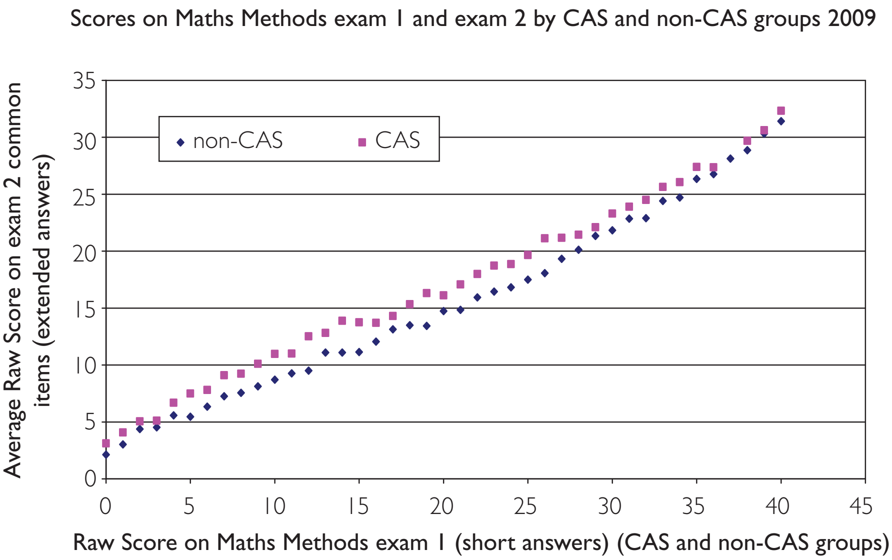

Figure 1: Average score with respect to Examination 1 (technology free) score

This is perhaps not surprising – there is an a priori argument that use of CAS as an enabling technology which provides numerical, graphical and algebraic representation of functions and relations (and can move smoothly between these representations) affords additional support for learning compared to technology that provides for only numerical and graphical representation such as a graphics calculator. If one wishes to develop student *facility* with the product rule for differentiation $(fg)' = fg' + gf'$ then this is assisted by being able to readily generate and analyse correct patterns, for example, moving from the general form of the product rule to a form where f is left undetermined, and a variety of specific function rules for g used, to the form where the rule of f is specified, for example e^x and the same variety of specific function rules used.

In this context, evaluation of the derivative can be related directly to the gradient of the tangent to the graph of the product function at a particular point and represented graphically. Where dynamic functionality is also utilised, the graph of the corresponding derivative function, and the table of values for the derivative, can be generated together. Students could then employ this to compare their perception of the gradient of the function across its domain (and subsets of the domain) with what they are seeing as the point at which the derivative is being evaluated is moved along the curve that forms the graph of the function. Naturally, the general result is established by a proof of suitable level of formality for the student cohort.

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