VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

**Victorian Certificate of Education**
**2007**

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

**STUDENT NUMBER**

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Figures | | | | | | | | | | Letter |
| Words | | | | | | | | | | |

# MATHEMATICAL METHODS (CAS)
# Written examination 1

**Friday 9 November 2007**

**Reading time: 9.00 am to 9.15 am (15 minutes)**
**Writing time: 9.15 am to 10.15 am (1 hour)**

## QUESTION AND ANSWER BOOK

**Structure of book**

| *Number of questions* | *Number of questions to be answered* | *Number of marks* |
|---|---|---|
| 12 | 12 | 40 |

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

**Materials supplied**
- Question and answer book of 9 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

**Instructions**
- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

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## Instructions

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

**Question 1**

Let $f(x) = \dfrac{x^3}{\sin(x)}$. Find $f'(x)$.

2 marks

**Question 2**

**a.** Solve the equation $\log_e(3x + 5) + \log_e(2) = 2$, for $x$.

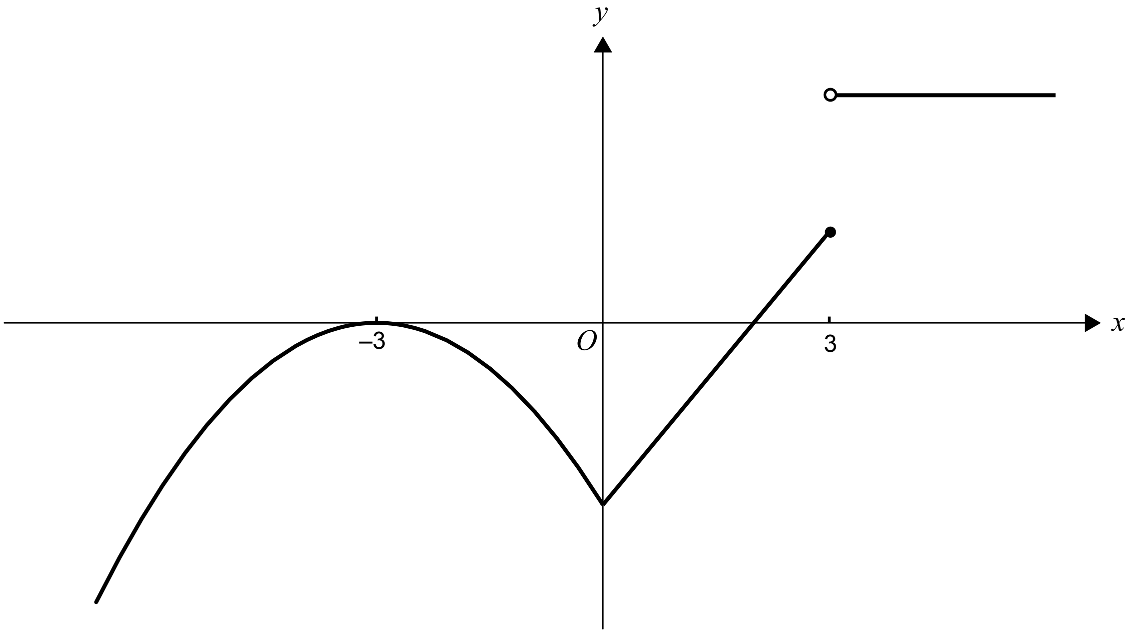
2 marks

**b.** Let $g(x) = \log_e(\tan(x))$. Evaluate $g'\left(\dfrac{\pi}{4}\right)$.

2 marks

**Question 3**

The diagram shows the graph of a function with domain $R$.




**a.** For the graph shown above, sketch on the same set of axes the graph of the derivative function.

3 marks

**b.** Write down the domain of the derivative function.

1 mark

**Question 4**

A wine glass is being filled with wine at a rate of $8\,\text{cm}^3/\text{s}$. The volume, $V\,\text{cm}^3$, of wine in the glass when the depth of wine in the glass is $x$ cm is given by $V = 4x^{\frac{3}{2}}$. Find the rate at which the depth of wine in the glass is increasing when the depth is 4 cm.

3 marks

**Question 5**

It is known that 50% of the customers who enter a restaurant order a cup of coffee. If four customers enter the restaurant, what is the probability that more than two of these customers order coffee? (Assume that what any customer orders is independent of what any other customer orders.)

2 marks

**Question 6**

Two events, $A$ and $B$, from a given event space, are such that $\Pr(A) = \frac{1}{5}$ and $\Pr(B) = \frac{1}{3}$.

**a.** Calculate $\Pr(A' \cap B)$ when $\Pr(A \cap B) = \frac{1}{8}$.

1 mark

**b.** Calculate $\Pr(A' \cap B)$ when $A$ and $B$ are mutually exclusive events.

1 mark

**Question 7**

If $f(x) = x\cos(3x)$, then $f'(x) = \cos(3x) - 3x\sin(3x)$.

Use this fact to find an antiderivative of $x\sin(3x)$.

3 marks

**Question 8**

Let $f: R \to R,\ f(x) = \sin\left(\frac{2\pi x}{3}\right)$.

**a.** Solve the equation $\sin\left(\frac{2\pi x}{3}\right) = -\frac{\sqrt{3}}{2}$ for $x \in [0, 3]$.

2 marks

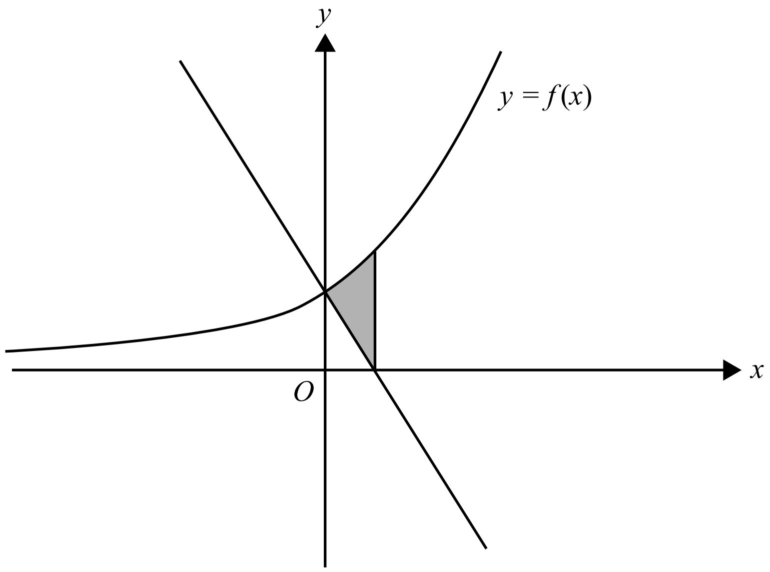
**b.** Let $g: R \to R,\ g(x) = 3f(x-1) + 2$.

Find the smallest positive value of $x$ for which $g(x)$ is a maximum.

2 marks

**Question 9**

The graph of $f: R \rightarrow R,\ f(x) = e^{\frac{x}{2}} + 1$ is shown. The normal to the graph of $f$ where it crosses the $y$-axis is also shown.




**a.** Find the equation of the **normal** to the graph of $f$ where it crosses the $y$-axis.

2 marks

**b.** Find the exact area of the shaded region.

3 marks

**Question 10**

The area of the region bounded by the curve with equation $y = kx^{\frac{1}{2}}$, where $k$ is a positive constant, the $x$-axis and the line with equation $x = 9$ is 27. Find $k$.

3 marks

**Question 11**

There is a daily flight from Paradise Island to Melbourne. The probability of the flight departing on time, given that there is fine weather on the island, is 0.8, and the probability of the flight departing on time, given that the weather on the island is not fine, is 0.6.

In March the probability of a day being fine is 0.4.

Find the probability that on a particular day in March

**a.** the flight from Paradise Island departs on time

2 marks

**b.** the weather is fine on Paradise Island, given that the flight departs on time.

2 marks

## Question 12

$P$ is the point on the line $2x + y - 10 = 0$ such that the length of $OP$, the line segment from the origin $O$ to $P$, is a minimum. Find the coordinates of $P$ **and** this minimum length.

4 marks

**END OF QUESTION AND ANSWER BOOK**

# MATHEMATICAL METHODS AND MATHEMATICAL METHODS (CAS)

## Written examinations 1 and 2

### FORMULA SHEET

**Directions to students**

| |
|---|
| Detach this formula sheet during reading time.<br>This formula sheet is provided for your reference. |

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# Mathematical Methods and Mathematical Methods CAS Formulas

## Mensuration

| | | | |
|---|---|---|---|
| area of a trapezium: | $\frac{1}{2}(a+b)h$ | volume of a pyramid: | $\frac{1}{3}Ah$ |
| curved surface area of a cylinder: | $2\pi rh$ | volume of a sphere: | $\frac{4}{3}\pi r^3$ |
| volume of a cylinder: | $\pi r^2 h$ | area of a triangle: | $\frac{1}{2}bc\sin A$ |
| volume of a cone: | $\frac{1}{3}\pi r^2 h$ | | |

## Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$$

$$\int \frac{1}{x} dx = \log_e|x| + c$$

$$\frac{d}{dx}\left(\sin(ax)\right) = a\ \cos(ax)$$

$$\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}\left(\cos(ax)\right) = -a\ \sin(ax)$$

$$\int \cos(ax)dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\ \sec^2(ax)$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + hf'(x)$

## Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

mean: $\mu = E(X)$

variance: $\text{var}(X) = \sigma^2 = E((X-\mu)^2) = E(X^2) - \mu^2$

| probability distribution | | mean | variance |
|---|---|---|---|
| discrete | $\Pr(X = x) = p(x)$ | $\mu = \sum x\, p(x)$ | $\sigma^2 = \sum (x-\mu)^2\, p(x)$ |
| continuous | $\Pr(a < X < b) = \int_a^b f(x)dx$ | $\mu = \int_{-\infty}^{\infty} x\, f(x)dx$ | $\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$ |

**END OF FORMULA SHEET**