



Victorian Certificate of Education 2008

FURTHER MATHEMATICS

Written examination 1

Monday 3 November 2008

Reading time: 11.45 am to 12.00 noon (15 minutes)

Writing time: 12.00 noon to 1.30 pm (1 hour 30 minutes)

MULTIPLE-CHOICE QUESTION BOOK

Structure of book

<i>Section</i>	<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of modules</i>	<i>Number of modules to be answered</i>	<i>Number of marks</i>
A	13	13			13
B	54	27	6	3	27
					Total 40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question book of 36 pages with a detachable sheet of miscellaneous formulas in the centrefold.
- Answer sheet for multiple-choice questions.
- Working space is provided throughout the book.

Instructions

- Detach the formula sheet from the centre of this book during reading time.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

At the end of the examination

- You may keep this question book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Working space

TURN OVER

SECTION A**Instructions for Section A**

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

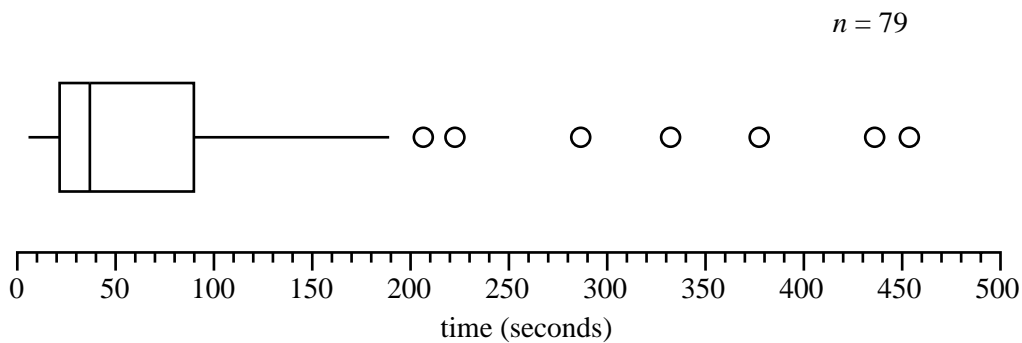
Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Core – Data analysis

The following information relates to Questions 1, 2, 3 and 4.

The box plot below shows the distribution of the time, in seconds, that 79 customers spent moving along a particular aisle in a large supermarket.



data source: www.stars.ac.uk

Question 1

The longest time, in seconds, spent moving along this aisle is closest to

- A. 40
- B. 60
- C. 190
- D. 450
- E. 500

Question 2

The shape of the distribution is best described as

- A. symmetric.
- B. negatively skewed.
- C. negatively skewed with outliers.
- D. positively skewed.
- E. positively skewed with outliers.

Question 3

The number of customers who spent more than 90 seconds moving along this aisle is closest to

- A. 7
- B. 20
- C. 26
- D. 75
- E. 79

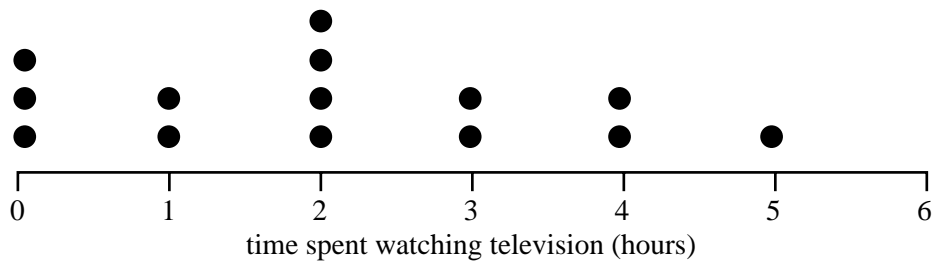
Question 4

From the box plot, it can be concluded that the median time spent moving along the supermarket aisle is

- A. less than the mean time.
- B. equal to the mean time.
- C. greater than the mean time.
- D. half of the interquartile range.
- E. one quarter of the range.

Question 5

A sample of 14 people were asked to indicate the time (in hours) they had spent watching television on the previous night. The results are displayed in the dot plot below.



Correct to one decimal place, the mean and standard deviation of these times are respectively

- A. $\bar{x} = 2.0$ $s = 1.5$
- B. $\bar{x} = 2.1$ $s = 1.5$
- C. $\bar{x} = 2.1$ $s = 1.6$
- D. $\bar{x} = 2.6$ $s = 1.2$
- E. $\bar{x} = 2.6$ $s = 1.3$

The following information relates to Questions 6 and 7.

The pulse rates of a large group of 18-year-old students are approximately normally distributed with a mean of 75 beats/minute and a standard deviation of 11 beats/minute.

Question 6

The percentage of 18-year-old students with pulse rates less than 75 beats/minute is closest to

- A. 32%
- B. 50%
- C. 68%
- D. 84%
- E. 97.5%

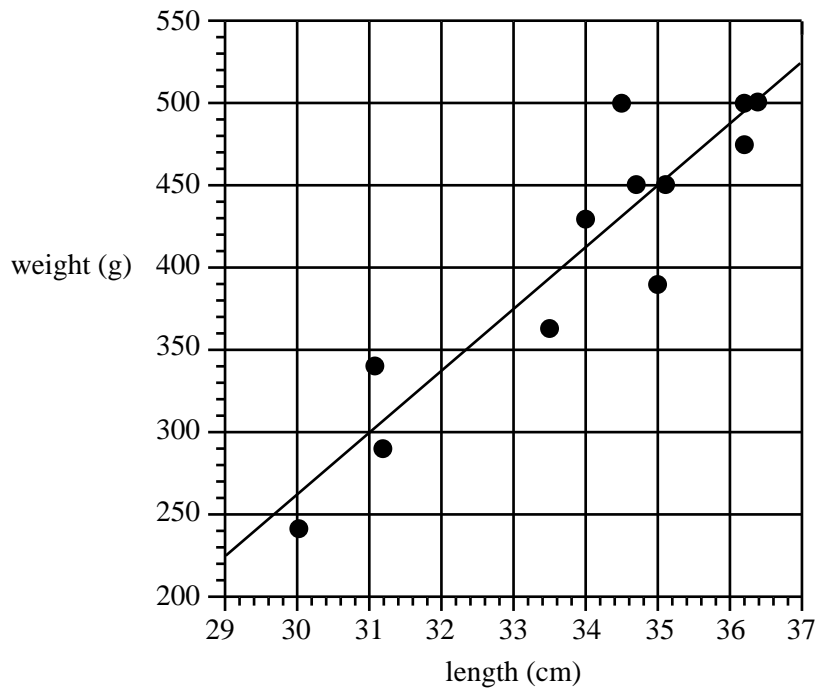
Question 7

The percentage of 18-year-old students with pulse rates less than 53 beats/minute or greater than 86 beats/minute is closest to

- A. 2.5%
- B. 5%
- C. 16%
- D. 18.5%
- E. 21%

The following information relates to Questions 8 and 9.

The weights (in g) and lengths (in cm) of 12 fish were recorded and plotted in the scatterplot below. The least squares regression line that enables the weight of these fish to be predicted from their length has also been plotted.



Data source: Journal of Statistics Education Data Archive (www.amstat.org/publications/jse)

Question 8

The least squares regression line predicts that the weight (in g) of a fish of length 30 cm would be closest to

- A. 240
- B. 252
- C. 262
- D. 274
- E. 310

Question 9

The median weight (in g) of the 12 fish is closest to

- A. 346
- B. 375
- C. 440
- D. 450
- E. 475

Question 10

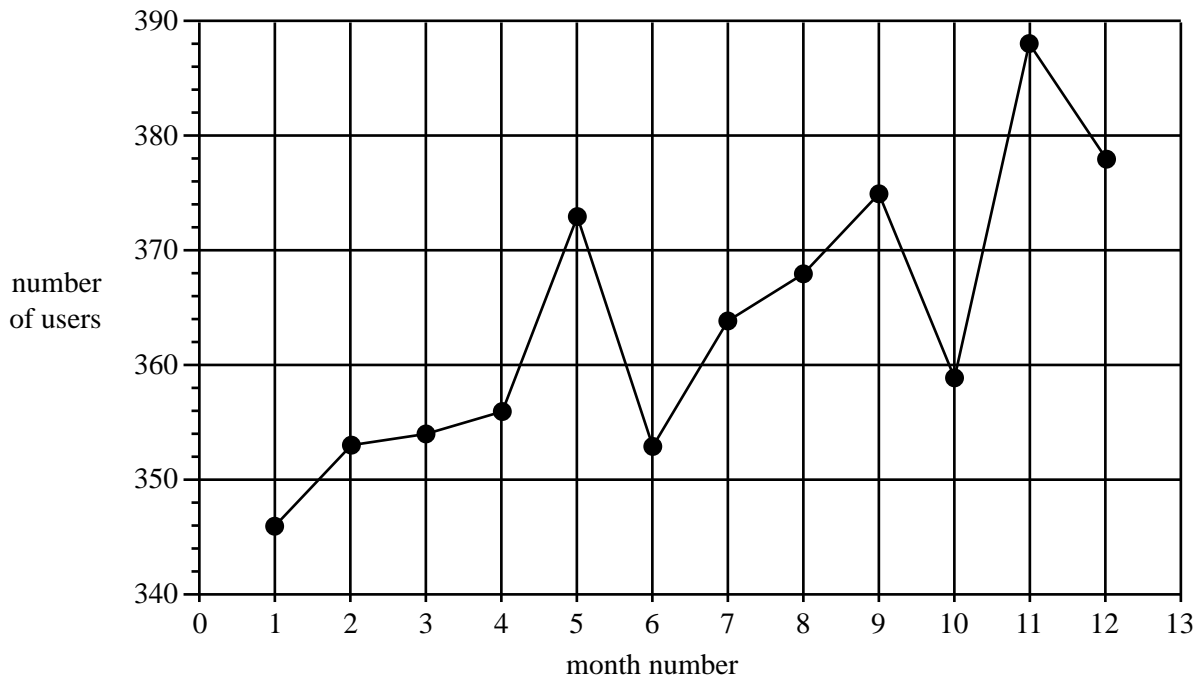
A large study of Year 12 students shows that there is a negative association between the time spent doing homework each week and the time spent watching television. The correlation coefficient is $r = -0.6$.

From this information it can be concluded that

- A. the time spent doing homework is 60% lower than the time spent watching television.
- B. 36% of students spend more time watching television than doing homework.
- C. the slope of the least squares regression line is 0.6.
- D. if a student spends less time watching television, they will do more homework.
- E. an increased time spent watching television is associated with a decreased time doing homework.

The following information relates to Questions 11, 12 and 13.

The time series plot below shows the number of users each month of an online help service over a twelve-month period.

**Question 11**

The time series plot has

- A. no trend.
- B. no variability.
- C. seasonality only.
- D. an increasing trend with seasonality.
- E. an increasing trend only.

Question 12

The data values used to construct the time series plot are given below.

Month number	1	2	3	4	5	6	7	8	9	10	11	12
Number of users	346	353	354	356	373	353	364	368	375	359	388	378

A four-point moving mean with centring is used to smooth timeline series.

The smoothed value of the number of users in month number 5 is closest to

- A. 357
- B. 359
- C. 360
- D. 365
- E. 373

Question 13

A least squares regression line is fitted to the time series plot.

The equation of this least squares regression line is

$$\text{number of users} = 346 + 2.77 \times \text{month number}$$

Let month number 1 = January 2007, month number 2 = February 2007, and so on.

Using the above information, the regression line predicts that the number of users in December 2009 will be closest to

- A. 379
- B. 412
- C. 443
- D. 446
- E. 448

SECTION B**Instructions for Section B**

Select **three** modules and answer **all** questions within the modules selected in pencil on the answer sheet provided for multiple-choice questions.

Show the modules you are answering by shading the matching boxes on your multiple-choice answer sheet **and** writing the name of the module in the box provided.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Module	Page
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Module 1: Number patterns

Before answering these questions you must **shade** the Number patterns box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

A sequence is generated by a first-order linear difference equation.

The first four terms of this sequence are 1, 3, 7, 15.

The next term in the sequence is

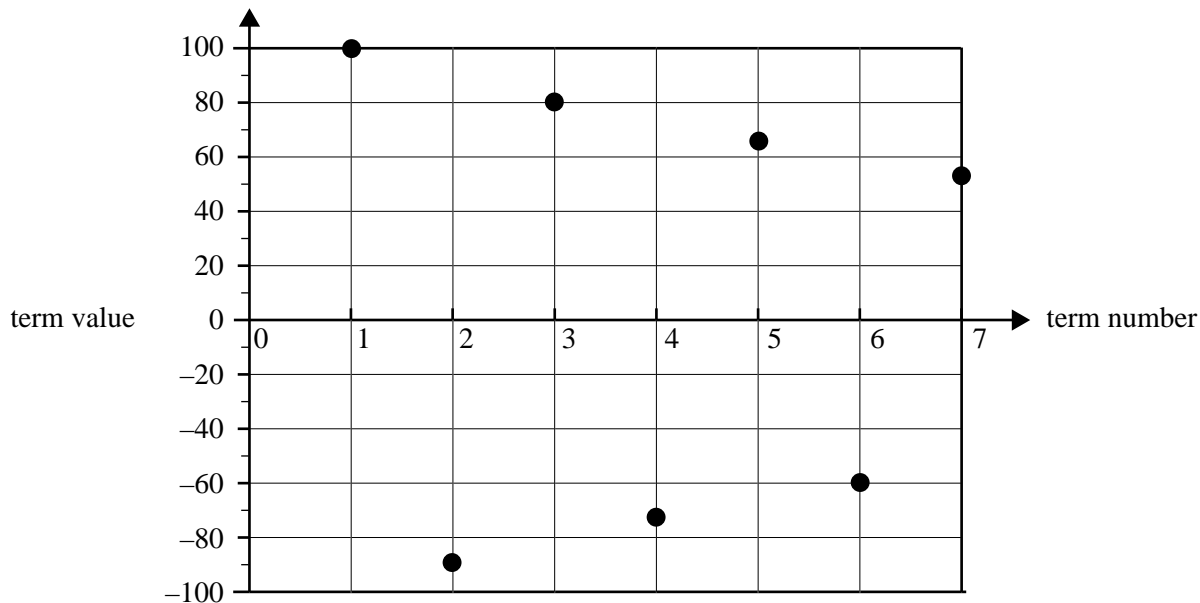
- A. 17
- B. 19
- C. 22
- D. 23
- E. 31

Question 2

For an examination, 8600 examination papers are to be printed at a rate of 25 papers per minute.

After one hour, the number of examination papers that still need to be printed is

- A. 1600
- B. 2500
- C. 6100
- D. 7100
- E. 8575

Question 3

The values of the first seven terms of a geometric sequence are plotted on the graph above.

Values of a and r that could apply to this sequence are respectively

- A. $a = 90$ $r = -0.9$
- B. $a = 100$ $r = -0.9$
- C. $a = 100$ $r = -0.8$
- D. $a = 100$ $r = 0.8$
- E. $a = 90$ $r = 0.9$

Question 4

In 2008, there are 800 bats living in a park.

After 2008, the number of bats living in the park is expected to increase by 15% per year.

Let B_n represent the number of bats living in the park n years after 2008.

A difference equation that can be used to determine the number of bats living in the park n years after 2008 is

- A. $B_n = 1.15B_{n-1} - 800$ $B_0 = 2008$
- B. $B_n = B_{n-1} + 1.15 \times 800$ $B_0 = 2008$
- C. $B_n = B_{n-1} - 0.15 \times 800$ $B_0 = 800$
- D. $B_n = 0.15B_{n-1}$ $B_0 = 800$
- E. $B_n = 1.15B_{n-1}$ $B_0 = 800$

Question 5

The first term of a geometric sequence is 9.

The third term of this sequence is 121.

The second term of this sequence could be

- A. -65
- B. -33
- C. 56
- D. 65
- E. 112

Question 6

Kai commenced a 12-day program of daily exercise. The time, in minutes, that he spent exercising on each of the first four days of the program is shown in the table below.

Day number	1	2	3	4
Time (minutes)	15	19	23	27

If this pattern continues, the total time (in minutes) that Kai will have spent exercising after 12 days is

- A. 59
- B. 180
- C. 354
- D. 444
- E. 468

Question 7

The sequence

12, 15, 27, 42, 69, 111 . . .

can best be described as

- A. fibonacci-related
- B. arithmetic with $d > 1$
- C. arithmetic with $d < 1$
- D. geometric with $r > 1$
- E. geometric with $r < 1$

Question 8

When placed in a pond, the length of a fish was 14.2 centimetres.

During its first month in the pond, the fish increased in length by 3.6 centimetres.

During its n th month in the pond, the fish increased in length by G_n centimetres, where $G_{n+1} = 0.75G_n$

The maximum length this fish can grow to (in cm) is closest to

- A. 14.4
- B. 16.9
- C. 19.0
- D. 28.6
- E. 71.2

Question 9

When full, a swimming pool holds 50 000 litres of water.

Due to evaporation and spillage the pool loses, on average, 2% of the water it contains each week.

To help to make up this loss, 500 litres of water is added to the pool at the end of each week.

Assume the pool is full at the start of Week 1.

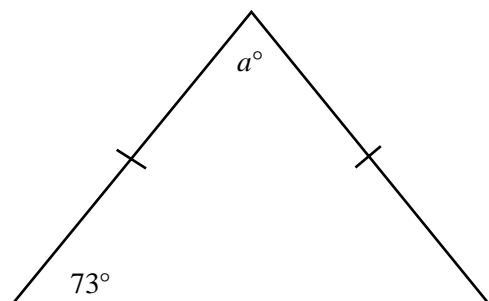
At the start of Week 5 the amount of water (in litres) that the pool contains will be closest to

- A. 47 500
- B. 47 600
- C. 48 000
- D. 48 060
- E. 48 530

Module 2: Geometry and trigonometry

Before answering these questions you must **shade** the Geometry and trigonometry box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

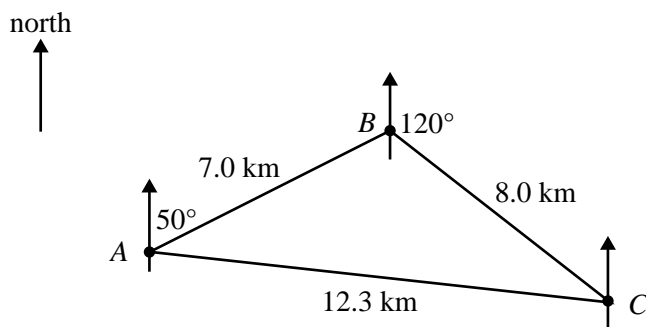


For the isosceles triangle shown above, the value of a is

- A. 17
- B. 34
- C. 73
- D. 90
- E. 107

The following information relates to Questions 2 and 3.

An orienteering course is triangular in shape and is marked by three points, A , B and C , as shown in the diagram below.



Question 2

In this course, the bearing of B from A is 050° and the bearing of C from B is 120° .

The bearing of B from C is

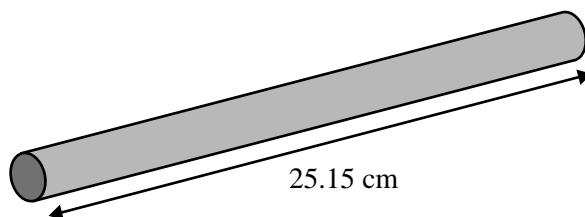
- A. 060°
- B. 120°
- C. 240°
- D. 300°
- E. 310°

Question 3

In this course, B is 7.0 km from A , C is 8.0 km from B and A is 12.3 km from C .

The area (in km^2) enclosed by this course is closest to

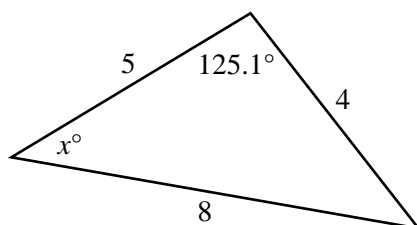
- A. 21
- B. 24
- C. 25
- D. 26
- E. 28

Question 4

The solid cylindrical rod shown above has a volume of 490.87 cm^3 . The length is 25.15 cm.

The radius (in cm) of the cross-section of the rod, correct to one decimal place, is

- A. 2.5
- B. 5.0
- C. 6.3
- D. 12.5
- E. 19.6

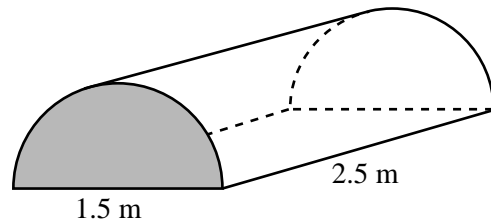
Question 5

For the triangle shown, the value of $\sin x^\circ$ is given by

- A. $\frac{\sin 125.1^\circ}{2}$
- B. $\frac{5^2 + 4^2 - 8^2}{2 \times 5 \times 4}$
- C. $2 \times \sin 125.1^\circ$
- D. $\frac{5^2 + 8^2 - 4^2}{2 \times 5 \times 8}$
- E. $\frac{5 \times \sin 125.1^\circ}{8}$

Question 6

A tent with semicircular ends is in the shape of a prism. The diameter of the ends is 1.5 m. The tent is 2.5 m long.



The total surface area (in m^2) of the tent, including the base, is closest to

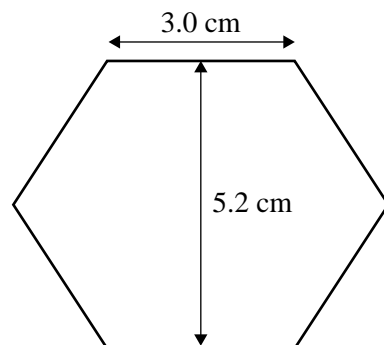
- A. 5.5
- B. 7.7
- C. 8.8
- D. 11.4
- E. 15.3

Question 7

Sand is poured out of a truck and forms a pile in the shape of a right circular cone. The diameter of the base of the pile of sand is 2.6 m. The height is 1.2 m.

The volume (in m^3) of sand in the pile is closest to

- A. 2.1
- B. 3.1
- C. 6.4
- D. 8.5
- E. 25.5

Question 8

A regular hexagon has side length 3.0 cm and height 5.2 cm as shown in the diagram above.

The area (in cm^2) of the hexagon is closest to

- A. 11.7
- B. 13.5
- C. 15.6
- D. 18.0
- E. 23.4

Question 9

Two hikers, Anton and Beth, walk in different directions from the same camp.

Beth walks for 12 km on a bearing of 135° to a picnic ground.

Anton walks for 6 km on a bearing of 045° to a lookout tower.

On what bearing (to the nearest degree) should Anton walk from the lookout tower to meet Beth at the picnic ground?

- A. 063°
- B. 108°
- C. 153°
- D. 162°
- E. 180°

Module 3: Graphs and relations

Before answering these questions you must **shade** the Graphs and relations box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

The concentration (in mg/L) of a particular chemical in a swimming pool is graphed over a four-week period.



For this four-week period, the concentration of the chemical was greater than 3 mg/L for

- A. exactly four weeks.
- B. between three and four weeks.
- C. exactly two weeks.
- D. exactly one week.
- E. less than one week.

Question 2

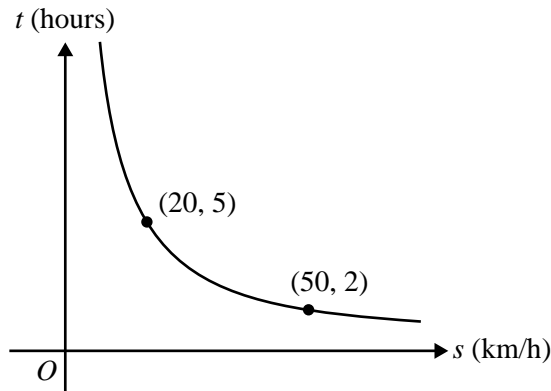
Initially there are 5000 litres of water in a tank. Water starts to flow out of the tank at the constant rate of 2 litres per minute until the tank is empty.

After t minutes, the number of litres of water in the tank, V , will be

- A. $V = 5000 - 2t$
- B. $V = 2t - 5000$
- C. $V = 5000 + 2t$
- D. $V = 2 - 5000t$
- E. $V = 5000t - 2$

Question 3

The graph below shows the time t , in hours, taken to travel 100 km at an average speed of s km/h.



Which statement is false?

- A. As average speed increases, the time taken to travel 100 km decreases.
- B. It will take 2 hours to travel 100 km at an average speed of 50 km/h.
- C. The relationship between time and average speed is linear.
- D. When travelling at an average speed of 20 km/h, the 100 km journey takes 5 hours to complete.
- E. A formula that relates s and t is $t = \frac{100}{s}$, $s > 0$

Question 4

When shopping, Betty can use either Easypark or Safepark to park her car.

At Easypark, cars can be parked for up to 8 hours per day.

The fee structure is as follows.

$$Fee = \begin{cases} \$5.00, & 0 < hours \leq 2 \\ \$8.00, & 2 < hours \leq 5 \\ \$11.00, & 5 < hours \leq 8 \end{cases}$$

Safepark charges fees according to the formula

$$Fee = \$2.50 \times hours$$

Betty wants to park her car for 5 hours on Monday and 3 hours on Tuesday.

The minimum total fee that she can pay for parking for the two days is

- A. \$7.50
- B. \$11.00
- C. \$15.50
- D. \$16.00
- E. \$20.00

Question 5

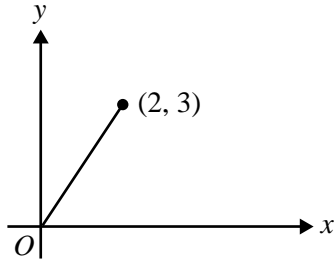
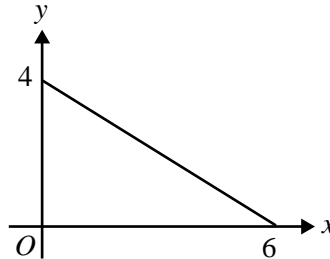
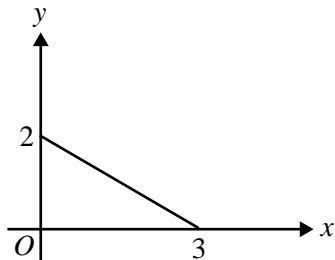
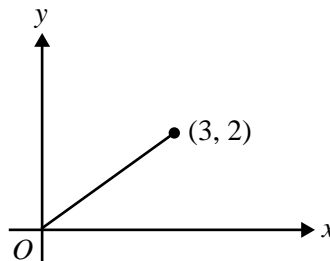
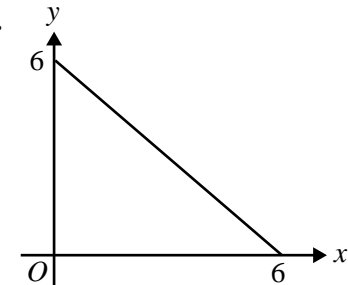
A mixture contains two liquids, A and B .

Liquid A costs \$2 per litre and liquid B costs \$3 per litre.

Let x be the volume (in litres) of liquid A purchased.

Let y be the volume (in litres) of liquid B purchased.

Which graph below shows all possible volumes of liquid A and liquid B that can be purchased for exactly \$12?

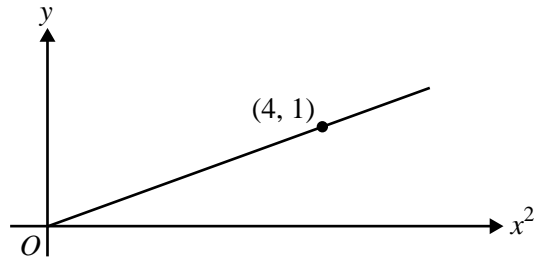
A.**B.****C.****D.****E.****Question 6**

At the local bakery, the cost of four donuts and six buns is \$14.70.

The cost of three donuts and five buns is \$11.90.

At this bakery, the cost of one donut and two buns will be

- A.** \$2.80
- B.** \$3.80
- C.** \$3.85
- D.** \$4.55
- E.** \$4.85

Question 7

The graph above shows the relationship between y and x^2 .

The relationship between y and x is

- A. $y = 4x$
- B. $y = \frac{1}{4}x$
- C. $y = \frac{1}{4}x^2$
- D. $y = 16x^2$
- E. $y = \frac{1}{16}x^2$

Question 8

A region is defined by the following inequalities

$$y \geq -4x + 10$$

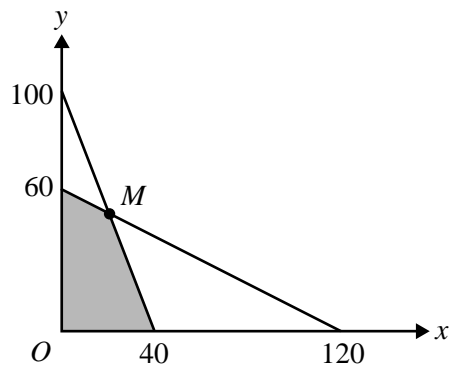
$$y - x \geq 1$$

A point that lies within this region is

- A. (1, 3)
- B. (2, 1)
- C. (3, 2)
- D. (4, 6)
- E. (5, 1)

Question 9

The shaded region in the graph below represents the feasible region for a linear programming problem.



Which objective function, Z , has its maximum value at the point M ?

- A. $Z = x + y$
- B. $Z = x - y$
- C. $Z = 3x + y$
- D. $Z = 3x - 2y$
- E. $Z = x + 4y$

Module 4: Business-related mathematics

Before answering these questions you must **shade** the Business-related mathematics box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

A plumber quoted \$300, excluding GST (Goods and Services Tax), to complete a job.

A GST of 10% is added to the price.

The full price for the job will be

- A. \$3
- B. \$30
- C. \$303
- D. \$310
- E. \$330

Question 2

Pia invests \$800 000 in an ordinary perpetuity to provide an ongoing fortnightly pension for her retirement. The interest rate for this investment is 5.8% per annum.

Assuming there are 26 fortnights per year, the amount she will receive at the end of each fortnight is closest to

- A. \$464
- B. \$892
- C. \$1422
- D. \$1785
- E. \$3867

Question 3

A computer originally purchased for \$6000 is depreciated each year using the reducing balance method.

If the computer is valued at \$2000 after four years, then the annual rate of depreciation is closest to

- A. 17%
- B. 24%
- C. 25%
- D. 28%
- E. 33%

The following information relates to Questions 4 and 5.

Sandra has purchased a \$4200 plasma television under a hire-purchase agreement. She paid \$600 deposit and will pay the balance in equal monthly instalments over one year.

A flat interest rate of 6% per annum is charged.

Question 4

The amount of each monthly instalment is

- A. \$300
- B. \$303
- C. \$318
- D. \$350
- E. \$371

Question 5

The annual effective interest rate that Sandra pays under this agreement is closest to

- A. 10%
- B. 11%
- C. 12%
- D. 13%
- E. 14%

Question 6

Sam and Charlie each invest \$5000 for three years.

Sam's investment earns simple interest at the rate of 7.5% per annum.

Charlie's investment earns interest at the rate of 7.5% per annum compounding annually.

At the conclusion of three years, correct to the nearest cent, Sam will have

- A. \$86.48 less than Charlie.
- B. \$86.48 more than Charlie.
- C. \$132.23 less than Charlie.
- D. \$132.23 more than Charlie.
- E. the same as Charlie.

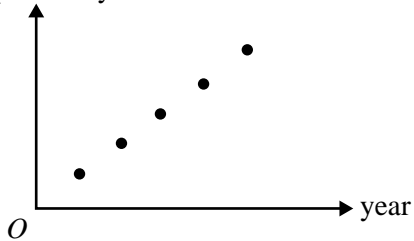
Question 7

Ernie took out a reducing balance loan to buy a new family home.

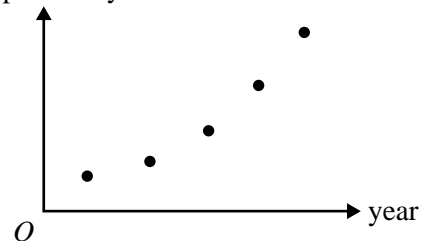
He correctly graphed the amount **paid off** the principal of his loan each year for the first five years.

The shape of this graph (for the first five years of the loan) is best represented by

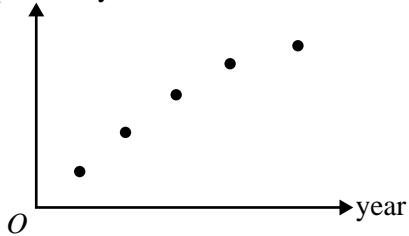
A.
amount paid off the
principal each year



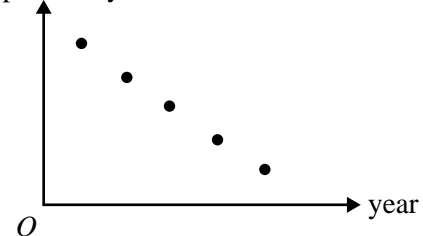
B.
amount paid off the
principal each year



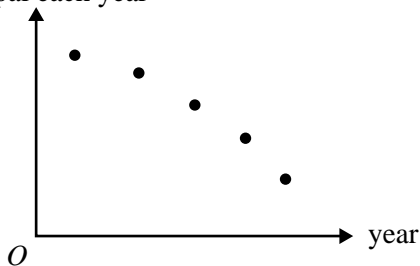
C.
amount paid off the
principal each year



D.
amount paid off the
principal each year



E.
amount paid off the
principal each year

**Question 8**

A loan of \$300 000 is taken out to finance a new business venture.

The loan is to be repaid fully over twenty years with quarterly payments of \$6727.80.

Interest is calculated quarterly on the reducing balance.

The annual interest rate for this loan is closest to

- A. 4.1%
- B. 6.5%
- C. 7.3%
- D. 19.5%
- E. 26.7%

Question 9

An amount of \$8000 is invested for a period of 4 years.

The interest rate for this investment is 7.2% per annum compounding quarterly.

The interest earned by the investment in the fourth year (in dollars) is given by

- A. $4 \times \left(\frac{7.2}{100} \times 8000 \right)$
- B. $8000 \times 1.018^4 - 8000 \times 1.018^3$
- C. $8000 \times 1.018^{16} - 8000 \times 1.018^{12}$
- D. $8000 \times 1.072^4 - 8000 \times 1.072^3$
- E. $8000 \times 1.072^{16} - 8000 \times 1.072^{12}$

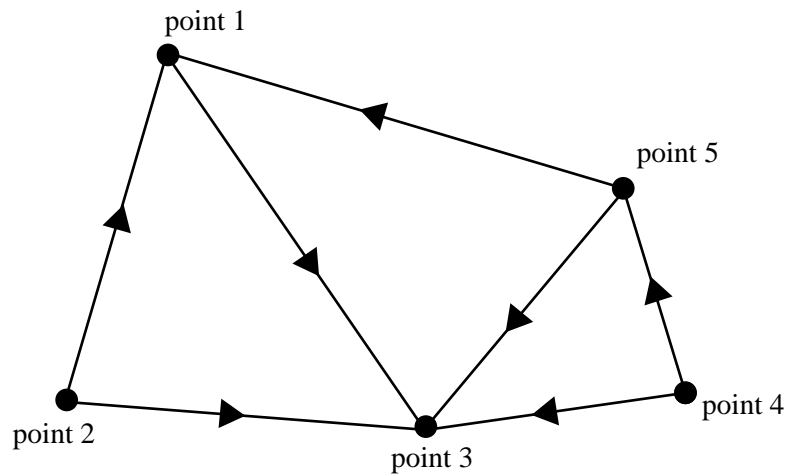
Module 5: Networks and decision mathematics

Before answering these questions you must **shade** the Networks and decision mathematics box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

Steel water pipes connect five points underground.

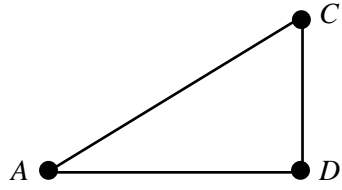
The directed graph below shows the directions of the flow of water through these pipes between these points.



The directed graph shows that water can flow from

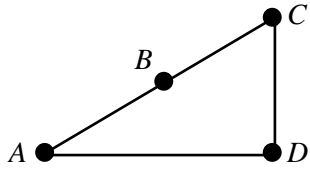
- A. point 1 to point 2.
- B. point 1 to point 4.
- C. point 4 to point 1.
- D. point 4 to point 2.
- E. point 5 to point 2.

Question 2

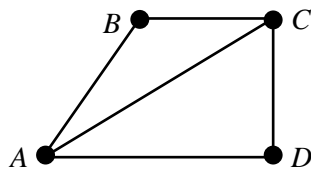


The graph above is a subgraph of which one of the following graphs?

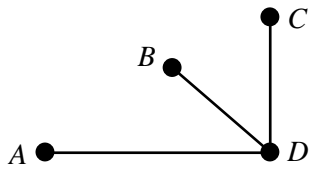
A.



B.



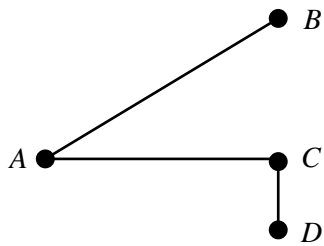
C.

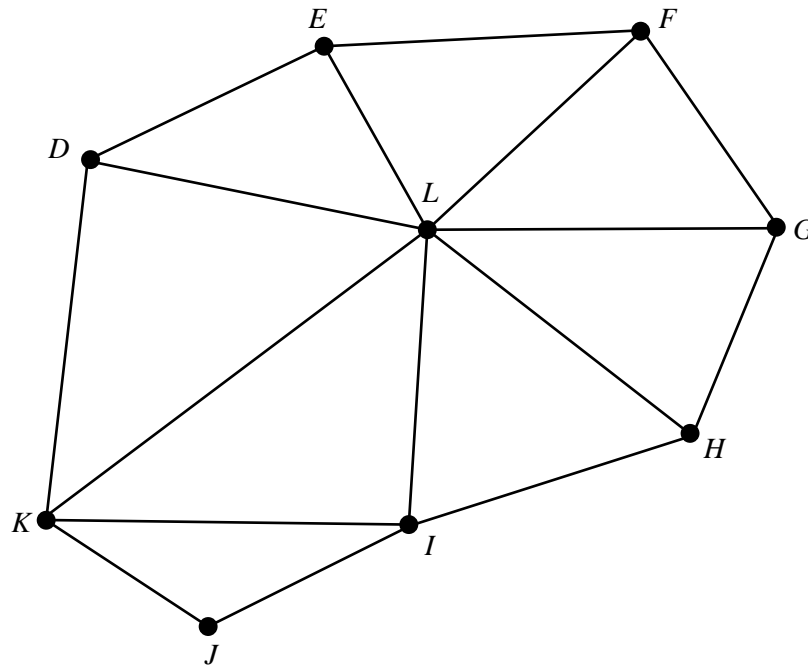


D.



E.



Question 3

A Hamiltonian circuit for the graph above is

- A. *KJIHGLFEDK*
- B. *DKLIJHGFED*
- C. *DEFGHIJKD*
- D. *JIKDLHGFE*
- E. *GHILKJILDEFG*

Question 4

A simple connected graph with 3 edges has 4 vertices.

This graph must be

- A. a complete graph.
- B. a tree.
- C. a non-planar graph.
- D. a graph that contains a loop.
- E. a graph that contains a circuit.

Question 5

A connected planar graph has five vertices, A, B, C, D and E .

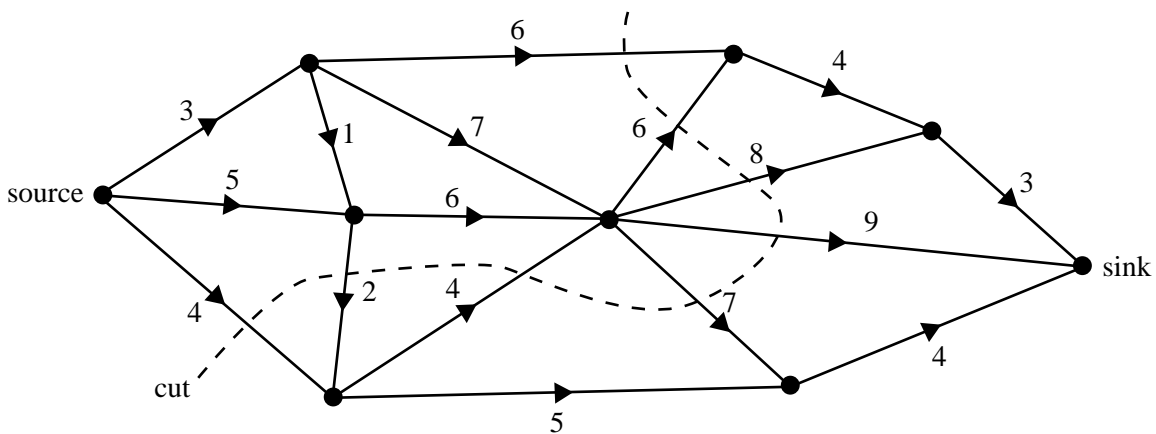
The degree of each vertex is given in the following table.

Vertex	Degree
A	3
B	4
C	3
D	5
E	3

Which one of the following statements regarding this planar graph is true?

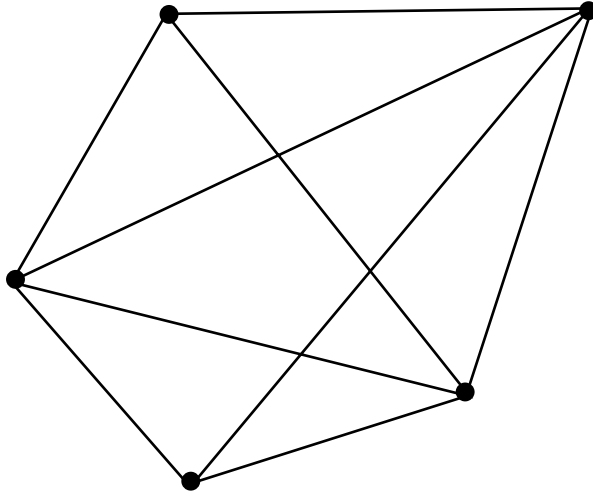
- A. The sum of degrees of the vertices equals 15.
- B. It contains more than one Eulerian path.
- C. It contains an Eulerian circuit.
- D. Euler's formula $v + f = e + 2$ could not be used.
- E. The addition of one further edge could create an Eulerian path.

Question 6



For the graph above, the capacity of the cut shown is

- A. 33
- B. 36
- C. 40
- D. 42
- E. 46

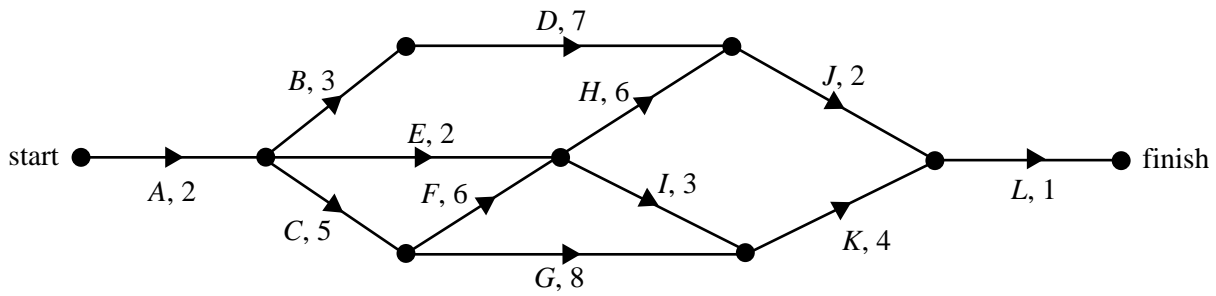
Question 7

The graph above has

- A. 4 faces.
- B. 5 faces.
- C. 6 faces.
- D. 8 faces.
- E. 9 faces.

The following information relates to Questions 8 and 9.

The network below shows the activities that are needed to finish a particular project and their completion times (in days).



Question 8

The earliest start time for Activity *K*, in days, is

- A. 7
- B. 15
- C. 16
- D. 19
- E. 20

Question 9

This project currently has one critical path.

A second critical path, in addition to the first, would be created by

- A. increasing the completion time of *D* by 7 days.
- B. increasing the completion time of *G* by 1 day.
- C. increasing the completion time of *I* by 2 days.
- D. decreasing the completion time of *C* by 1 day.
- E. decreasing the completion time of *H* by 2 days.

Module 6: Matrices

Before answering these questions you must **shade** the Matrices box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

If $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 3 & 7 \\ 8 & d \end{bmatrix} = \begin{bmatrix} 4 & 7 \\ 8 & 11 \end{bmatrix}$ then d is equal to

- A. -11
- B. -10
- C. 7
- D. 10
- E. 11

Question 2

Apples cost \$3.50 per kg, bananas cost \$4.20 per kg and carrots cost \$1.89 per kg.

Ashley buys 3 kg of apples, 2 kg of bananas and 1 kg of carrots.

A matrix product to calculate the total cost of these items is

A. $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 3.50 \\ 4.20 \\ 1.89 \end{bmatrix}$

B. $[3 \ 2 \ 1] [3.50 \ 4.20 \ 1.89]$

C. $[3.50 \times 2 \ 4.20 \times 3 \ 1.89 \times 1]$

D. $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} [3.50 \ 4.20 \ 1.89]$

E. $[3.50 \ 4.20 \ 1.89] \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

Question 3

The cost prices of three different electrical items in a store are \$230, \$290 and \$310 respectively.

The selling price of each of these three electrical items is 1.3 times the cost price plus a commission of \$20 for the salesman.

A matrix that lists the selling price of each of these three electrical items is determined by evaluating

A. $1.3 \times \begin{bmatrix} 230 \\ 290 \\ 310 \end{bmatrix} + [20]$

B. $1.3 \times \begin{bmatrix} 230 \\ 290 \\ 310 \end{bmatrix} + 1.3 \times 20$

C. $1.3 \times \begin{bmatrix} 230 \\ 290 \\ 310 \end{bmatrix} + \begin{bmatrix} 20 \\ 20 \\ 20 \end{bmatrix}$

D. $1.3 \times \begin{bmatrix} 230 \\ 290 \\ 310 \end{bmatrix} + 1.3 \times \begin{bmatrix} 20 \\ 20 \\ 20 \end{bmatrix}$

E. $1.3 \times \begin{bmatrix} 230 + 20 \\ 290 + 20 \\ 310 + 20 \end{bmatrix}$

Question 4

Matrix A is a 1×3 matrix.

Matrix B is a 3×1 matrix.

Which one of the following matrix expressions involving A and B is defined?

A. $A + \frac{1}{3}B$

B. $2B \times 3A$

C. A^2B

D. B^{-1}

E. $B - A$

Question 5

The determinant of $\begin{bmatrix} 3 & 2 \\ 6 & x \end{bmatrix}$ is equal to 9.

The value of x is

A. -7

B. -4.5

C. 1

D. 4.5

E. 7

Question 6

The solution of the matrix equation $\begin{bmatrix} 0 & -3 & 2 \\ 1 & 1 & 1 \\ -2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ 5 \\ 8 \end{bmatrix}$ is

A. $\begin{bmatrix} 1 \\ 24 \\ 2 \end{bmatrix}$

B. $\begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$

C. $\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$

D. $\begin{bmatrix} -11 \\ 4 \\ \frac{4}{3} \\ 8 \end{bmatrix}$

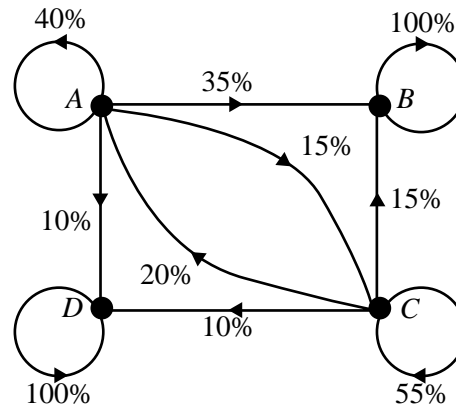
E. $\begin{bmatrix} 11 \\ 5 \\ 8 \end{bmatrix}$

The following information relates to Questions 7, 8 and 9.

A large population of mutton birds migrates each year to a remote island to nest and breed. There are four nesting sites on the island, A , B , C and D .

Researchers suggest that the following transition matrix can be used to predict the number of mutton birds nesting at each of the four sites in subsequent years. An equivalent transition diagram is also given.

$$T = \begin{array}{c} \text{this year} \\ \begin{array}{cccc} A & B & C & D \\ \left[\begin{array}{cccc} 0.4 & 0 & 0.2 & 0 \\ 0.35 & 1 & 0.15 & 0 \\ 0.15 & 0 & 0.55 & 0 \\ 0.1 & 0 & 0.1 & 1 \end{array} \right] \\ \begin{array}{l} A \\ B \\ C \\ D \end{array} \end{array} \end{array} \begin{array}{l} \\ \\ \\ \text{next year} \end{array}$$



Question 7

Two thousand eight hundred mutton birds nest at site C in 2008.

Of these 2800 mutton birds, the number that nest at site A in 2009 is predicted to be

- A. 560
- B. 980
- C. 1680
- D. 2800
- E. 3360

Question 8

This transition matrix predicts that, in the long term, the mutton birds will

- A. nest only at site A .
- B. nest only at site B .
- C. nest only at sites A and C .
- D. nest only at sites B and D .
- E. continue to nest at all four sites.

Question 9

Six thousand mutton birds nest at site B in 2008.

Assume that an equal number of mutton birds nested at each of the four sites in 2007. The same transition matrix applies.

The total number of mutton birds that nested on the island in 2007 was

- A. 6000
- B. 8000
- C. 12000
- D. 16000
- E. 24000

FURTHER MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Further Mathematics Formulas

Core: Data analysis

standardised score:
$$z = \frac{x - \bar{x}}{s_x}$$

least squares line:
$$y = a + bx \quad \text{where } b = r \frac{s_y}{s_x} \quad \text{and} \quad a = \bar{y} - b\bar{x}$$

residual value:
$$\text{residual value} = \text{actual value} - \text{predicted value}$$

seasonal index:
$$\text{seasonal index} = \frac{\text{actual figure}}{\text{deseasonalised figure}}$$

Module 1: Number patterns

arithmetic series:
$$a + (a + d) + \dots + (a + (n - 1)d) = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

geometric series:
$$a + ar + ar^2 + \dots + ar^{n-1} = \frac{a(1 - r^n)}{1 - r}, \quad r \neq 1$$

infinite geometric series:
$$a + ar + ar^2 + ar^3 + \dots = \frac{a}{1 - r}, \quad |r| < 1$$

Module 2: Geometry and trigonometry

area of a triangle:
$$\frac{1}{2}bc \sin A$$

Heron's formula:
$$A = \sqrt{s(s-a)(s-b)(s-c)} \quad \text{where } s = \frac{1}{2}(a + b + c)$$

circumference of a circle:
$$2\pi r$$

area of a circle:
$$\pi r^2$$

volume of a sphere:
$$\frac{4}{3}\pi r^3$$

surface area of a sphere:
$$4\pi r^2$$

volume of a cone:
$$\frac{1}{3}\pi r^2 h$$

volume of a cylinder:
$$\pi r^2 h$$

volume of a prism:
$$\text{area of base} \times \text{height}$$

volume of a pyramid:
$$\frac{1}{3} \text{area of base} \times \text{height}$$

Pythagoras' theorem: $c^2 = a^2 + b^2$

sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab \cos C$

Module 3: Graphs and relations

Straight line graphs

gradient (slope): $m = \frac{y_2 - y_1}{x_2 - x_1}$

equation: $y = mx + c$

Module 4: Business-related mathematics

simple interest: $I = \frac{PrT}{100}$

compound interest: $A = PR^n$ where $R = 1 + \frac{r}{100}$

hire purchase: effective rate of interest $\approx \frac{2n}{n+1} \times \text{flat rate}$

Module 5: Networks and decision mathematics

Euler's formula: $v + f = e + 2$

Module 6: Matrices

determinant of a 2×2 matrix: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}; \det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

inverse of a 2×2 matrix: $A^{-1} = \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $\det A \neq 0$