## STUDENT NUMBER

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| Figures |  |
| Words |  | y

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# SPECIALIST MATHEMATICS <br> Written examination 1 

Monday 5 November 2007<br>Reading time: 3.00 pm to 3.15 pm ( 15 minutes)<br>Writing time: 3.15 pm to 4.15 pm (1 hour)

## QUESTION AND ANSWER BOOK

Structure of book

| Number of <br> questions | Number of questions <br> to be answered | Number of <br> marks |
| :---: | :---: | :---: |
| 10 | 10 | 40 |

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are not permitted to bring into the examination room: notes of any kind, a calculator of any type, blank sheets of paper and/or white out liquid/tape.


## Materials supplied

- Question and answer book of 12 pages with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.


## Instructions

- Detach the formula sheet from the centre of this book during reading time.
- Write your student number in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

## Instructions

Answer all questions in the spaces provided.
A decimal approximation will not be accepted if an exact answer is required to a question.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this book are not drawn to scale.
Take the acceleration due to gravity to have magnitude $g \mathrm{~m} / \mathrm{s}^{2}$, where $g=9.8$.

## Question 1

Express $\frac{2 \sqrt{3}+2 i}{1-\sqrt{3} i}$ in polar form.
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$\qquad$
$\qquad$
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$\qquad$
$\qquad$
$\qquad$
4 marks

## Question 2

a. Show that $\sqrt{5}-i$ is a solution of the equation $z^{3}-(\sqrt{5}-i) z^{2}+4 z-4 \sqrt{5}+4 i=0$.
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$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
b. Find all other solutions of the equation $z^{3}-(\sqrt{5}-i) z^{2}+4 z-4 \sqrt{5}+4 i=0$.
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$\qquad$
2 marks

## Question 3

Find the equation of the tangent to the curve $x^{3}-2 x^{2} y+2 y^{2}=2$ at the point $P(2,3)$.
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## Question 4

Find the volume generated when the region enclosed by the curve $y=\frac{1}{\sqrt{1-x^{2}}}$, the $x$-axis, the $y$-axis and the line $x=-\frac{1}{2}$ is rotated about the $x$-axis to form a solid of revolution.
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## Question 5

A block of mass 6 kg is given an initial push. As a result of this push, the block's initial velocity is $4 \mathrm{~m} / \mathrm{s}$ and it travels across a horizontal floor in a straight line. It comes to rest 3 metres from where it was pushed due to the frictional force, $F$, between the block and the floor.

a. Calculate the acceleration of the block across the floor.
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$\qquad$
$\qquad$
$\qquad$
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$\qquad$
$\qquad$
b. Calculate the value of $\mu$, the coefficient of friction between the block and the floor. Give your answer in the form $\frac{b}{c g}$ where $b$ and $c$ are positive integers.
$\qquad$
$\qquad$
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$\qquad$
$\qquad$

## Question 6

A particle moves so that its velocity at time $t$ is given by

$$
\underset{\sim}{\mathrm{v}}(t)=-4 \sin (2 t) \underset{\sim}{\mathrm{i}}+6 \cos (2 t) \underset{\sim}{\mathrm{j}} \text { for } 0 \leq t \leq \frac{\pi}{2} .
$$

a. Given that $\underset{\sim}{r}(0)=2 \underset{\sim}{i}$, find the position vector $\underset{\sim}{r}(t)$ of the particle at any time $t$.
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$\qquad$
$\qquad$
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$\qquad$
2 marks
b. Find the cartesian equation of the path followed by the particle.
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$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
2 marks
c. Sketch the path followed by the particle on the axes below.


## Question 7

a. Use Euler's method to find $y_{2}$ if $\frac{d y}{d x}=\frac{1}{x}$, given that $y_{0}=y(1)=1$ and $h=0.1$.

Express your answer as a fraction.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
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$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
2 marks
b. Solve the differential equation given in part a. to find the value of $y$ which is estimated by $y_{2}$. Express your answer in the form $\log _{e}(a)+b$, where $a$ and $b$ are positive real constants.
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$\qquad$

## Question 8

a. Sketch the slope field of the differential equation $\frac{d y}{d x}=\frac{1+y^{2}}{2}$ for $y=-2,-1,0,1,2$ at each of the values $x=-2,-1,0,1,2$ on the axes below.

b. If $y=-1$ when $x=0$, solve the differential equation given in part a. to find $y$ in terms of $x$.
$\qquad$
$\qquad$
$\qquad$
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$\qquad$
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$\qquad$
c. Sketch the graph of the solution curve found in part b. on the slope field in part a.

## Question 9

A particle moves in the cartesian plane with position vector $\underset{\sim}{r}=x \underset{\sim}{i}+y j$ where $x$ and $y$ are functions of $t$. If its velocity vector is $\underset{\sim}{v}=-y \underset{\sim}{i}+x \underset{\sim}{j}$, find the acceleration vector of the particle in terms of the position vector ${ }_{\sim}^{r}$.
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$\qquad$
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$\qquad$
$\qquad$
3 marks

## Question 10

Given that $\tan (2 x)=\frac{4 \sqrt{2}}{7}$ where $x \in\left[0, \frac{\pi}{4}\right)$, find the exact value of $\sin (x)$.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
3 marks

## SPECIALIST MATHEMATICS

## Written examinations 1 and 2

## FORMULA SHEET

## Directions to students

Detach this formula sheet during reading time.
This formula sheet is provided for your reference.

## Specialist Mathematics Formulas

## Mensuration

area of a trapezium:
curved surface area of a cylinder:
volume of a cylinder:
volume of a cone:
volume of a pyramid:
volume of a sphere:
area of a triangle:
sine rule:
cosine rule:
$\frac{1}{2}(a+b) h$
$2 \pi r h$
$\pi r^{2} h$
$\frac{1}{3} \pi r^{2} h$
$\frac{1}{3} A h$
$\frac{4}{3} \pi r^{3}$
$\frac{1}{2} b c \sin A$
$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$
$c^{2}=a^{2}+b^{2}-2 a b \cos C$

## Coordinate geometry

ellipse: $\quad \frac{(x-h)^{2}}{a^{2}}+\frac{(y-k)^{2}}{b^{2}}=1 \quad$ hyperbola: $\quad \frac{(x-h)^{2}}{a^{2}}-\frac{(y-k)^{2}}{b^{2}}=1$

## Circular (trigonometric) functions

$\cos ^{2}(x)+\sin ^{2}(x)=1$
$1+\tan ^{2}(x)=\sec ^{2}(x)$
$\cot ^{2}(x)+1=\operatorname{cosec}^{2}(x)$
$\sin (x+y)=\sin (x) \cos (y)+\cos (x) \sin (y)$
$\cos (x+y)=\cos (x) \cos (y)-\sin (x) \sin (y)$
$\sin (x-y)=\sin (x) \cos (y)-\cos (x) \sin (y)$
$x(x+y)=\cos (x) \cos (y)-\sin (x) \sin (y)$
$\cos (x-y)=\cos (x) \cos (y)+\sin (x) \sin (y)$
$\tan (x+y)=\frac{\tan (x)+\tan (y)}{1-\tan (x) \tan (y)}$
$\tan (x-y)=\frac{\tan (x)-\tan (y)}{1+\tan (x) \tan (y)}$
$\cos (2 x)=\cos ^{2}(x)-\sin ^{2}(x)=2 \cos ^{2}(x)-1=1-2 \sin ^{2}(x)$
$\sin (2 x)=2 \sin (x) \cos (x)$
$\tan (2 x)=\frac{2 \tan (x)}{1-\tan ^{2}(x)}$

| function | $\sin ^{-1}$ | $\cos ^{-1}$ | $\tan ^{-1}$ |
| :--- | :---: | :---: | :---: |
| domain | $[-1,1]$ | $[-1,1]$ | $R$ |
| range | $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ | $[0, \pi]$ | $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ |

## Algebra (complex numbers)

$z=x+y i=r(\cos \theta+i \sin \theta)=r \operatorname{cis} \theta$
$|z|=\sqrt{x^{2}+y^{2}}=r$

$$
\begin{aligned}
& -\pi<\operatorname{Arg} z \leq \pi \\
& \frac{z_{1}}{z_{2}}=\frac{r_{1}}{r_{2}} \operatorname{cis}\left(\theta_{1}-\theta_{2}\right)
\end{aligned}
$$

$z^{n}=r^{n} \operatorname{cis}(n \theta)$ (de Moivre's theorem)

## Calculus

$\frac{d}{d x}\left(x^{n}\right)=n x^{n-1}$
$\int x^{n} d x=\frac{1}{n+1} x^{n+1}+c, n \neq-1$
$\frac{d}{d x}\left(e^{a x}\right)=a e^{a x}$
$\int e^{a x} d x=\frac{1}{a} e^{a x}+c$
$\frac{d}{d x}\left(\log _{e}(x)\right)=\frac{1}{x}$
$\int \frac{1}{x} d x=\log _{e}|x|+c$
$\frac{d}{d x}(\sin (a x))=a \cos (a x)$
$\int \sin (a x) d x=-\frac{1}{a} \cos (a x)+c$
$\frac{d}{d x}(\cos (a x))=-a \sin (a x)$
$\int \cos (a x) d x=\frac{1}{a} \sin (a x)+c$
$\frac{d}{d x}(\tan (a x))=a \sec ^{2}(a x)$
$\int \sec ^{2}(a x) d x=\frac{1}{a} \tan (a x)+c$
$\frac{d}{d x}\left(\sin ^{-1}(x)\right)=\frac{1}{\sqrt{1-x^{2}}}$
$\int \frac{1}{\sqrt{a^{2}-x^{2}}} d x=\sin ^{-1}\left(\frac{x}{a}\right)+c, a>0$
$\frac{d}{d x}\left(\cos ^{-1}(x)\right)=\frac{-1}{\sqrt{1-x^{2}}}$
$\int \frac{-1}{\sqrt{a^{2}-x^{2}}} d x=\cos ^{-1}\left(\frac{x}{a}\right)+c, a>0$
$\frac{d}{d x}\left(\tan ^{-1}(x)\right)=\frac{1}{1+x^{2}}$
$\int \frac{a}{a^{2}+x^{2}} d x=\tan ^{-1}\left(\frac{x}{a}\right)+c$
product rule:

$$
\frac{d}{d x}(u v)=u \frac{d v}{d x}+v \frac{d u}{d x}
$$

quotient rule:

$$
\frac{d}{d x}\left(\frac{u}{v}\right)=\frac{v \frac{d u}{d x}-u \frac{d v}{d x}}{v^{2}}
$$

chain rule:

$$
\frac{d y}{d x}=\frac{d y}{d u} \frac{d u}{d x}
$$

Euler's method: If $\frac{d y}{d x}=f(x), x_{0}=a$ and $y_{0}=b$, then $x_{n+1}=x_{n}+h$ and $y_{n+1}=y_{n}+h f\left(x_{n}\right)$
acceleration:

$$
a=\frac{d^{2} x}{d t^{2}}=\frac{d v}{d t}=v \frac{d v}{d x}=\frac{d}{d x}\left(\frac{1}{2} v^{2}\right)
$$

constant (uniform) acceleration: $\quad v=u+a t \quad s=u t+\frac{1}{2} a t^{2} \quad v^{2}=u^{2}+2 a s \quad s=\frac{1}{2}(u+v) t$

## Vectors in two and three dimensions

$$
\underset{\sim}{\mathrm{r}}=x \underset{\sim}{\mathrm{i}}+y \underset{\sim}{\mathrm{j}}+z \underset{\sim}{\mathrm{k}}
$$

$|\underset{\sim}{\mathbf{r}}|=\sqrt{x^{2}+y^{2}+z^{2}}=r$
$\underset{\sim}{r}{ }_{1} \cdot \underset{\sim}{r}{ }_{2}=r_{1} r_{2} \cos \theta=x_{1} x_{2}+y_{1} y_{2}+z_{1} z_{2}$
$\underset{\sim}{\mathrm{i}}=\frac{d \underset{\sim}{\mathrm{r}}}{d t}=\frac{d x}{d t} \mathrm{i}+\frac{d y}{d t} \underset{\sim}{\mathrm{j}}+\frac{d z}{d t} \underset{\sim}{\mathrm{k}}$

## Mechanics

momentum:
equation of motion:
friction:
$\underset{\sim}{p}=m \underset{\sim}{v}$
$\mathrm{R}=m \mathrm{a}$
$F \leq \mu N$

