VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education 2018

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

Letter

STUDENT NUMBER

ALGORITHMICS (HESS)

Written examination

Tuesday 20 November 2018

Reading time: 3.00 pm to 3.15 pm (15 minutes)
Writing time: 3.15 pm to 5.15 pm (2 hours)

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	18	18	80
			Total 100

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers and one scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 23 pages
- Answer sheet for multiple-choice questions

Instructions
- Write your **student number** in the space provided above on this page.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- All written responses must be in English.

At the end of the examination
- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION A – Multiple-choice questions

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** or that **best answers** the question.
A correct answer scores 1; an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Use the Master Theorem to solve recurrence relations of the form shown below.

$$T(n) = \begin{cases} aT\left(\frac{n}{b}\right) + kn^c & \text{if } n > 1 \\ d & \text{if } n = 1 \end{cases} \quad \text{where } a > 0,\ b > 1,\ c \geq 0,\ d \geq 0,\ k > 0$$

and its solution $T(n) = \begin{cases} O(n^c) & \text{if } \log_b a < c \\ O(n^c \log n) & \text{if } \log_b a = c \\ O(n^{\log_b a}) & \text{if } \log_b a > c \end{cases}$

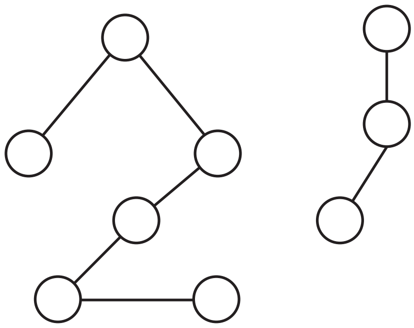
Question 1
Which method or methods can be used to check for the correctness of algorithms?
A. induction only
B. contradiction only
C. induction and contradiction
D. once written, an algorithm is already known to be correct

Question 2
A requirement of Cobham's thesis is that it
A. assumes all P problems are easy and all NP problems are hard.
B. only considers algorithms that are determined to be practically feasible.
C. considers the size of the input to be the most important component for solvability.
D. considers constant factors and lower-order terms as important components for solvability.

Question 3
A manager is attempting to coordinate several impatient customers waiting in line.
To be fairest to the customers, which abstract data type (ADT) should the manager **not** use?
A. decision tree
B. directed list
C. queue
D. stack

Question 4

Consider the following graph.



Which of the following properties does this graph have?

A. directed and acyclic

B. cyclic and weighted

C. cyclic and fully connected

D. non-directed and non-connected

Question 5

Which one of the following must exist in the definition of an ADT?

A. a unique identifier

B. a set of operations

C. a concrete representation of data

D. the implementation of operations

Question 6

Which one of the following best states the difference between a list ADT and a dictionary ADT?

A. Finding the minimum item in a sorted list is usually faster than finding the minimum item in a dictionary.

B. Finding the maximum item in a dictionary is usually faster than finding the maximum item in a sorted list.

C. A list can have its values accessed via an integer index, while a dictionary cannot.

D. Priorities are defined within a list ADT, while a dictionary has no priorities in the ADT.

Question 7

When testing to find out whether a graph has multiple connected components, which one of the following modifications to a basic implementation of Prim's algorithm is the most appropriate?

A. Choose the maximum weighted edge to connect a node to the growing tree.

B. Run Prim's algorithm on every node and keep track of unique sets of nodes.

C. Delete all existing edges of the existing graph and create new edges to connect each node.

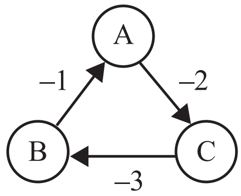
D. No modifications need to take place as Prim's algorithm can already test for multiple disconnected components.

DO NOT WRITE IN THIS AREA

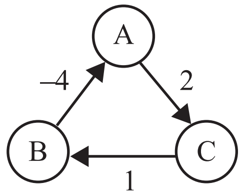
Question 8

For which one of the following graphs would the Floyd-Warshall algorithm return all-pair shortest paths?

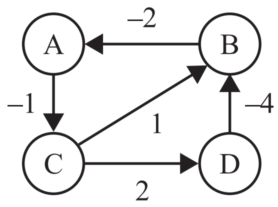
A.



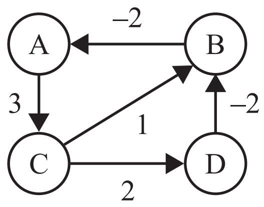
B.



C.



D.



Question 9

Assume quicksort always chooses the rightmost element as a pivot when creating partitions. The sorting maintains the relative order of elements while partitioning and stops the recursion at the empty array.

Given the array {1,2,3,4,5,6,7,8}, how many pivots will be chosen before the algorithm returns a sorted array?

A. 0
B. 1
C. 3
D. 8

Question 10

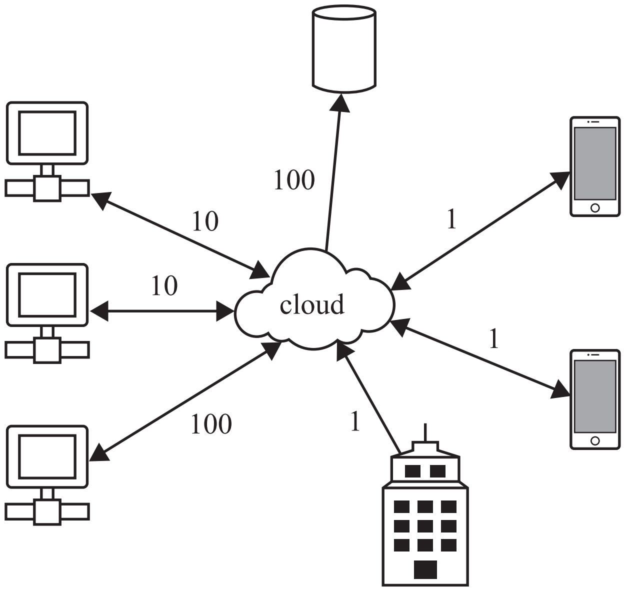
Consider the following pseudocode for the algorithm `unknown(x)`, where `x` is a node of a graph.

```
Algorithm unknown(x)
        Let x.discovered = True
        For eachNode in x.adjacentNodes
            If eachNode.discovered is not True Then
                unknown(eachNode)
            EndIf
        EndFor
```

Which algorithm does the pseudocode best represent?

A. best-first search
B. depth-first search
C. brute force search
D. breadth-first search

Question 11



Which one of the following ADTs would be best suited to model the network topology diagram shown above?

A. a weighted, directed graph

B. a stack with keys

C. a directed graph

D. an array

Use the following information to answer Questions 12 and 13.

Consider the following three algorithms, which all operate on inputs of size n elements, and their Big-O or Big-Θ resource requirements:

- Algorithm A: $O(n)$ worst case time
- Algorithm B: $\Theta(2^n)$ best case time
- Algorithm C: $\Theta(n \log n + n)$ best case time and worst case time

Question 12

Which one of the following statements about algorithms A, B and C is correct?

A. Algorithms A and B are in complexity class P.

B. Algorithms B and C are in complexity class P.

C. Algorithms A and C are in complexity class P.

D. Algorithms A and C are not in complexity class P.

Question 13

Which one of the following statements about algorithms A, B and C is correct?

A. No algorithm will terminate in O(1) time on any large n inputs.

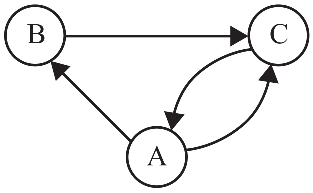
B. Algorithm A could terminate in O(1) time on some large n inputs.

C. Algorithm B could terminate in O(1) time on some large n inputs.

D. Algorithm C could terminate in O(1) time on some large n inputs.

Question 14

Sameera runs an algorithm on the graph below to compute a path from A to B. The algorithm becomes trapped in a cycle.



Which one of the following algorithms has Sameera used?

A. PageRank algorithm

B. Dijkstra's algorithm

C. breadth-first search without marking previous nodes

D. depth-first search without marking previous nodes

Question 15

Cyclist Sergei has a clever plan to win every race that he enters. He will start at the front of the race pedalling slowly at 5 km/h. Every time someone catches up to him, Sergei will double his speed.

Assuming that the maximum speed for any cyclist is 60 km/h, how many people will catch Sergei before he cannot go any faster?

A. 2

B. 3

C. 4

D. 5

Question 16

An algorithm takes a set of inputs and performs calculations on it. The output from the algorithm consists of a set of numbers and letters that are related to the initial inputs. The output from the algorithm can be verified in worst case $O(n^2)$ time.

Given this information, it can be said that the algorithm is

A. decidable and in complexity class NP.

B. undecidable and in complexity class NP.

C. decidable and in complexity class P and not in complexity class NP.

D. undecidable and in complexity class P and not in complexity class NP.

Question 17

A neural network can be used to

A. find exact solutions to computationally undecidable problems.

B. demonstrate that Searle's Chinese Room Argument has merit.

C. process complex inputs with no known relationships.

D. always solve problems faster than DNA computing.

DO NOT WRITE IN THIS AREA

Use the following information to answer Questions 18 and 19.

The following is a proof by contradiction that Prim's algorithm generates a minimal spanning tree (MST), with some details omitted:

- Let G be a graph with vertices, V, and a minimal spanning tree, T^*.
- Let T be a spanning tree generated by Prim's algorithm on graph G.
- Assume $T \neq T^*$. Let the edge (u, v) be in T but not in T^*.
- When (u, v) was added to T by Prim's algorithm, it was the minimum edge crossing from the set of vertices, S, that had already been added to T, to the set of vertices, $V - S$, that had not been added to T.
- Let (x, y) be an edge on the path from u to v in T^* such that x is in S and y is in $V - S$.
- Form a new tree, T^{**}, by adding (u, v) to T^* and removing (x, y) from T^*.

Question 18

Which one of the following statements is **not** correct?

A. The edge (u, v) exists because $T \neq T^*$.

B. There is a path from u to v in T^* because it is a spanning tree.

C. T^{**} is not a tree because adding (u, v) to T^* and removing (x, y) from T^* creates a cycle.

D. T^{**} is a tree because adding (u, v) to T^* and removing (x, y) from T^* does not create a cycle.

Question 19

Which one of the following statements generates the contradiction needed to complete the proof?

A. T^{**} has a higher weight than T^*, thus T^* could not be the MST of G.

B. T^{**} has a lower weight than T^*, thus T^* could not be the MST of G.

C. T^{**} is not a tree, thus T^* could not be the MST of G.

D. T^{**} is not a tree, thus T^* is the MST of G.

Question 20

Simulated annealing has been suggested as a useful technique to compute a global minimum rainfall over a large set of rainfall data.

Which heuristic applied to simulated annealing is more likely to return a global minimum rainfall?

A. increase the probability of choosing a worse solution at random as running time increases

B. decrease the probability of choosing a worse solution at random as running time increases

C. allow the system temperature to cool quickly to find a solution in a short amount of time

D. keep the system at a constant temperature to find the best solution within a short amount of time

END OF SECTION A
TURN OVER

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.

Use the Master Theorem to solve recurrence relations of the form shown below.

$$T(n) = \begin{cases} aT\left(\frac{n}{b}\right) + kn^c & \text{if } n > 1 \\ d & \text{if } n = 1 \end{cases} \quad \text{where } a > 0, b > 1, c \geq 0, d \geq 0, k > 0$$

and its solution $T(n) = \begin{cases} O(n^c) & \text{if } \log_b a < c \\ O(n^c \log n) & \text{if } \log_b a = c \\ O(n^{\log_b a}) & \text{if } \log_b a > c \end{cases}$

Question 1 (2 marks)

Describe **one** similarity and **one** difference between a Turing machine and a modern computer.

Question 2 (4 marks)

Elise needs to use a priority queue abstract data type (ADT).

Write a priority queue ADT specification for Elise.

Question 3 (4 marks)

The following algorithm finds the maximum element in a list.

```
Algorithm FIND_MAX(L):
     Input: L, a non-empty list of elements

     If L has only one element
          return first element of L as the maximum
     EndIf

     m = FIND_MAX(L without the first element)
     If m is greater than the first element in L
          return m
     Else
          return the first element of L
     EndIf
```

a. Write a recurrence relation and its solution to describe the worst case running time of the algorithm when the list contains *n* elements. 3 marks

b. Would the Big-O for the best case running time of the algorithm be smaller than the worst case running time of the algorithm? Justify your answer. 1 mark

DO NOT WRITE IN THIS AREA

Question 4 (2 marks)

Consider the following recurrence relations.

$$S(n)=\begin{cases}2S\left(\dfrac{n}{3}\right)+3\sqrt{n} & \text{if } n>3\\ O(1) & n<4\end{cases}$$

$$T(n)=\begin{cases}2T\left(\dfrac{n}{2}\right)+O(n) & \text{if } n>1\\ O(1) & n<2\end{cases}$$

a. What is the Big-O solution to $S(n)$? 1 mark

b. What is the Big-O solution to $T(n)$? 1 mark

Question 5 (3 marks)

Maryam invests in companies on the stock market. She is currently considering 50 potential companies she can invest in and has estimated the potential profit she is likely to receive from each company over the next 12 months. The investment in each company has an initial cost and Maryam has a fixed budget that she cannot exceed.

Describe a dynamic programming algorithm that Maryam can use to select a set of companies to invest in that will provide maximum profit for her budget.

DO NOT WRITE IN THIS AREA

Question 6 (5 marks)

A new shopping centre has been built and has attracted many customers. The centre's management has noted that many of these customers are getting lost within the shopping centre. The centre's management wants to design and build an interactive map application that will allow customers to find their way to each shop from fixed information kiosks around the shopping centre.

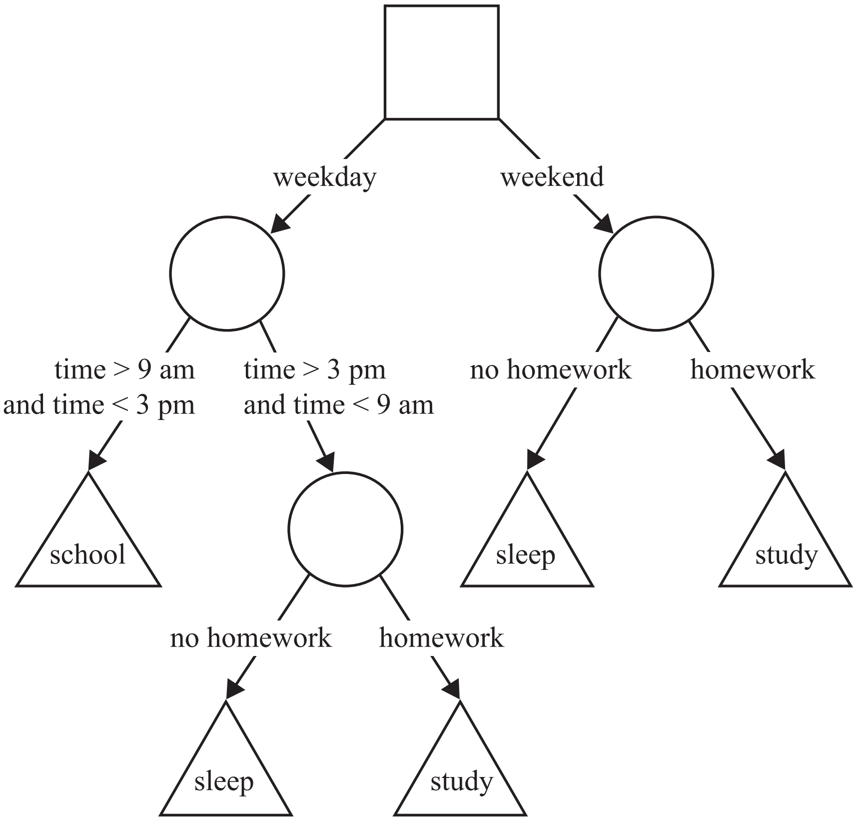
a. Name and justify a graph ADT that would best suit the interactive map application. 3 marks

b. The centre's management has decided to install an additional information kiosk.

Explain how the additional information kiosk could be integrated within the interactive map application. What changes to the graph ADT named in **part a.** would have to occur? 2 marks

Question 7 (4 marks)

Consider the decision tree below.



Represent this decision tree using logical operations in pseudocode.

Question 8 (4 marks)

Prove via induction that the sum of the first n odd numbers is the same as n^2, for all $n \geq 1$.

DO NOT WRITE IN THIS AREA

Question 9 (9 marks)

Devices that are capable of wireless transmission are becoming cheaper and easier to integrate into existing technology, such as vacuum cleaners and refrigerators. These devices typically communicate using a network. The network below shows devices named A–O.

Grid	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
a									
b		O		_		F		M	
c			_	_	_			N	
d		_	_	A _	_	E _			
e			_	_	_			L	
f		C	B	_	K	I			
g							G		
h			D						
i	H		J						

The network above has the following conditions:

- Each device is aware of its coordinates and there is a complete list of devices.
- Each square represents a 50 m × 50 m area.
- For successful connection over wireless transmission, devices must be within range: two squares vertically or horizontally and one square diagonally. For example, device A can communicate with those squares with an underscore.
- Each device can relay data between nodes within range. For example, device A can communicate with device E, which can then communicate with device F.
- Each device can return temperature values measured within its square.

a. Which device(s) cannot successfully connect over wireless transmission with any other device? 1 mark

b. Describe how the Floyd-Warshall algorithm can be used to check if all devices can communicate with any other device. 2 marks

c. Write an algorithm to compute the average temperature of all devices in the network, given all temperatures in an input list, `temperature_list`. Some devices may be defective and return –255 as their temperature value. The algorithm should **not** include these temperatures in the calculation. 6 marks

Question 10 (4 marks)

Joe delivers groceries to the same five houses every day as part of his delivery business. To conserve fuel, he computed the length of all possible routes between his home and those five houses to find the minimum route.

a. How many routes did Joe compute? 1 mark

b. Joe's algorithm to determine the minimum route is an example of what algorithmic design pattern? 1 mark

c. As Joe's delivery business grows and more houses are added to the delivery list, will Joe's algorithmic approach still be useful for computing the optimal delivery route? Justify your answer. 2 marks

Question 11 (4 marks)

Consider the following algorithm for sorting a list of integers in ascending order, which makes use of another algorithm called SORT2. Assume that SORT2 runs in $\Theta(n \log n)$ time on a list of length n, unless the list is sorted in descending order, in which case SORT2 runs in $\Theta(n^2)$ time.

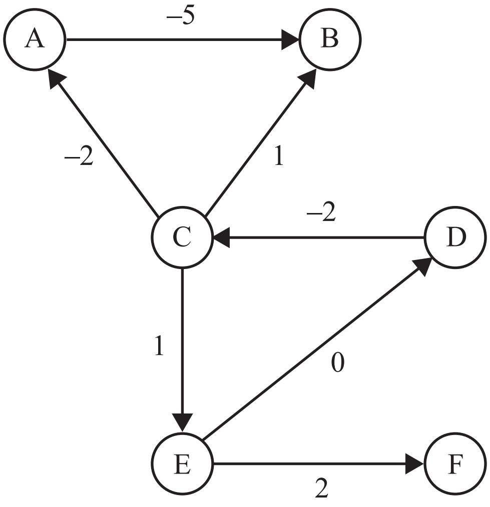
```
Algorithm SORT(L)
    INPUT: L, a list of integers

    Let sorted = True
    For each element x except the last in L
        If x is greater than its right neighbour in L
            Let sorted = False
    EndFor
    If sorted is False Then
        Return the result of SORT2(L)
    Else
        Return L
    EndIf
```

a. What is the best case running time of SORT? Write down a list of 10 integers that gives the best case running time. 2 marks

b. What is the worst case running time of SORT? Write down a list of 10 integers that gives the worst case running time. 2 marks

Question 12 (2 marks)



Explain why the graph above is not a suitable input to a naive implementation of the Bellman-Ford algorithm.

Question 13 (3 marks)

Describe DNA computing and explain how it can be used as an alternative method of computation. Provide an example as part of your explanation.

Question 14 (9 marks)

The strategy used for climbing a rock wall is very important. For a given rock wall, there is a set of numbered handholds and footholds that can be chosen in sequence to allow the climber to get to the top of the wall. Long routes with handholds and footholds that are spaced far apart can cause the climber to tire quickly and increase their risk of falling. Short routes with closely spaced handholds and footholds allow climbers to preserve energy and make it to the top.

You have been selected as the strategy coach of the Australian team for the Climbing World Championships. In particular, you have to plot a course for each climber on each wall.

a. What information would you need about each wall and each climber? What ADT would be best to use to model this problem? Why? 3 marks

b. What algorithm would you use to determine the optimal climbing route using the ADT from **part a.**? In your answer, describe how you would translate the output from your algorithm into meaningful information that the climber could use. Also include an estimation of the time complexity of your approach and how it compares to a brute force approach to the problem. 6 marks

Question 15 (3 marks)

Social media profiles have been received by a company that plans to use the data to infer which users it should target first in order to influence large communities. The data contains user profiles, their lists of friends and their public comments.

Describe an algorithm that could be used to identify the most influential profiles in the data.

Question 16 (4 marks)

Bernie is developing a mobile robot for a national park. The robot will travel through the park using existing paths and deposit animal food at feeding stations. These feeding stations will be changed every season to match the movement of animals during the year. Bernie is currently designing the algorithm that the robot will use to select the most efficient path it should take to visit all of the feeding stations while minimising the distance it travels.

What is the best algorithm for Bernie to use to decide the order in which the robot will visit each feeding station? Will it guarantee an optimal solution? Explain why or why not.

Question 17 (4 marks)

A farmer shares a field with another farmer. Every year each farmer can sell 10 cows and reduce their income by 10% or buy 10 cows and increase their income by 10% or do nothing.

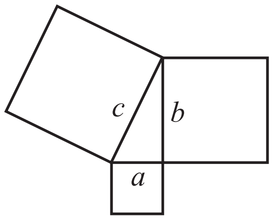
At the beginning of each year, the available resources in the field, as a percentage, change by 100 minus the total number of cows. For example, assume that the field has 100% resources and, at the beginning of the year, Farmer 1 has 50 cows and Farmer 2 has 70 cows, thus the resources change to 80%. If next year the total number of cows is 110, the resources change to 70%. If next year the number of cows is 90, the resources change to 80%.

If the available resources at the beginning of a year drop below 50%, all cows must be removed and both farmers lose their businesses.

Describe an algorithm that will select a strategy that maximises one farmer's income at the end of two years, without losing all of this farmer's cows.

Question 18 (10 marks)

Two pirates, Monty and Blackbeard, have decided to divide their fortunes. Monty makes the following proposition to Blackbeard: Choose between taking the square made of gold whose side is the hypotenuse (side *c*) of the right-angle triangle or the two squares made of gold whose sides are adjacent to the right-angle triangle (sides *a* and *b*), as shown in the diagram below.



If Blackbeard decides to take the square with the hypotenuse (side *c*), he must give a certain percentage of the gold to Monty.

a. Write an algorithm, `square_area`, in pseudocode that will return the area of any golden square. 2 marks

b. Assume `square_area` exists. Write an algorithm in pseudocode that will determine the best decision for Blackbeard to make in order to maximise the amount of gold he receives. If Blackbeard should choose the square with the hypotenuse (side *c*) then the algorithm should return `True`, otherwise it should return `False`. 4 marks

c. What is the best decision for Blackbeard to make to maximise the amount of gold he receives? Justify your answer. 2 marks

d. Instead of squares made of gold, Monty and Blackbeard must share cubes made of gold.

What is the best decision for Blackbeard to make in this instance? Justify your answer. 2 marks

END OF QUESTION AND ANSWER BOOK