Mathematical Methods (CAS)
Written examinations 1 and 2 – October/November

Introduction

Mathematical Methods (CAS) Examination 1 is designed to assess students' knowledge of mathematical concepts, their skills in carrying out mathematical algorithms and their ability to apply concepts and skills in standard ways without the use of technology. Mathematical Methods Examination 1 and Mathematical Methods Examination 1 CAS will be a common examination.

Mathematical Methods Examination 2 is designed to assess students' ability to understand and communicate mathematical ideas, and to interpret, analyse and solve both routine and non-routine problems. Students are required to respond to multiple-choice questions in Part I of the paper and to extended answer questions, involving multi-stage solutions of increasing complexity, in Part II of the paper.

A formula sheet will be provided with each examination. Details of the formulas to be provided are published with the examination. The formula sheets for Mathematical Methods and Mathematical Methods (CAS) are exactly the same and common to both examinations 1 and 2.

Structure and format

Examination 1

The examination will consist of short answer questions which are to be answered without the use of technology.

The examination will be out of a total of 40 marks.

Examination 2

The examination will consist of two parts. Part 1 will be a multiple-choice section containing 22 questions and Part II will consist of extended answer questions, involving multi-stage solutions of increasing complexity. Part II will be worth 58 marks and examination 2 will be out of a total of 80 marks.

Approved materials

Examination 1

The following materials are permitted in this examination.

- Normal stationery: this includes pens, pencils, highlighters, erasers, sharpeners and rulers.
- A calculator is not allowed in this examination.
- Notes are not permitted in this examination.

Note: protractors, set squares, aids for curve sketching are no longer required for this examination and have been **removed** from the list of approved materials.

Examination 2

The following materials are permitted in this examination.

- Normal stationery: this includes pens, pencils, highlighters, erasers, sharpeners and rulers.
- One bound reference that may be annotated. The reference may be a textbook.
- Protractors, set squares, aids for curve sketching.
- CAS calculator or CAS software and, if desired, one scientific calculator.

The memories of calculators do not need to be cleared for this examination.

For approved schools only, CAS software is permitted for use by students enrolled in Mathematical Methods (CAS). The above students will be permitted to use computerbased CAS software Derive, Maple, Math-CAD, Mathematica and TI-Interactive and stored files up to 1.44 Mb on a floppy disc or CD for examinations in these studies, where the use of technology is permitted.

The VCAA publishes details of approved technology for use in mathematics examinations annually. Details of approved calculators for 2006 were published in the October 2005 *VCAA Bulletin*, No. 31. Details concerning VCAA approved reference material and technology for use in the 2006 Mathematical Methods (CAS) examinations were published in the October 2005 *VCAA Bulletin*, No. 31 and November 2005 *VCAA Bulletin*, No. 32.

Other resources

Teachers should refer to the Examination section of the *VCE and VCAL Administrative Handbook 2006, VCE Mathematics Assessment Handbook*, the VCE Mathematical Methods (CAS) Study page on the VCAA website and to the *VCAA Bulletin* for further advice during the year.

Sample examinations

The sample examination papers for Mathematical Methods (CAS) examinations 1 and 2 address content that remains unchanged and new content areas including continuous probability distributions, composition of functions, the modulus function and related rates of change.

Victorian Certificate of Education
2006

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STUDENT NUMBER

									Letter
Figures									
Words									

MATHEMATICAL METHODS (CAS)
Written examination 1

Day Date 2006

Reading time: *. to *.** (15 minutes)**
Writing time: *. to *.** (1 hour)**

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
11	11	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper and/or white out liquid/tape, a calculator or CAS.

Materials supplied
- Question and answer book of 7 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

For the function $f: (-1, \infty) \rightarrow R, f(x) = 2\log_e(x+1)$,

a. find the rule of the inverse function f^{-1}.

b. find the domain of the inverse function f^{-1}.

2 + 1 = 3 marks

Question 2

a. Find $\frac{dy}{dx}$ if $y = 3x^4\tan(x)$.

b. If $f: (-\infty, 2) \rightarrow R$ is such that $f'(x) = \frac{1}{x-2}$ and $f(1) = 6$. Find the rule of f.

2 + 2 = 4 marks

Question 3

Solve the equation $\sin(x) = \sqrt{3}\cos(x)$ for $x \in [-\pi, \pi]$, giving exact values in terms of π.

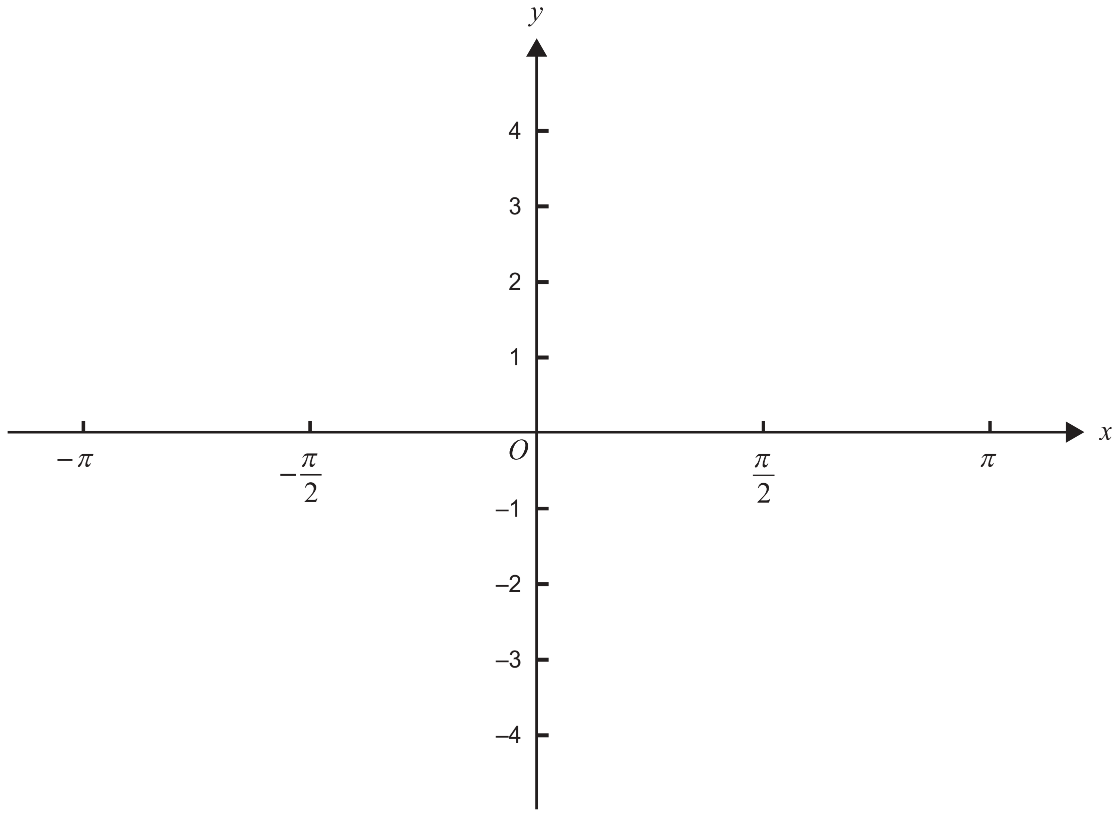
3 marks

Question 4

For the function $f:[-\pi,\pi] \rightarrow R,\ f(x) = 3\sin\left(2\left(x + \frac{\pi}{3}\right)\right)$

a. write down the amplitude and period of the function.

b. sketch the graph of the function f on the set of axes below. Label axes intercepts and endpoints with their coordinates.



2 + 3 = 5 marks

Question 5

Let X be a random variable with a normal distribution with mean 4 and standard deviation 2 and let Z be a random variable with the standard normal distribution.

a. Find $\Pr(X > 4)$.

b. Find b such that $\Pr(X > 5) = \Pr(Z < b)$.

1 + 1 = 2 marks

Question 6

The random variable X has a distribution with probability density function given by

$$f(x) = \begin{cases} ax(2-x) & \text{if } 0 \le x \le 2 \\ 0 & \text{if } x < 0 \text{ or } x > 2 \end{cases}$$

where a is a positive constant.

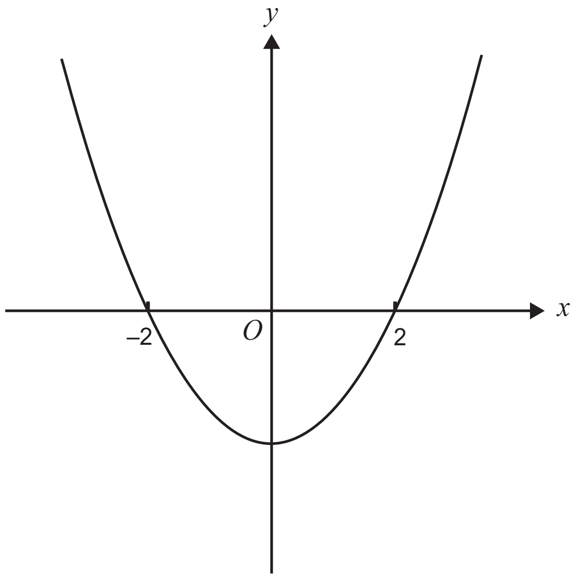
a. Find the value of a.

b. Find $\Pr\left(X < \frac{1}{2}\right)$.

3 + 2 = 5 marks

Question 7

Part of the graph of $y = x^2 - 4$ is shown below.



a. On the same set of axes sketch the graph of $y = |x^2 - 4|$.

b. Hence find the area of the region bounded by the two curves.

1 + 2 = 3 marks

Question 8

Let $f(x) = x^2 + 1$ and $g(x) = \log_e(x)$.

a. Write down the rule of $g(f(x))$.

b. Find the derivative of $g(f(x))$.

c. Hence find an anti-derivative of $\frac{x}{x^2+1}$.

1 + 1 + 1 = 3 marks

Question 9

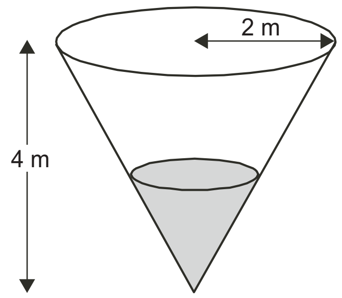
The line $y = 4x - 1$ is a tangent to the curve $y = x^4 + c$.

Find the exact value of c.

4 marks

Question 10

A right conical vessel with base radius 2 m and height 4 m is being filled with water at a constant rate of 3 m^3/min. At what rate is the water rising when the depth is 3 m?



4 marks

Question 11

Suppose that the probability of snow at a resort is dependent on whether or not it has snowed on the previous day. If it has snowed the day before the probability of snow is 0.6. If it has not snowed on the previous day then the probability of snow is 0.1.

If it has snowed on Thursday, what is the probability that it will **not** snow on the following Saturday?

4 marks

END OF QUESTION AND ANSWER BOOK

Victorian Certificate of Education
2006

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STUDENT NUMBER Letter

Figures

Words

MATHEMATICAL METHODS (CAS)

Written examination 2

Day Date 2006

Reading time: *. to *.** (15 minutes)**
Writing time: *. to *.** (2 hours)**

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	4	4	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator. For approved computer based CAS, their full functionality may be used and stored files up to 1.44 Mb on a floppy disk or CD.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 22 pages with a detachable sheet of miscellaneous formulas in the centrefold.
- Answer sheet for multiple-choice questions.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- All written responses must be in English.

At the end of the examination
- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Version 2 – March 2006

SECTION 1

Instructions for Section I

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Question 1

The average value of the function $y = \sin(x)$ over the interval $[0, \pi]$ is

A. $\frac{2}{\pi}$

B. $\frac{\pi}{2}$

C. 0.5

D. 0

E. π

Question 2

The simultaneous equations $(m - 2)x + 3y = 6$ and $2x + (m + 2)y = m$, have a unique solution for

A. $m \in R\backslash\{0\}$

B. $m \in R\backslash\{-1, 1\}$

C. $m \in R\backslash\left\{-\sqrt{10}, \sqrt{10}\right\}$

D. $m \in R\backslash[-1, 1]$

E. $m = 2$ or $m = -2$

Question 3

The largest set of real values of p for which $|p + 3| > 3$ is

A. $p > 0$ or $p < -6$

B. $p > 0$

C. $p < -6$

D. $p > -6$

E. $p > 0$ or $p < -3$

Question 4

The transformation $T : R^2 \to R^2$ is defined by

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 3 \\ 2 \end{bmatrix}.$$

The equation of the image of the curve $y = x^2$ under T is

A. $y = 2(x - 3)^2 + 2$

B. $2y = (x - 3)^2 + 2$

C. $y = 2(x + 3)^2 - 2$

D. $2y = 2(x + 3)^2 - 2$

E. $y = \left(\dfrac{x}{2} - 3\right)^2 + 2$

Question 5

A small object is oscillating up and down at the end of a vertical spring. The object is h metres above its starting point at time t seconds, where

$$h = 0.5\left(1 - e^{-0.05t}\cos\left(\frac{3\pi t}{2}\right)\right) \quad \text{and} \quad t > 0$$

The rate in m/s, correct to two decimal places, at which the object is rising 2.5 seconds after motion starts is

A. 0.03
B. –0.43
C –0.40
D. –1.45
E. 1.67

Question 6

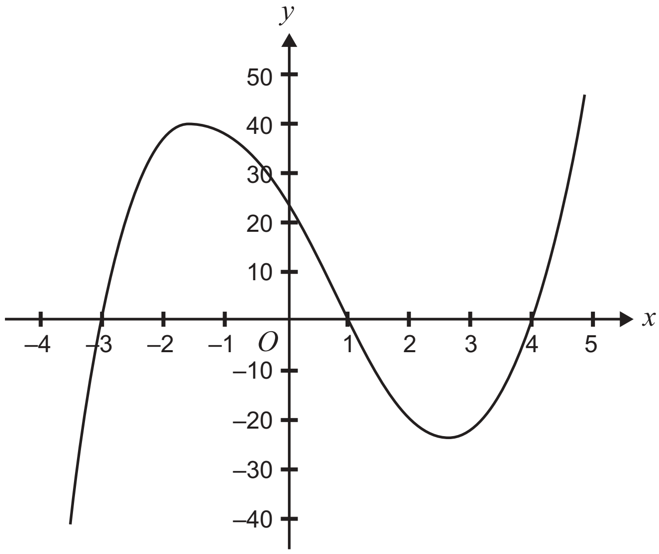
The number of ants, N, in a colony varies with time according to the rule $N(t) = 1000e^{0.1t}$, where t is the time measured in days, and $t \geq 0$.

The average rate of change in the number of ants over the first 10 days is closest to

A. 172
B. 183
C. 272
D. 1718
E. 2718

Question 7

Part of the graph of $y = ax^3 + bx^2 + cx + d$ is shown below.



The values of *a, b, c* and *d* could be

	a	*b*	*c*	*d*
A.	1	–2	–11	12
B.	2	–4	–22	24
C.	1	2	–22	12
D.	–2	–6	–2	24
E.	3	–1	1	24

Question 8

If $f(x) = e^{2x}$ for all $x \in R$, then $[f(x)]^2 = f(y)$, where y is equal to

A. $2x$

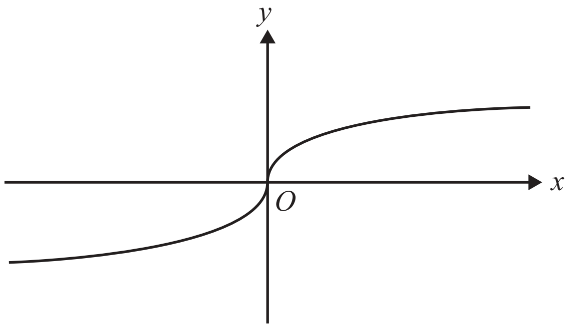
B. $4x$

C. x^2

D. $2x^2$

E. $4x^2$

Question 9



The rule of the graph shown could be

A. $y = \frac{1}{x^3}$

B. $y = x^{\frac{3}{2}}$

C. $y = x^{\frac{2}{3}}$

D. $y = x^{\frac{1}{3}}$

E. $y = |x|^{\frac{1}{3}}$

Question 10

The maximal domain, D, of the function $f : D \to R$ with rule $f(x) = \log_e(|x|) + 1$ is

A. $R \setminus \{0\}$

B. $(-1, \infty)$

C. R

D. $(0, \infty)$

E. $(-\infty, 0)$

Question 11

The function $f : [a, \infty) \to R$ with rule $f(x) = 2x^3 - 3x^2 + 6$ will have an inverse function provided

A. $a \geq 1$

B. $a \geq 0$

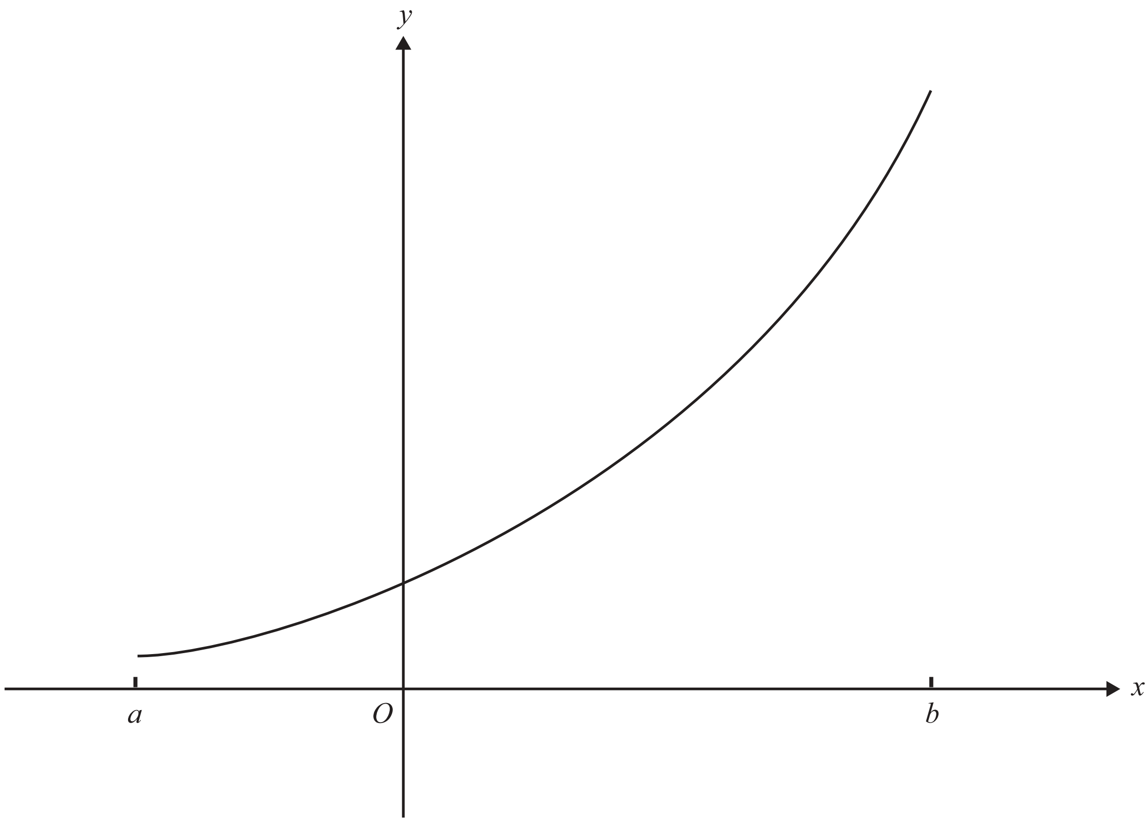
C. $a \leq 0$

D. $a \leq \frac{3}{2}$

E. $a \leq 1$

Question 12

The graph of the function $y = f(x)$, where f: $[a, b] \to R$ and a is a negative constant and b a positive constant, is shown below.



The function $F: [a, b] \to R$ is defined by $F(t) = \int_0^t f(x)dx$.

Then $F(t) > 0$ for

A. $t \in [a, b]$

B. $t \in [a, b] \backslash \{0\}$

C. $t \in [0, b]$ only

D. $t \in (0, b]$ only

E. $t \in [a, 0)$ only

Question 13

The interval [0, 4] is divided into n equal subintervals by the points $x_0, x_1, \ldots x_{n-1}, x_n$ where $0 = x_0 < x_1 < \ldots < x_{n-1} < x_n = 4$. Let $\delta x = x_i - x_{i-1}$ for $i = 1, 2, \ldots, n$.

Then $\lim_{\delta x \to 0} \sum_{i=1}^{n} \left(x_i \, \delta x\right)$ is equal to

A. $\int_4^0 x \; dx$

B. $\int_0^4 \frac{x^2}{2} \, dx$

C. 0

D. 4

E. 8

Question 14

If $y = |\sin(x)|$, then the rate of change of y with respect to x at $x = k$, $\pi < k < 2\pi$, is

A. $-\cos(k)$

B. $\cos(k)$

C. $-\sin(k)$

D. $\sin(k)$

E. $k \cos(1)$

Question 15

A function $f: R \to R$ is such that

$f'(x) = 0$ at $x = 0$ and $x = 2$

$f'(x) < 0$ for $0 < x < 2$ and $x > 2$

$f'(x) > 0$ for $x < 0$

Which one of the following is true?

A. The graph of f has a stationary point of inflection at $x = 0$.

B. The graph of f has a local maximum point at $x = 2$.

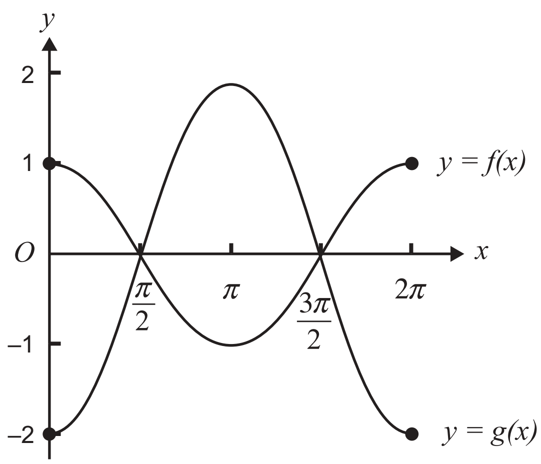
C. The graph of f has a stationary point of inflection at $x = 2$.

D. The graph of f has a local minimum point at $x = 0$.

E. The graph of f has a local minimum point at $x = 2$.

Question 16

The diagram below shows the graph of two circular functions, f and g.



The graph of the function with equation $y = f(x)$ is transformed into the graph of the function $y = g(x)$ by

A. a dilation by a scale factor of $\frac{1}{2}$ from the y-axis and a reflection in the x-axis.

B. a dilation by a scale factor of $\frac{1}{2}$ from the x-axis and a reflection in the x-axis.

C. a dilation by a scale factor of 2 from the x-axis and a reflection in the x-axis.

D. a dilation by a scale factor of 2 from the y-axis and a reflection in the y-axis.

E. a dilation by a scale factor of 2 from the x-axis and a reflection in the y-axis.

Question 17

The equation of the normal to the curve with equation $y = 2x^{\frac{3}{2}}$ at the point where $x = 4$ is

A. $y = -\frac{1}{6}x + \frac{50}{3}$

B. $y = 6x - 18$

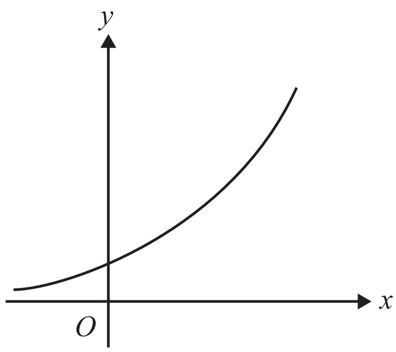
C. $y = 6x - 8$

D. $y = 6x + 40$

E. $y = 160x - 62$

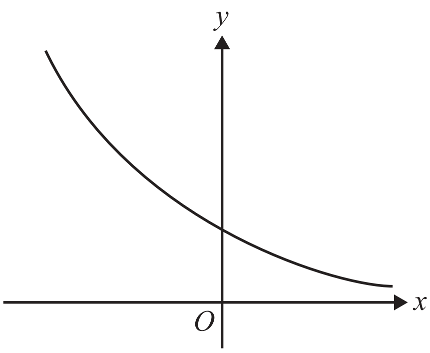
Question 18

The graph of the function f, with rule $y = f(x)$, is shown below.

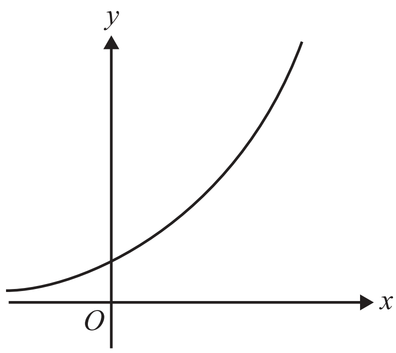


Which one of the following could be the graph of the curve with equation $y = f'(x)$?

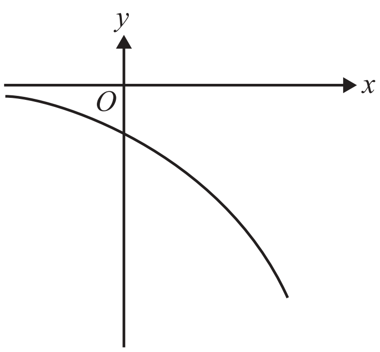
A.



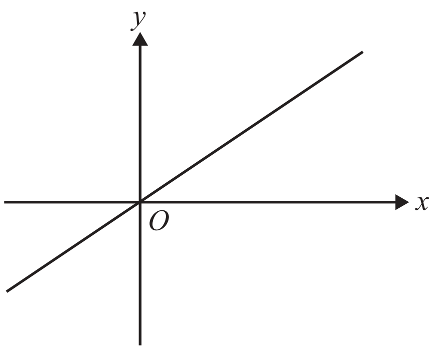
B.



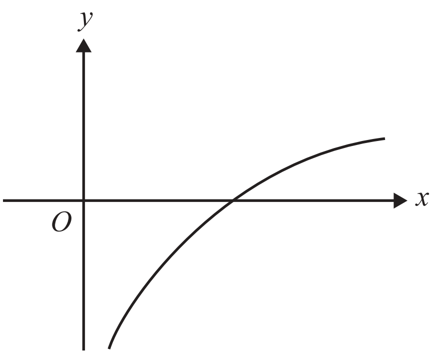
C.



D.



E.



Question 19

The random variable X has a normal distribution with mean 12.2 and standard deviation 1.4.

If Z has the standard normal distribution, then the probability that X is greater than 15 is equal to

A. $\Pr(Z < 2)$

B. $\Pr(Z > 2)$

C. $\Pr(Z > -2)$

D. $1 - \Pr(Z > 2)$

E. $1 - \Pr(Z < -2)$

Question 20

The probability of winning a single game of chance is 0.15, and whether or not the game is won is independent of any other game. Suppose Jodie plays a sequence of n games.

If the probability of Jodie winning at least one game is more than 0.95, then the smallest value n can take is closest to

A. 19

B. 15

C. 8

D. 7

E. 4

Question 21

The number, X, of children in a family is a random variable with the following probability distribution.

x	0	1	2	3
$\Pr(X = x)$	0.4	0.3	0.2	0.1

If two families are selected at random, the probability that they have the same number of children is

A. 0.10

B. 0.20

C. 0.30

D. 0.40

E. 0.50

Question 22

The continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} \frac{1}{2}\sin(x) & \text{if} \quad 0 < x < \pi \\ 0 & \text{elsewhere} \end{cases}$$

The value of a such that $\Pr(X > a) = 0.25$ is closest to

A. 0.25

B. 0.75

C. 1.04

D. 1.05

E. 2.09

END OF SECTION 1

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

a. Consider the function $f: [0, 2] \to R, f(x) = ax^3 + bx^2 + cx + 2$.

The graph of $y = f(x)$ has a turning point at the point with coordinates (1, 1).

i. Find the values of a and b in terms of c.

ii. If also $f(2) = 0$, find the value of c.

2 + 1 = 3 marks

b. Consider the function $f: [0, 2] \to R, f(x) = (x-1)^2(x-2) + 1$.

i. Find $f'(x)$.

ii. The coordinates of the turning point of the graphs of $y = f(x)$ occur at $(m, 1)$ and $\left(n, \frac{23}{27}\right)$. Find the values of m and n.

iii. State the absolute maximum and minimum values of this function.

iv. Find the real values of p for which the equation $f(x) = p$, where $x \in [0, 2]$, has exactly one solution.

1 + 2 + 2 + 2 = 7 marks

c. Consider the function $f: R \to R, f(x) = (x-1)^2(x-2) + 1$.

i. Describe a sequence of transformations which transforms the graph of $y = f(x)$ into the graph of $y = f\left(\frac{x}{k}\right) - 1$.

ii. Find the x-axis intercepts of the graph of $y = f\left(\frac{x}{k}\right) - 1$.

iii. Find the real values of h for which **only one** of the solutions of $f(x+h) = 1$ is positive.

2 + 2 + 2 = 6 marks

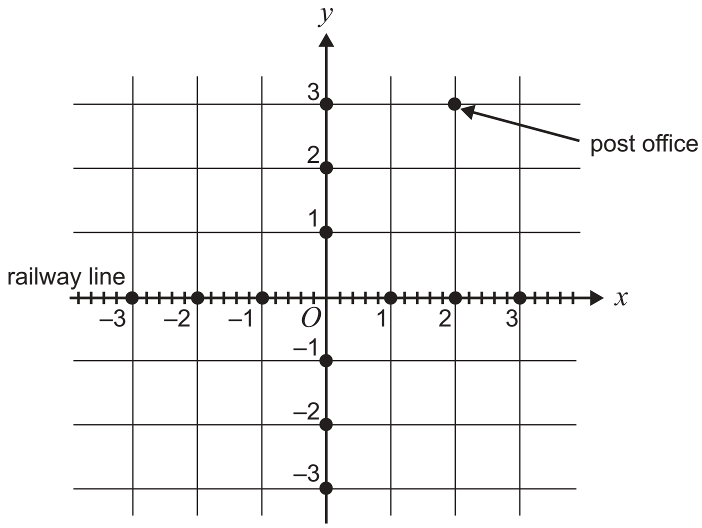
Total 16 marks

Question 2

In a country town, it is decided that a new road should be built.

The grid below shows the positions of the railway line and the post office.

In each direction, 1 unit represents 1 kilometre.

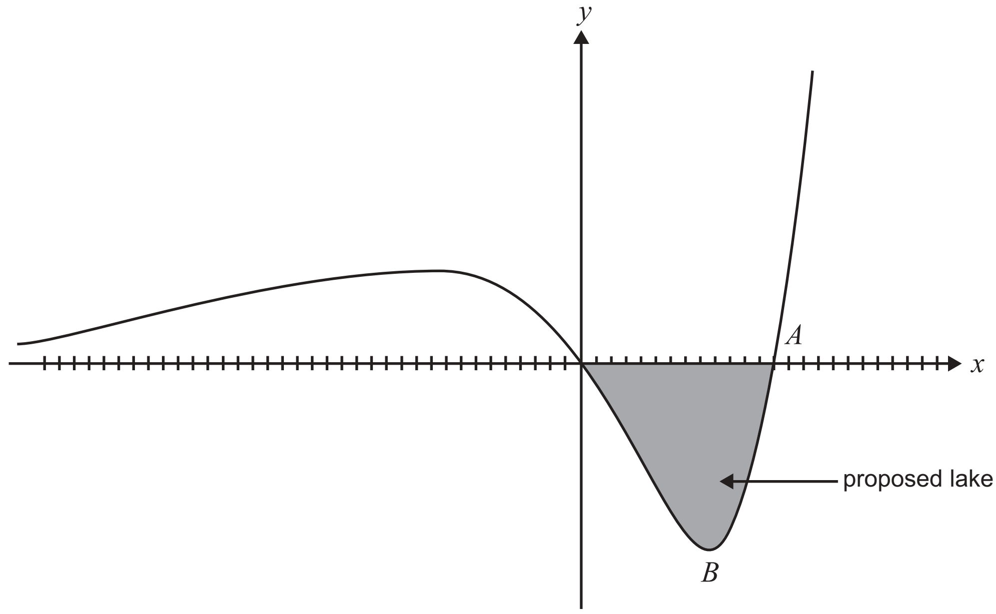


It is decided that the road should follow the path whose equation is

$$y = (2x^2 - 3x)e^{ax} \qquad \text{where } a > 0.$$

a. Find the exact value of a for which the road will pass through the post office.

2 marks



b. The town council wishes to develop the shaded region bounded by the road and the railway line as a lake for native water birds, as shown in the diagram above.

i. Find the x-coordinate of the point A where the road crosses the railway line.

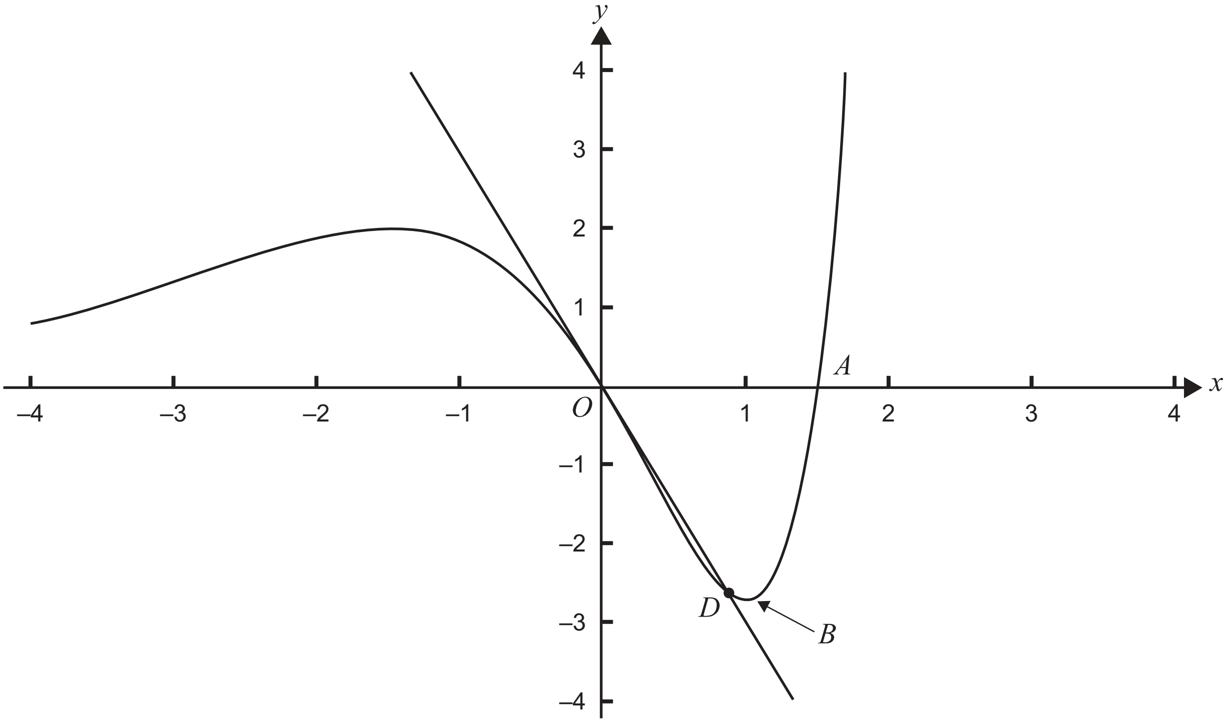
ii. For what value of a is the area of the lake 10 square units? Give your answer correct to three decimal places.

2 + 4 = 6 marks

c. In fact, they decide to build the road for which $a = 1$. Find the exact coordinates of the turning point B.

3 marks

d. In the actual construction of the road with $a = 1$, a straight line segment is taken from the origin to a point D on the curve, between the origin and B, before the road again follows the curve. This straight line segment is part of the tangent to the curve at the origin.



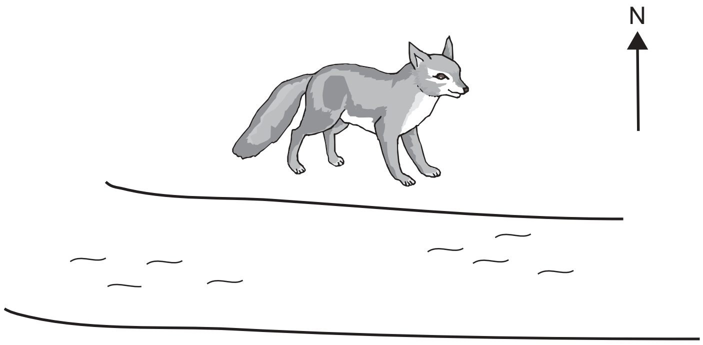
i. Find the equation of the tangent at the origin.

ii. Find the coordinates of the point D. Give values correct to two decimal places.

2 + 2 = 4 marks

Total 15 marks

Question 3



A fox hunts each night in one of two areas, either on the north side of a creek or on the south side. The side it hunts on each night depends only on the side it hunted on the night before. If the fox hunts on the north side of the creek one night, then the probability of the fox hunting on the north side of the creek the next night is $\frac{2}{5}$.

The transition matrix for the probabilities of the fox hunting on either side of the creek given the side of the creek hunted on the previous night is $\begin{bmatrix} \frac{2}{5} & \frac{4}{5} \\ \frac{3}{5} & \frac{1}{5} \end{bmatrix}$.

a. If the fox hunts on the south side of the creek one night, what is the exact probability that it hunts on the north side of the creek the next night?

1 mark

b. Suppose the fox hunts on the north side of the creek on Monday night. What is the exact probability that it will hunt on the north side of the creek on Thursday night of the same week?

3 marks

c. In the long term, what percentage of nights, to the nearest per cent, will the fox hunt on the north side of the creek?

3 marks

The time, t, in hours that the fox spends hunting each night is independent of the area it hunts in and is a random variable with probability density function

$$f(x)=\begin{cases}\frac{3}{32}t(4-t) & \text{if} \quad 0\le t\le 4\\ 0 & \text{otherwise}\end{cases}$$

d. What is the exact probability that the fox spends longer than 3 hours hunting on a night?

2 marks

e. What is the probability, correct to three decimal places, that the fox spends longer than 3 hours hunting on at least two out of three nights?

2 marks

f. On 10.4% of nights the fox hunts for less than n minutes. Find the value of n.

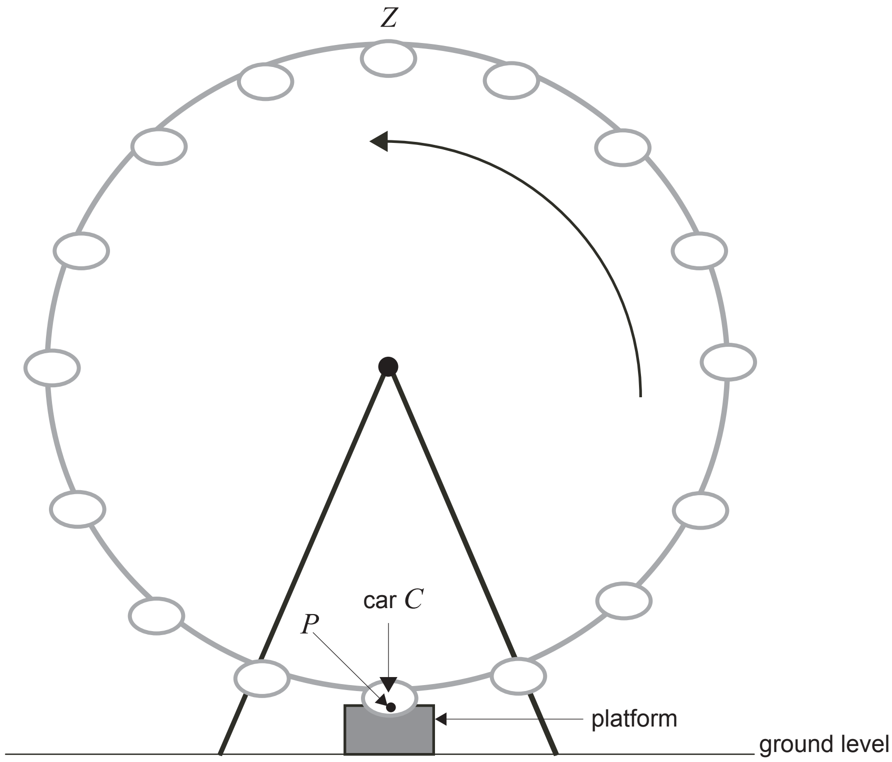
3 marks

Total 14 marks

Question 4

A Ferris wheel at a theme park rotates in an anticlockwise direction at a constant rate. People enter the cars of the Ferris wheel from a platform which is above ground level. The Ferris wheel does not stop at any time.

The Ferris wheel has 16 cars, spaced evenly around the circular structure.



A spider attached itself to the point P on the side of car C when the point P was at its lowest point at 1.00 pm.

The height, h metres, of the point P above ground level, at time t hours after 1.00 pm, is given by

$$h(t) = 62 + 60 \sin\left(\frac{(5t-1)\pi}{2}\right)$$

a. Write down the maximum height, in metres, of the point P above ground level.

1 mark

b. Write down the minimum height, in metres, of the point P above ground level.

1 mark

c. At what time, after 1.00 pm, does point P first return to its lowest point?

1 mark

d. **i.** Find the time, after 1.00 pm, when the point P first reaches a height of 92 metres above ground level.

ii. Find the **number of minutes** during one rotation when the point P is at least 92 metres above ground level.

2 + 1 = 3 marks

e. **i.** Write down an expression, in terms of t, for the rate of change of h with respect to time.

ii. At what rate (in m/h) is h changing when $t = 1$?

1 + 1 = 2 marks

When point P first reaches position Z, that is, its highest point (see diagram page 20), the spider becomes frightened. It drops down from the car on a thread (which remains vertical at all times) at a rate of 5 metres per minute until it reaches the ground.

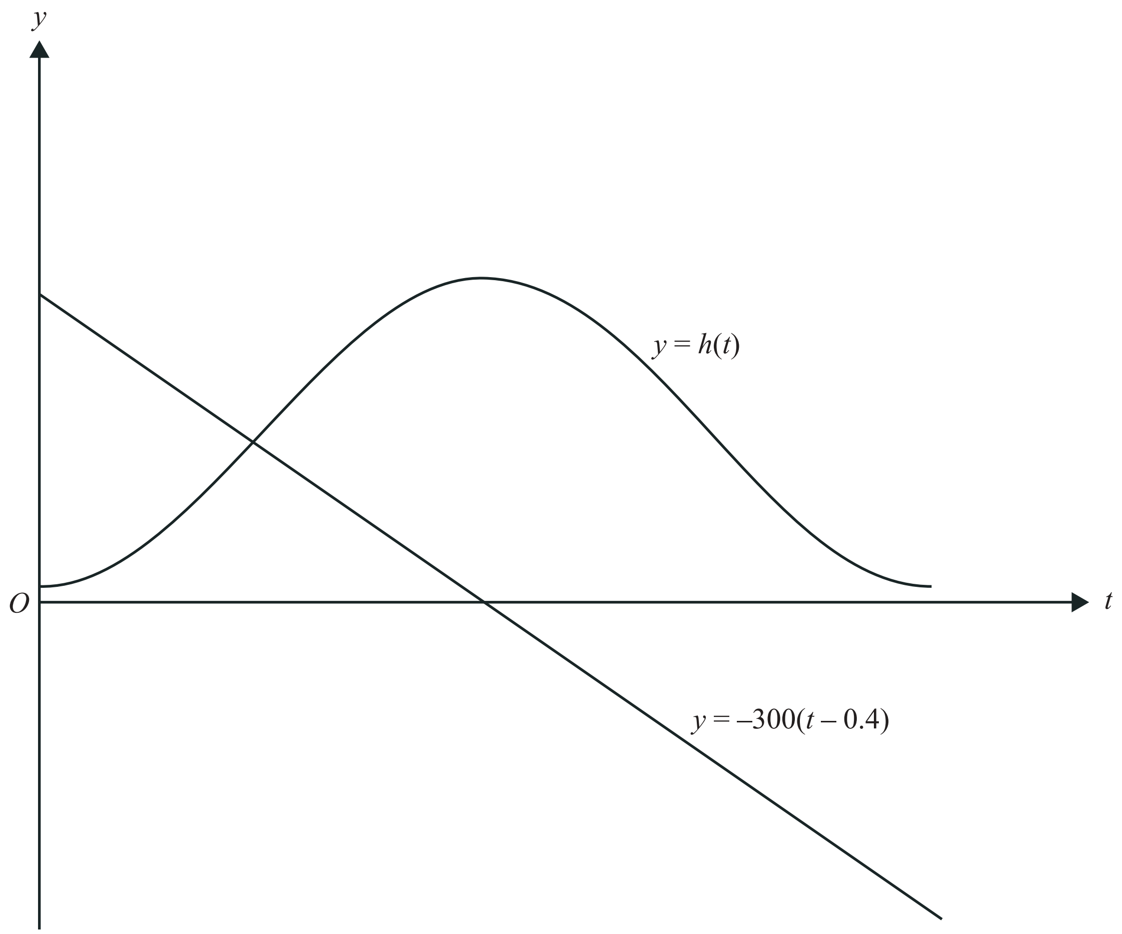
As it drops, the spider's height $s(t)$ metres above ground level at time t (where t is the time in hours after 1.00 pm) is given by

$$s(t) = h(t) - 300(t - 0.4)$$

f. The graph of $y = h(t)$ for the first revolution of the Ferris wheel after 1.00 pm and the graph of $y = -300(t - 0.4)$ are shown together on the axes below.

i. On the diagram label the local maximum point of the graph of $y = h(t)$ with its coordinates.

ii. On the diagram draw a graph which shows the height of the spider above ground level at time t.



iii. Find, to the nearest minute, the time taken from when the spider leaves car C to when it reaches the ground.

1 + 2 + 2 = 5 marks

Total 13 marks

END OF QUESTION AND ANSWER BOOK

MATHEMATICAL METHODS AND MATHEMATICAL METHODS (CAS)

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Version 2 – March 2006

Mathematical Methods and Mathematical Methods CAS Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$	volume of a pyramid:	$\frac{1}{3}Ah$
curved surface area of a cylinder:	$2\pi rh$	volume of a sphere:	$\frac{4}{3}\pi r^3$
volume of a cylinder:	$\pi r^2 h$	area of a triangle:	$\frac{1}{2}bc\sin A$
volume of a cone:	$\frac{1}{3}\pi r^2 h$		

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$$

$$\frac{d}{dx}\left(\sin(ax)\right) = a\ \cos(ax)$$

$$\frac{d}{dx}\left(\cos(ax)\right) = -a\ \sin(ax)$$

$$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\ \sec^2(ax)$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\int \frac{1}{x} dx = \log_e|x| + c$$

$$\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$$

$$\int \cos(ax)dx = \frac{1}{a}\sin(ax) + c$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + hf'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

mean: $\mu = E(X)$

variance: $\mathrm{var}(X) = \sigma^2 = E((X-\mu)^2) = E(X^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x-\mu)^2\, p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$

END OF FORMULA SHEET