2005 Specialist Mathematics GA 2: Written examination 1

GENERAL COMMENTS

The number of students who sat for the 2005 Specialist Maths examination 1 was 5626, 533 fewer than in 2004. As in previous years, students were required to answer 30 multiple-choice questions in Part I, and five questions worth a total of 20 marks in Part II.

Students' overall performance on the 2005 examination was similar to the improved performance that was shown on the 2004 paper. The mean score for Part I was 18.6 out of 30, down from 19.5 in 2004 but higher than the 17.5 mean in 2003. Five questions were answered correctly by less than 50% of students, which was one more question than in both 2004 and 2003; however, three of these five questions were answered correctly by at least 46% of students. For Part II, the mean score was 10.9 out of 20, up from 10.8 in 2004 and 10.3 in 2003. Three out of nine question parts had a mean score of less than 50% of the maximum possible, which was the same as in 2004 but down from six out of nine in 2003.

The overall mean and median scores were 30.2 and 31 out of 50, compared with 31.0 and 32.5 in 2004 and 28.1 and 28 in 2003. This is the second successive year that both the mean and the median marks have exceeded 60% of the maximum score. About eight per cent of students scored less than 25% of the available marks, compared to six per cent in 2004 and seven per cent in 2003. Four students scored less than 4 marks out of 50, compared with six students in 2004 and ten in 2003. Very high scores were more common than in any previous year. About twelve per cent of students scored more than 90% of the available marks (compared with ten per cent in 2004 and four per cent in 2003), and 77 students scored full marks, compared with 40 in 2004 and 12 in 2003.

In the comments on specific questions in the next section, many common mistakes that are made year after year are highlighted. These mistakes should be brought to the attention of students so that they can avoid them in future. In particular, students must be aware of the need to use the chain rule when differentiating a function of a function, and to use the change of variable rule when changing the variable of integration (see comments on Question 15).

Areas of weakness included:

- vector resolutes
- 'alternative forms' for acceleration
- poor algebraic skills. This was most evident in Question 2 of Part II, where an inability to simplify expressions by taking out common factors often prevented students from correctly solving for k
- showing a given result, which was required by four of the nine question parts (1a., 3a., 4a. and 5a.) in Part II. The onus is on students to include sufficient relevant working to convince the assessors that they do know how to derive the result. Just as importantly, students should be reminded that they **can** use a given value in the remaining part(s) of the question, **whether or not** they were able to derive it.

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

SPECIFIC INFORMATION

Part I – Multiple-choice questions

The table below indicates the percentage of students who chose each option. The correct answer is indicated by shading.

Question	% A	% B	% C	% D	% E	% No Answer
1	4	3	4	26	63	0
2	13	15	14	47	11	1
3	8	9	66	6	11	0
4	4	4	15	72	5	0
5	9	8	17	19	46	1
6	4	12	5	72	6	1
7	83	1	7	1	8	0
8	12	16	53	13	5	1
9	7	66	15	5	6	1
10	12	47	25	8	8	1
11	66	20	6	5	4	0
12	3	12	74	6	3	0
13	71	20	3	4	1	0
14	71	8	3	5	12	0
15	4	6	20	60	10	0
16	8	7	9	9	65	1
17	3	5	9	80	3	1
18	71	7	11	5	5	1
19	11	69	11	5	4	0
20	15	9	25	10	39	1
21	28	51	5	8	7	0
22	7	43	28	16	5	2
23	7	9	67	9	8	1
24	3	7	21	13	55	1
25	80	11	3	2	3	0
26	3	24	63	7	2	1
27	3	11	19	13	53	1
28	46	23	12	12	6	1
29	4	18	65	10	3	1
30	5	68	16	7	4	1

The mean score for Part I was 18.6 and the standard deviation was 6.7. In general, students performed consistently across the various Areas of Study.

Five questions (Questions 2, 5, 10, 20 and 22) were answered correctly by less than 50% of students, but only Question 20, which required the use of an 'alternative form' for acceleration, was answered correctly by less than 40% of students. These questions are all commented on below. Other questions, which were answered correctly by a majority of students, are also commented on because of the high popularity of one or more of the distractors.

Question 1
Twenty-six per cent of students chose option D, indicating that they did not express the equation in 'canonical' form (by dividing both sides by three, to give one on the right-hand side) before deducing the length of the semi-major axis.

Question 2
This was the first question on the examination that was not answered correctly by a majority of students. To answer this question, students needed to recognise that the graph of a reciprocal quadratic function has no vertical asymptotes if the quadratic expression has no real roots, and that this is the case if its discriminant is negative. As can be seen in the table above, each of the four distractors was chosen by a significant proportion of the students who gave an incorrect response. It is unlikely that these responses were based on any particular misconception; rather, it appears that many of these students guessed the answer.

Question 5

Question 5, in which the derivative of $y = \text{Tan}^{-1}\left(\sqrt{3x}\right)$ had to be found, was answered correctly by 46% of students, with options C and D the most popular distractors. Option C was obtained when $\sqrt{3x}$ was taken to equal $\left(\sqrt{3}\right)x$; a similar mistake was common in Question 1a. of Part II of the 2004 examination. Option D came from writing $\frac{d}{dx}\left(\sqrt{3x}\right) = \frac{1}{2}(3x)^{-\frac{1}{2}}$.

Question 10

In Question 10, which was answered correctly by 47% of students, one quarter of students chose option C. Students should know that relations of the form $(z-w)(\overline{z}-\overline{w}) = r^2$ represent circles in the complex plane, and should have realised that option B represents an ellipse, with foci (1, 0) and –1, 0), **not** a circle.

Question 11

In Question 11, 20% of students chose option B, presumably because substituting 2*x* for *x* in the rule for the derivative of $\text{Sin}^{-1}(x)$ gives $\frac{6}{\sqrt{1-4x^2}}$.

Question 13

In Question 13, 20% of students chose option B, a common misconception when dealing with solids of revolution. However, this question was answered far better than the similar question (Question 16) on last year's paper.

Question 15

Twenty per cent of students chose option C, indicating that they either omitted $\frac{dx}{du}$ when changing the variable of integration or that they took it to be 1 instead of –1. This is another common mistake that occurs year after year.

Question 20

Question 20 was the worst-answered question in Part I, with only 25% of students answering it correctly. A large percentage of students chose option E, which is the expression for $\frac{dv}{dx}$, whereas acceleration is given by $v\frac{dv}{dx}$. Students also performed poorly on Question 30 of Part I in 2004, which similarly required knowledge of the various ways for expressing acceleration. This indicates that teachers and students need to give these 'alternative forms' more attention.

Question 22

This was the final question that was answered incorrectly by a majority of students. Half of the students who answered incorrectly chose option C, which either followed from poor algebra $(5-y=1 \Rightarrow y=6)$ or possibly from equating $\overrightarrow{PQ}$ to $\overrightarrow{RS}$ instead of to $\overrightarrow{SR}$, which gives $x = -3$, $y = 6$.

Question 24

Twenty-one per cent of students chose option C, which was obtained simply by putting $x = \cos(t)$ and $y = \sin(t)$.

Question 26

Option B was chosen by 24% of students, which was obtained by omitting the **vector** constant of integration when antidifferentiating the acceleration vector. Once again, this is a very common mistake whenever a question of this type is included in the examination.

Question 27

Nineteen per cent of students chose option C, indicating that they took the maximum acceleration to be 10 = 20 – 10, rather than 30 = 20 – (–10).

Question 28
Another regular mistake occurred in Question 28, in which 23% of students chose option B. This answer was obtained when the vertical component, 14 cos 60º, of the pulling force was ignored.

The vertical component could be read as acting either upwards or downwards, hence two answers were accepted for this question. The most common interpretation was that it was acting upwards.

Question 29
Eighteen per cent of students chose Option B, which is the **resultant** of forces $\underset{\sim}{P}$ and $\underset{\sim}{Q}$. Since the particle is held in equilibrium, $\underset{\sim}{R}$ must act in the **opposite** direction to the resultant.

Part II – Short-answer questions

Question 1a.

Marks	0	1	Average
%	37	63	**0.7**

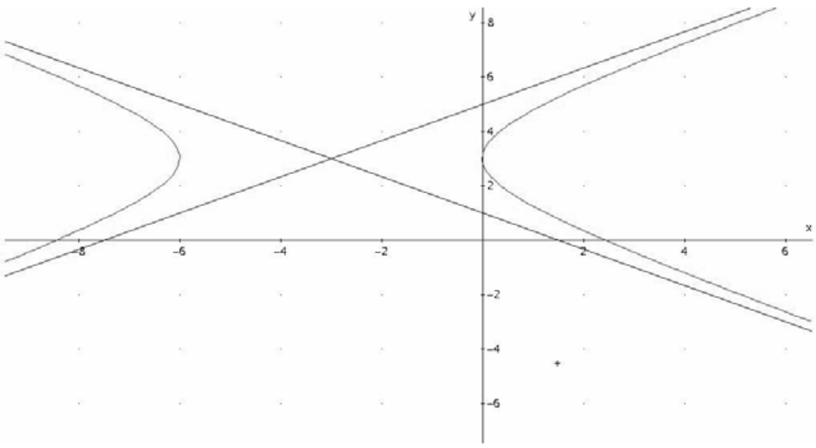
$c = -3$ [given]

The simplest way to show that $c = -3$ was to recognise from the equation of the hyperbola that the asymptotes go through (c, 3), and then to substitute $x = c$, $y = 3$ in the given asymptote equation, $y = \frac{2}{3}x + 5$.

However, most students made use of the general form for the equations of the asymptotes of a hyperbola, $y = \pm\frac{b}{a}(x-h)+k$. Many of these students floundered because they were unsure of what to do with the '±', while the 'proof' of many other students was unconvincing.

Question 1b.

Marks	0	1	2	3	Average
%	27	13	22	38	**1.8**



Students needed to draw a 'horizontal' hyperbola with centre (–3, 3) and vertices (–6, 3) and (0, 3). The asymptotes had to cross at the centre, with one passing through (–7.5, 0) and (0, 5) and the other one passing through (0, 1) and (1.5, 0).

Many students drew a hyperbola with centre (0, 5). Others had trouble correctly positioning the second asymptote or the two vertices. Some students had both asymptotes correct, but one or both of their hyperbola branches clearly did not exhibit asymptotic behaviour.

Question 2

Marks	0	1	2	3	4	Average
%	16	2	22	18	42	**2.8**

$k = -2$

Most students recognised that the product rule had to be used to differentiate $e^{2x}\cos(x)$ and over 60% of the students were able to obtain a correct expression for $\frac{d^2y}{dx^2}$. Many students seemed unaware that they could 'equate coefficients' to obtain the value of *k* once they had substituted for the derivatives in the differential equation. Instead, they tried to solve for *k*, sometimes exhibiting poor algebra and interesting ways of obtaining the correct result; for example, $k = \frac{2\sin(x) - 4\cos(x)}{-\sin(x) + 2\cos(x)} = \frac{2-4}{-1+2} = \frac{-2}{1} = -2$ (where sin (*x*) and cos (*x*) in the numerator have been cancelled with their counterparts in the denominator). Overall, though, this question was well done.

Question 3a.

Marks	0	1	Average
%	49	51	**0.6**

$x = 4$ [given], from $\underset{\sim}{u}.\underset{\sim}{v} = 6 - 2x + 2 = 0$

Surprisingly, many students did not attempt this routine question.

Question 3b.

Marks	0	1	2	Average
%	46	11	43	**1.0**

$\underset{\sim}{u} = 5\underset{\sim}{i} + 2\underset{\sim}{j} + 3\underset{\sim}{k}$

Many students had no idea how to proceed in this question, apparently having no real understanding of the concept of a vector resolute. Students should be encouraged to start with a quick sketch of the situation in questions like this. Parentheses were often omitted or ignored when (unnecessarily) subtracting vectors, leading to common mistakes such as $\underset{\sim}{u} - (3\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}) = \underset{\sim}{u} - 3\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$.

Question 4a.

Marks	0	1	Average
%	44	56	**0.6**

$v = 8$ [given]

All that was required for this question was to substitute $t = 12$ in the expression for *v*. Disappointingly, however, almost half of the students scored zero. Many students thought they had to find $\frac{dv}{dt}$, while another common mistake was to substitute $t = 8$ (instead of 12).

Question 4b.

Marks	0	1	2	3	Average
%	54	8	11	27	**1.2**

36.6 s

This question was poorly done. A common misconception was that the cyclists pass when their graphs intersect (after 10.9 s). Another common incorrect answer was 24.6 s, the time taken for the cyclists to pass **after** cyclist B attains the speed of 8 m/s (although this answer was often the result of an incorrect equation or confused logic, rather than a deliberate attempt to find this time). Some students did not realise that they needed to use their graphics calculator to evaluate the integral numerically, but those who did usually obtained the correct answer.

Question 5a.

Marks	0	1	2	3	Average
%	29	7	4	59	**2.1**

$\mu = 0.41$ [given]

This question was reasonably well done. Students who treated the two boxes separately often made the mistake of including the pulling force in their horizontal equation of motion for the 18 kg box; that is, they obtained $135 + T - \mu(18g) = 18 \times 0.5$, rather than $T - \mu(18g) = 18 \times 0.5$. A small minority of students treated the system as a whole and were usually successful.

Question 5b.

Marks	0	1	2	Average
%	51	8	41	**1.0**

81 N

Many of the students who completed part a. by treating the system as a whole subsequently had trouble with part b. because their force diagram did not show all the forces acting on **each** box. Common mistakes were the omission of the frictional force acting on the 12 kg box and, once again, the inclusion of the pulling force in the equation of motion for the 18 kg box.