VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education 2006

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

Figures									Letter
Words									

MATHEMATICAL METHODS
Written examination 1

Friday 3 November 2006

Reading time: 9.00 am to 9.15 am (15 minutes)
Writing time: 9.15 am to 10.15 am (1 hour)

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
11	11	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied
- Question and answer book of 10 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

Let $f(x) = x^2 + 1$ and $g(x) = 2x + 1$. Write down the rule of $f(g(x))$.

1 mark

Question 2

For the function $f: R \to R, f(x) = 3e^{2x} - 1$,

a. find the rule for the inverse function f^{-1}

2 marks

b. find the domain of the inverse function f^{-1}.

1 mark

Question 3

a. Let $f(x) = e^{\cos(x)}$. Find $f'(x)$

1 mark

b. Let $y = x\tan(x)$. Evaluate $\frac{dy}{dx}$ when $x = \frac{\pi}{6}$.

3 marks

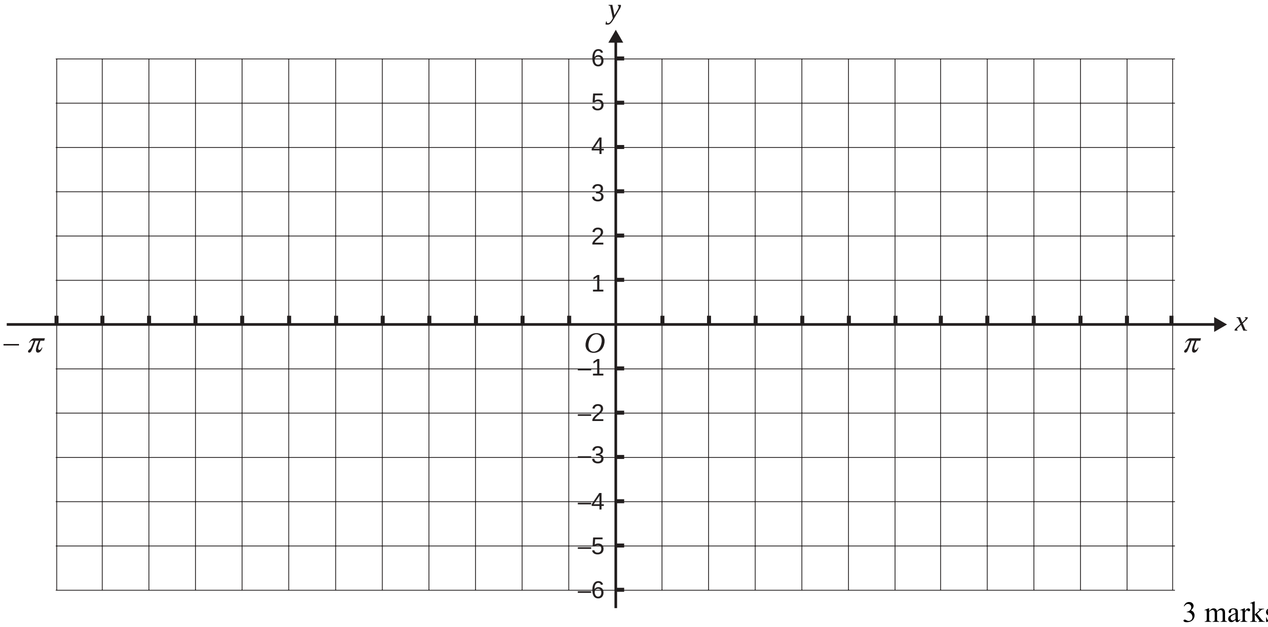
Question 4

For the function $f:[-\pi, \pi] \rightarrow R,\ f(x) = 5\cos\left(2\left(x + \frac{\pi}{3}\right)\right)$

a. write down the amplitude and period of the function

2 marks

b. sketch the graph of the function f on the set of axes below. Label axes intercepts with their coordinates. Label endpoints of the graph with their coordinates.



3 marks

Question 5

Let X be a normally distributed random variable with a mean of 72 and a standard deviation of 8. Let Z be the standard normal random variable. **Use the result that Pr(Z < 1) = 0.84, correct to two decimal places**, to find

a. the probability that X is greater than 80

1 mark

b. the probability that $64 < X < 72$

1 mark

c. the probability that $X < 64$ given that $X < 72$.

2 marks

Question 6

The probability density function of a continuous random variable X is given by

$$f(x) = \begin{cases} \frac{x}{12} & 1 \le x \le 5 \\ 0 & \text{otherwise} \end{cases}$$

a. Find Pr ($X < 3$).

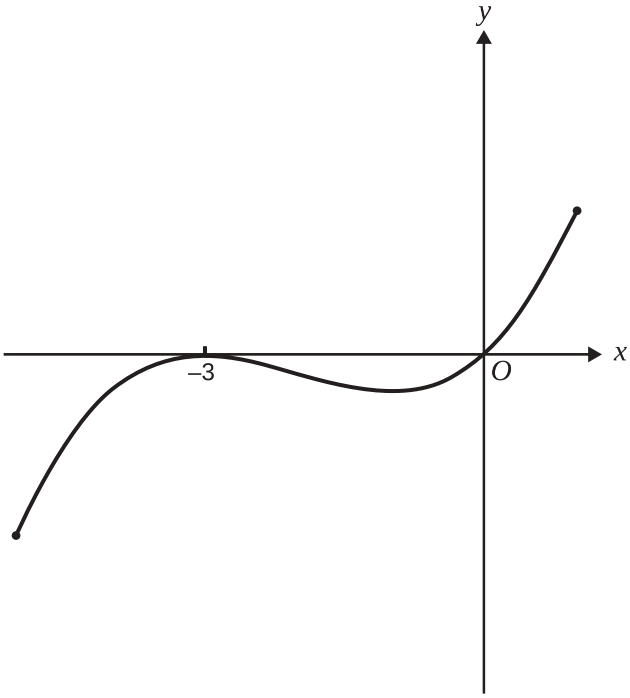
2 marks

b. If Pr ($X \ge a$) = $\frac{5}{8}$, find the value of a.

2 marks

Question 7

The graph of $f: [-5, 1] \to R$ where $f(x) = x^3 + 6x^2 + 9x$ is as shown.



a. On the same set of axes sketch the graph of $y = |f(x)|$.

2 marks

b. State the range of the function with rule $y = |f(x)|$ and domain $[-5, 1]$.

1 mark

Question 8

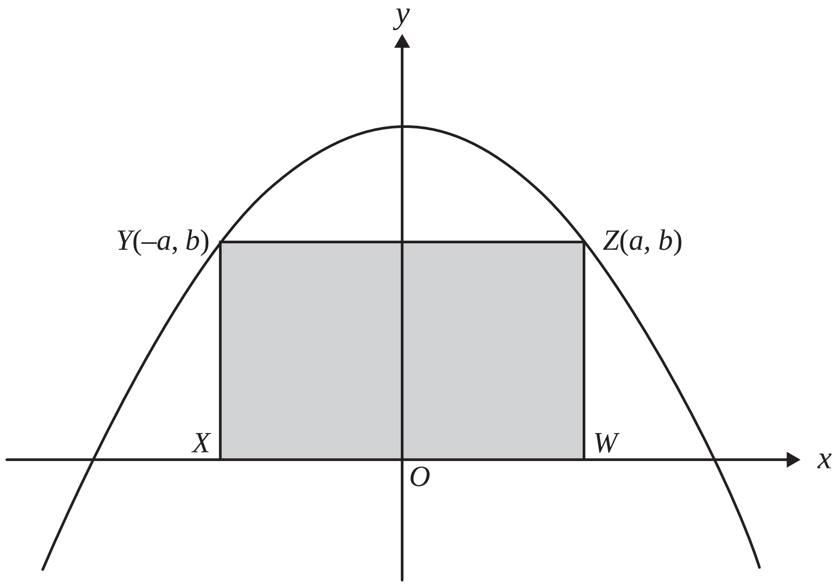
A **normal** to the graph of $y = \sqrt{x}$ has equation $y = -4x + a$, where a is a real constant. Find the value of a.

4 marks

TURN OVER

Question 9

A rectangle $XYZW$ has two vertices, X and W, on the x-axis and the other two vertices, Y and Z, on the graph of $y = 9 - 3x^2$, as shown in the diagram below. The coordinates of Z are (a, b) where a and b are positive real numbers.



a. Find the area, A, of rectangle $XYZW$ in terms of a.

1 mark

b. Find the maximum value of A and the value of a for which this occurs.

3 marks

Question 10

Jo has either tea or coffee at morning break. If she has tea one morning, the probability she has tea the next morning is 0.4. If she has coffee one morning, the probability she has coffee the next morning is 0.3. Suppose she has coffee on a Monday morning. What is the probability that she has tea on the following Wednesday morning?

3 marks

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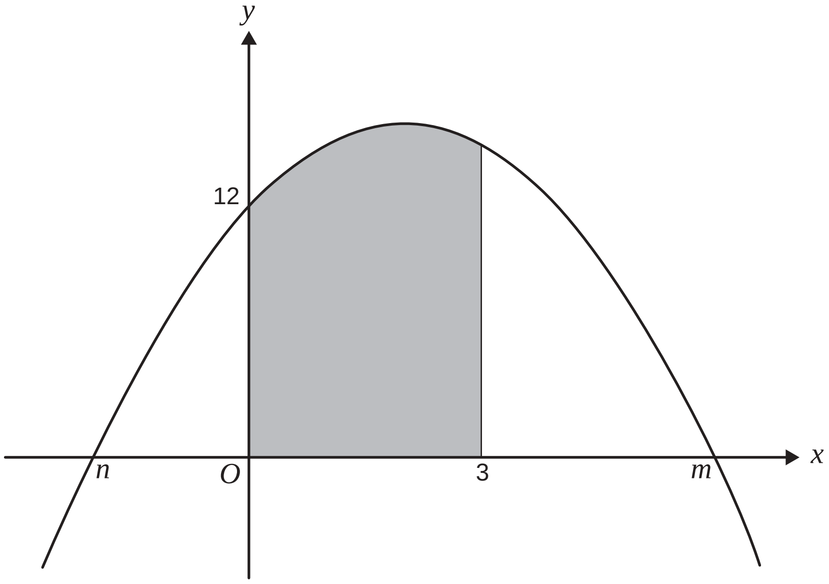
Question 11

Part of the graph of the function $f: R \to R,\ f(x) = -x^2 + ax + 12$ is shown below. If the shaded area is 45 square units, find the values of a, m and n where m and n are the x-axis intercepts of the graph of $y = f(x)$.



5 marks

END OF QUESTION AND ANSWER BOOK

MATHEMATICAL METHODS AND
MATHEMATICAL METHODS (CAS)

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

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Mathematical Methods and Mathematical Methods CAS Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$	volume of a pyramid:	$\frac{1}{3}Ah$
curved surface area of a cylinder:	$2\pi rh$	volume of a sphere:	$\frac{4}{3}\pi r^3$
volume of a cylinder:	$\pi r^2 h$	area of a triangle:	$\frac{1}{2}bc\sin A$
volume of a cone:	$\frac{1}{3}\pi r^2 h$		

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$$

$$\int \frac{1}{x} dx = \log_e |x| + c$$

$$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$$

$$\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$$

$$\int \cos(ax)dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + hf'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

mean: $\mu = E(X)$

variance: $\text{var}(X) = \sigma^2 = E((X-\mu)^2) = E(X^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x-\mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$

END OF FORMULA SHEET