VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education 2010

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

Figures									Letter:
Words									

FURTHER MATHEMATICS

Written examination 2

Wednesday 3 November 2010

Reading time: 11.45 am to 12.00 noon (15 minutes)
Writing time: 12.00 noon to 1.30 pm (1 hour 30 minutes)

QUESTION AND ANSWER BOOK

Structure of book

Core		
Number of questions	*Number of questions to be answered*	*Number of marks*
3	3	15
Module		
Number of modules	*Number of modules to be answered*	*Number of marks*
6	3	45
		Total 60

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 31 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

This examination consists of a core and six modules. Students should answer **all** questions in the core and then select **three** modules and answer **all** questions within the modules selected.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π, surds or fractions.

Diagrams are not to scale unless specified otherwise.

Core

Question 1

Table 1 shows the percentage of women ministers in the parliaments of 22 countries in 2008.

Table 1

Country	Percentage of women ministers
Norway	56
Sweden	48
France	47
Spain	44
Switzerland	43
Austria	38
Denmark	37
Iceland	36
Germany	33
Netherlands	33
New Zealand	32
Australia	24
Italy	24
United States	24
Belgium	23
United Kingdom	23
Ireland	21
Liechtenstein	20
Canada	16
Luxembourg	14
Japan	12
Singapore	0

a. What proportion of these 22 countries have a higher percentage of women ministers in their parliament than Australia?

____

1 mark

b. Determine the median, range and interquartile range of this data.

median ____

range ____

interquartile range ____

2 marks

Core – Question 1 – continued

The ordered stemplot below displays the distribution of the percentage of women ministers in parliament for 21 of these countries. The value for **Canada** is missing.

stem (10s)	leaf (units)
0	0
1	2 4
2	0 1 3 3 4 4 4
3	2 3 3 6 7 8
4	3 4 7 8
5	6

c. Complete the stemplot above by adding the value for Canada.

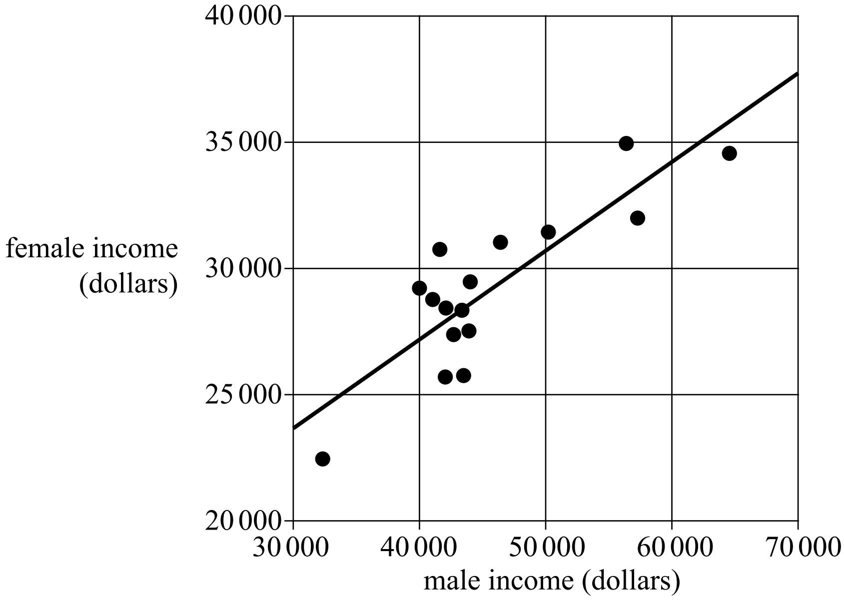
1 mark

d. Both the median and the mean are appropriate measures of centre for this distribution.
Explain why.

1 mark

Question 2

In the scatterplot below, average annual *female income*, in dollars, is plotted against average annual *male income*, in dollars, for 16 countries. A least squares regression line is fitted to the data.



The equation of the least squares regression line for predicting female income from male income is

$$\textit{female income} = 13\,000 + 0.35 \times \textit{male income}$$

a. What is the independent variable?

1 mark

b. Complete the following statement by filling in the missing information.

From the least squares regression line equation it can be concluded that, for these countries, on average, female income increases by $__________ for each $1000 increase in male income.

1 mark

c. **i.** Use the least squares regression line equation to predict the average annual female income (in dollars) in a country where the average annual male income is $15 000.

ii. The prediction made in **part c. i.** is not likely to be reliable.

Explain why.

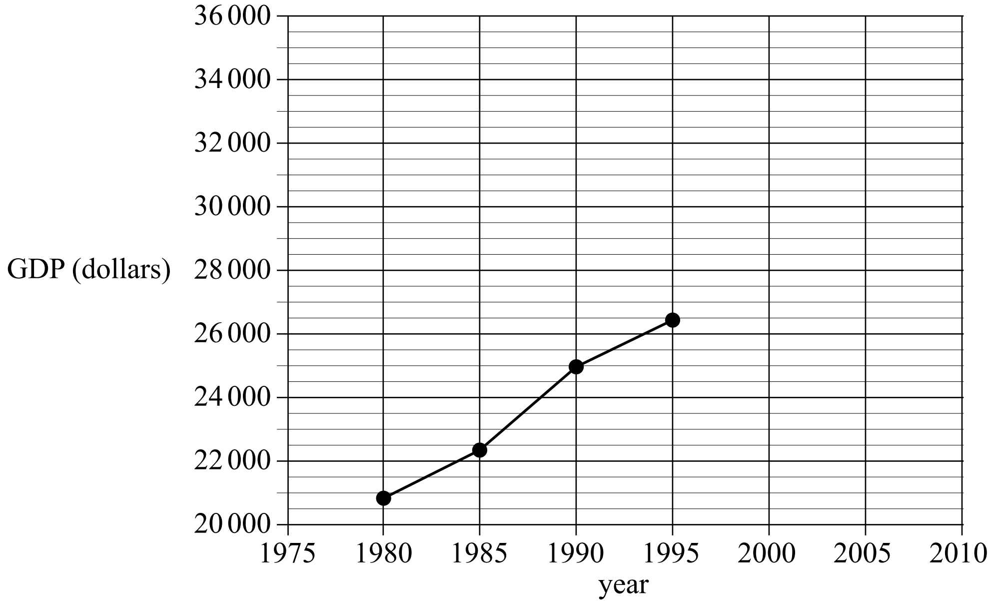
1 + 1 = 2 marks

Question 3

Table 2 shows the Australian gross domestic product (GDP) per person, in dollars, at five yearly intervals for the period 1980 to 2005.

Table 2

Year	1980	1985	1990	1995	2000	2005
GDP	20 900	22 300	25 000	26 400	30 900	33 800



a. Complete the time series plot above by plotting the GDP for the years 2000 and 2005.

1 mark

b. Briefly describe the general trend in the data.

1 mark

In Table 3, the variable *year* has been rescaled using 1980 = 0, 1985 = 5 and so on. The new variable is *time*.

Table 3

Year	1980	1985	1990	1995	2000	2005
Time	0	5	10	15	20	25
GDP	20 900	22 300	25 000	26 400	30 900	33 800

c. Use the variables *time* and *GDP* to write down the equation of the least squares regression line that can be used to predict *GDP* from *time*. Take *time* as the independent variable.

2 marks

d. In the year 2007, the *GDP* was $34 900. Find the error in the prediction if the least squares regression line calculated in **part c.** is used to predict *GDP* in 2007.

2 marks

Total 15 marks

END OF CORE

TURN OVER

Module 1: Number patterns

Question 1

The charge, in dollars, for a single trip on a tollway depends on the number of sections of road that a motorist travels and the type of toll pass that the motorist uses.

a. Using toll pass A, the charge for travelling along n sections of road in a single trip on the tollway is given by the nth term of the following arithmetic sequence.

$4.50, $6.20, $7.90 . . .

i. Show that the common difference for this sequence is $1.70.

ii. Find the charge for travelling along five sections of road in a single trip on the tollway using toll pass A.

iii. One motorist paid $16.40 for a single trip on the tollway using toll pass A.

How many sections of road did this motorist travel along?

iv. At one entry point, fifteen motorists entered the tollway. The first motorist travelled along one section of road. The second motorist travelled along two sections of road. The third motorist travelled along three sections of road and so on.

Find the **total** amount of money that these 15 motorists paid for their trips, assuming they all used toll pass A.

v. Using toll pass A, the charge, in dollars, A_n, for travelling along n sections of road in a single trip on the tollway is given by the difference equation

$$A_{n+1} = mA_n + k \qquad A_1 = 4.50$$

Write the values of m and k in the boxes below.

$m =$ ☐ $k =$ ☐

1 + 1 + 1 + 2 + 2 = 7 marks

Module 1: Number patterns – Question 1 – continued

b. Different charges apply when motorists use toll pass B. With toll pass B, the charge, in dollars, B_n, for travelling along n sections of road in a single trip on the tollway is given by the difference equation

$$B_{n+1} = 0.9B_n + 3 \qquad B_1 = 5$$

i. Explain the meaning of $B_1 = 5$ in terms of the context of this problem.

ii. Find B_3, the charge for travelling along three sections of road in a single trip using toll pass B.

iii. This difference equation indicates that there is a maximum charge which motorists who use toll pass B may pay. What is this maximum charge?

1 + 1 + 1 = 3 marks

c. A motorist wishes to get the best value for money when travelling on the tollway.

Compare the charges for a single trip using toll pass A and toll pass B.

Explain when it would be better for the motorist to use each pass.

2 marks

Question 2

Two signposts are 100 km apart on the tollway.

There are six complete sections of road between these two signposts.

The lengths of the successive sections of road increase by 5%.

a. Determine the length of the first section of road.

Write your answer in kilometres, correct to one decimal place.

2 marks

b. Let L_n be the length of the nth section of road between the two signposts.

Write a difference equation, in terms of L_n and L_{n+1}, that will generate the lengths of the six successive sections of road.

1 mark

Total 15 marks

END OF MODULE 1

Module 2: Geometry and trigonometry

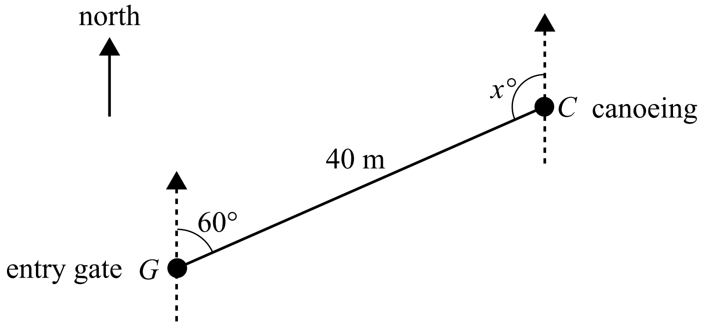
Question 1

In the plan below, the entry gate of an adventure park is located at point *G*.

A canoeing activity is located at point *C*.

The straight path *GC* is 40 metres long.

The bearing of *C* from *G* is 060°.



a. Write down the size of the angle that is marked $x°$ in the plan above.

1 mark

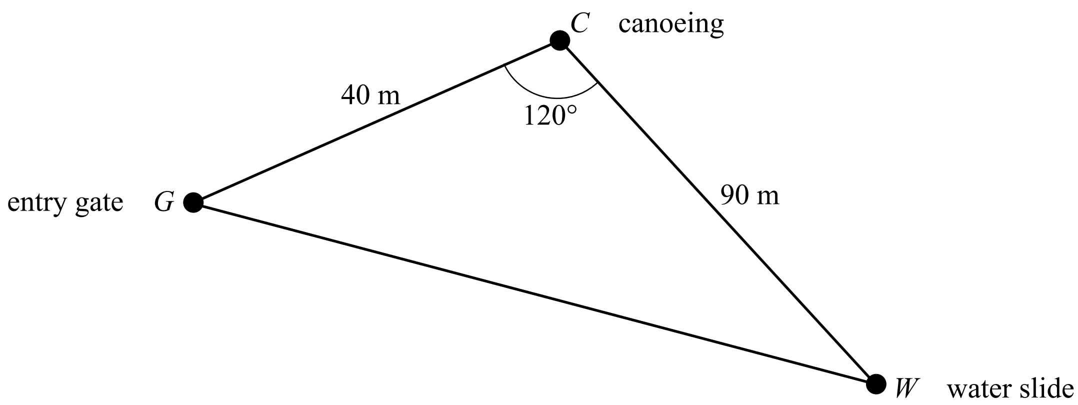
b. What is the bearing of the entry gate from the canoeing activity?

1 mark

c. How many metres **north** of the entry gate is the canoeing activity?

1 mark

CW is a 90 metre straight path between the canoeing activity and a water slide located at point *W*.
GW is a straight path between the entry gate and the water slide.
The angle *GCW* is 120°.

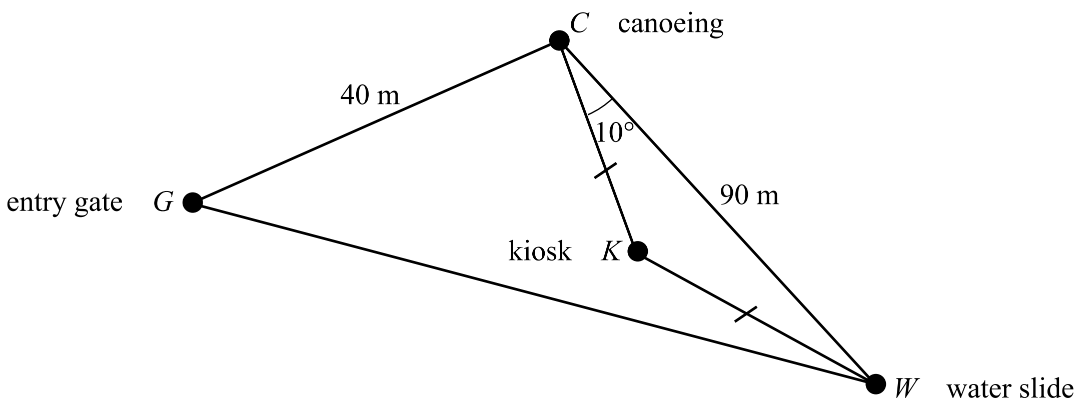


d. **i.** Find the area that is enclosed by the three paths, *GC*, *CW* and *GW*.
Write your answer in square metres, correct to one decimal place.

ii. Show that the length of path *GW* is 115.3 metres, correct to one decimal place.

1 + 1 = 2 marks

Straight paths *CK* and *WK* lead to the kiosk located at point *K*.
These two paths are of equal length.
The angle *KCW* is 10°.



e. **i.** Find the size of the angle *CKW*.

---

ii. Find the length of path *CK*, in metres, correct to one decimal place.

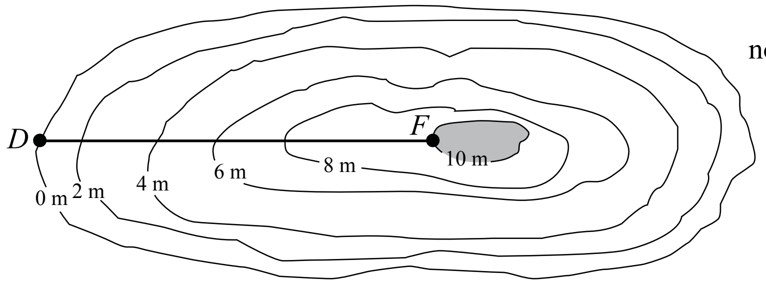
---

1 + 1 = 2 marks

Question 2

A hill in the adventure park has a flat top, shown shaded on the contour map below.

The flat top is reached by climbing a staircase from point *D* to point *F*.



On the contour map, 1 centimetre represents 2 metres on the horizontal level.

a. The length of the line *DF* on the contour map is 4.5 centimetres.

What is the horizontal distance, in metres, from *D* to *F*?

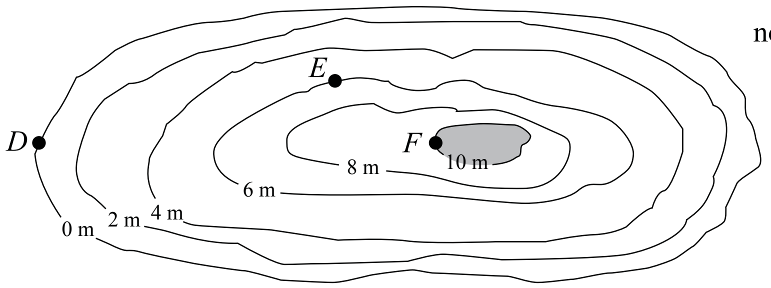
1 mark

b. Under new building regulations, a staircase is unsafe if its angle of elevation is greater than 45°.

Show that the staircase between points *D* and *F* on the contour map above would be considered unsafe.

1 mark

The unsafe staircase will be replaced by two new staircases between points *D* and *E*, and points *E* and *F*, shown on the contour map below.



c. The new staircase leading from *E* to *F* will have a slope of 0.8.

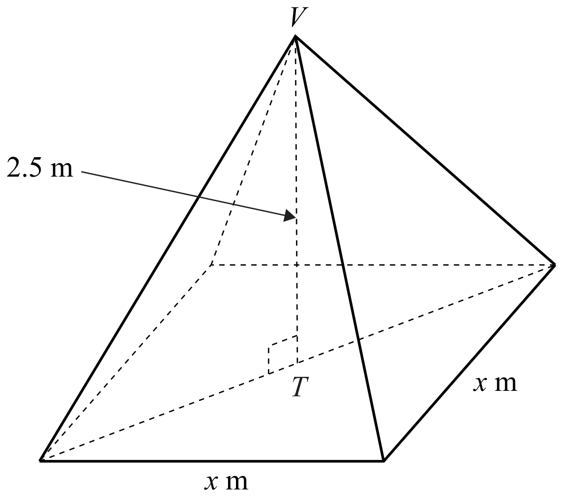
Calculate the length of a line drawn on the contour map joining points *E* and *F*.

2 marks

Question 3

A concrete square pyramid with volume 1.8 m^3 sits on the flat top of the hill.

The length of the square base of the pyramid is *x* metres. The height of the pyramid, *VT*, is 2.5 metres.

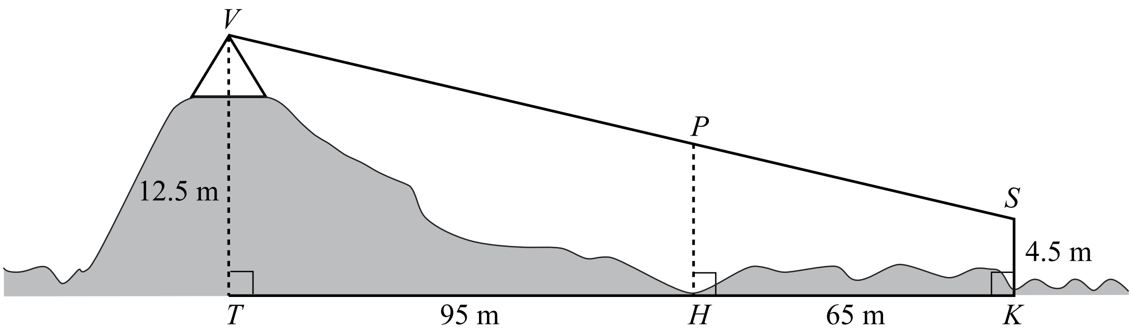


Find the value of *x*, correct to two decimal places.

2 marks

Question 4

A flying fox suspension wire begins at *V*, 12.5 metres above *T* as shown in the diagram below. It ends at *S*, 4.5 metres above *K*.



At *P*, the flying fox wire passes over *H*.

The horizontal distances *TH* and *HK* are 95 metres and 65 metres respectively.

Calculate the vertical distance, *PH*, in metres.

2 marks

Total 15 marks

END OF MODULE 2

Module 3: Graphs and relations

Question 1

Anne sells Softsleep pillows for $65 each.

a. Write an equation for the revenue, R dollars, that Anne receives from the sale of x Softsleep pillows.

1 mark

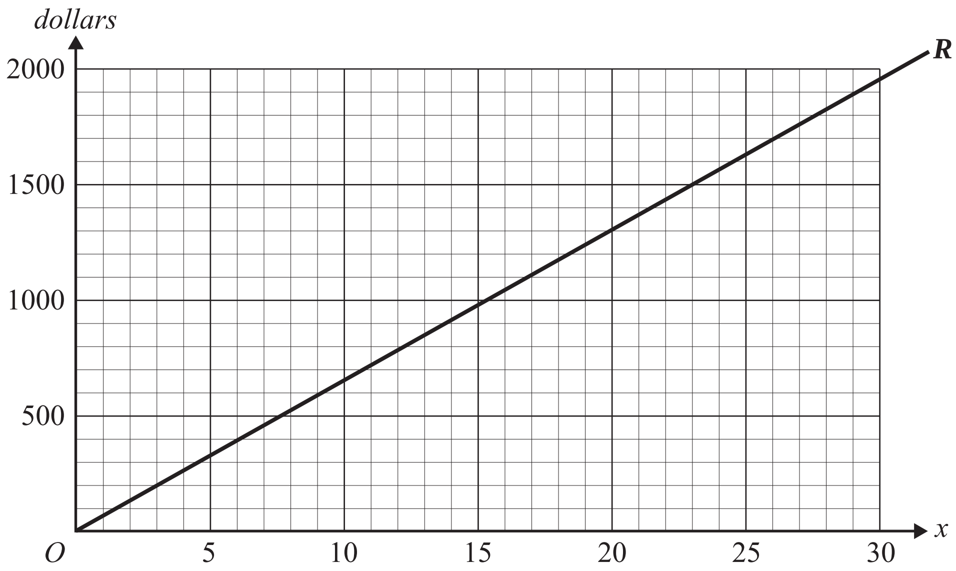
b. The cost, C dollars, of making x Softsleep pillows is given by

$$C = 500 + 40x$$

Find the cost of making 30 Softsleep pillows.

1 mark

The revenue, R, from the sale of x Softsleep pillows is graphed below.



c. Draw the graph of $C = 500 + 40x$ on the axes above.

1 mark

d. How many Softsleep pillows will Anne need to sell in order to break even?

1 mark

Question 2

Anne also sells Resteasy pillows.

Last week she sold 35 Softsleep and m Resteasy pillows.

The selling price per pillow is shown in Table 1 below.

Table 1

Type	**Selling price per pillow**	**Number sold**
Softsleep	\$65	35
Resteasy	\$50	m

The total revenue from pillow sales last week was \$4275.

Find m, the number of Resteasy pillows sold.

1 mark

Question 3

Let x be the number of Softsleep pillows that are sold each week

and y be the number of Resteasy pillows that are sold each week.

A constraint on the number of pillows that can be sold each week is given by

Inequality 1: $x + y \leq 150$

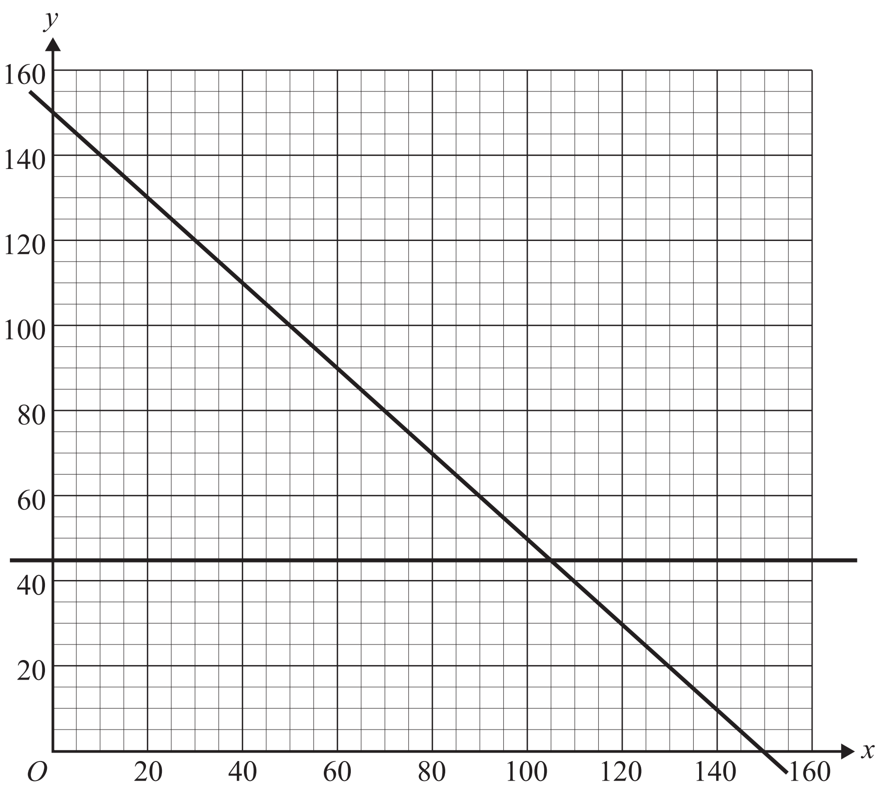
a. Explain the meaning of Inequality 1 in terms of the context of this problem.

1 mark

Each week, Anne sells at least 30 Softsleep pillows and at least k Resteasy pillows.
These constraints may be written as

Inequality 2: $x \geq 30$
Inequality 3: $y \geq k$

The graphs of $x + y = 150$ and $y = k$ are shown below.



b. State the value of k.

1 mark

c. On the axes above

i. draw the graph of $x = 30$

ii. shade the region that satisfies Inequalities 1, 2 and 3.

1 + 1 = 2 marks

d. Softsleep pillows sell for \$65 each and Resteasy pillows sell for \$50 each.
What is the maximum possible weekly revenue that Anne can obtain?

2 marks

Anne decides to sell a third type of pillow, the Snorestop.

She sells two Snorestop pillows for each Softsleep pillow sold. She cannot sell more than 150 pillows in total each week.

e. Show that a new inequality for the number of pillows sold each week is given by

Inequality 4: $3x + y \leq 150$

where x is the number of Softsleep pillows that are sold each week
and y is the number of Resteasy pillows that are sold each week.

1 mark

Softsleep pillows sell for \$65 each.

Resteasy pillows sell for \$50 each.

Snorestop pillows sell for \$55 each.

f. Write an equation for the revenue, R dollars, from the sale of all three types of pillows, in terms of the variables x and y.

1 mark

g. Use Inequalities 2, 3 and 4 to calculate the maximum possible weekly revenue from the sale of all three types of pillow.

2 marks

Total 15 marks

END OF MODULE 3

Module 4: Business-related mathematics

Question 1

The cash price of a large refrigerator is $2000.

a. A customer buys the refrigerator under a hire-purchase agreement.
She does not pay a deposit and will pay $55 per month for four years.

i. Calculate the total amount, in dollars, the customer will pay.

ii. Find the total interest the customer will pay over four years.

iii. Determine the annual flat interest rate that is applied to this hire-purchase agreement.
Write your answer as a percentage.

1 + 1 + 1 = 3 marks

b. Next year the cash price of the refrigerator will rise by 2.5%.
The following year it will rise by a further 2.0%.
Calculate the cash price of the refrigerator after these two price rises.

1 mark

Question 2

$360 000 is invested in a perpetuity at an interest rate of 5.2% per annum.

a. Find the monthly payment that the perpetuity provides.

1 mark

b. After six years of monthly payments, how much money remains invested in the perpetuity?

1 mark

Question 3

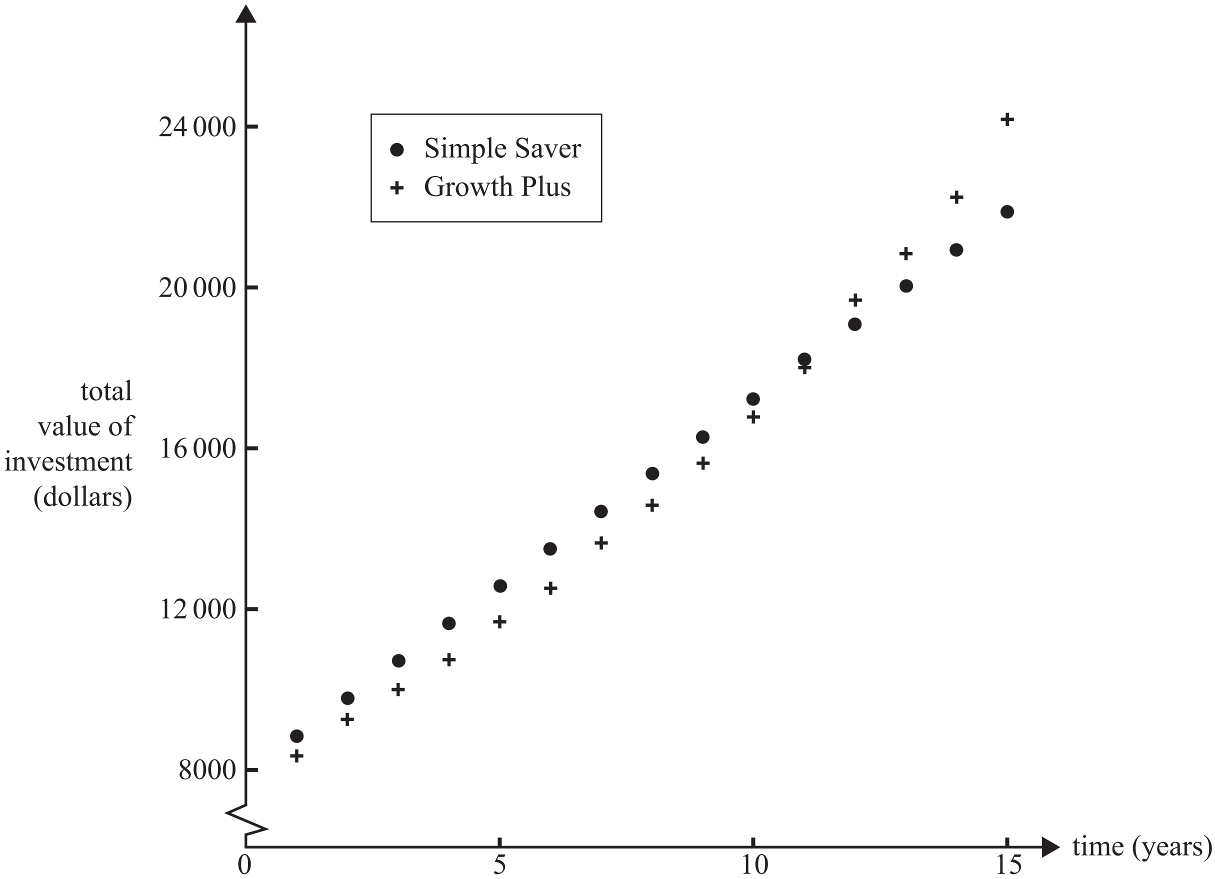
Simple Saver is a simple interest investment in which interest is paid annually.

Growth Plus is a compound interest investment in which interest is paid annually.

Initially, $8 000 is invested with both Simple Saver and Growth Plus.

The graph below shows the total value (principal and all interest earned) of each of these investments over a 15 year period.

The increase in the value of each investment over time is due to interest.



a. Which investment pays the highest annual interest rate, Growth Plus or Simple Saver?

Give a reason to justify your answer.

1 mark

b. After 15 years, the total value (principal and all interest earned) of the Simple Saver investment is $21 800.

Find the amount of interest paid annually.

1 mark

c. After 15 years, the total value (principal and all interest earned) of the Growth Plus investment is \$24 000.

i. Write down an equation that can be used to find the annual compound interest rate, r.

ii. Determine the annual compound interest rate.
Write your answer as a percentage correct to one decimal place.

1 + 1 = 2 marks

Question 4

A home buyer takes out a reducing balance loan of \$250 000 to purchase an apartment.

Interest on the loan will be calculated and paid monthly at the rate of 6.25% per annum.

a. The loan will be fully repaid in equal monthly instalments over 20 years.

i. Find the monthly repayment, in dollars, correct to the nearest cent.

ii. Calculate the total interest that will be paid over the 20 year term of the loan.
Write your answer correct to the nearest dollar.

1 + 2 = 3 marks

b. After 60 monthly repayments have been made, what will be the outstanding principal on the loan?
Write your answer correct to the nearest dollar.

1 mark

By making a lump sum payment after nine years, the home buyer is able to reduce the principal on his loan to \$100 000. At this time, his monthly repayment changes to \$1250. The interest rate remains at 6.25% per annum, compounding monthly.

c. With these changes, how many months, in total, will it take the home buyer to fully repay the \$250 000 loan?

1 mark

Total 15 marks

END OF MODULE 4

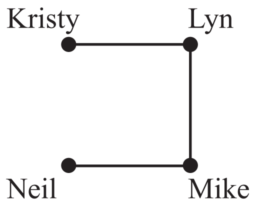
Module 5: Networks and decision mathematics

In a competition, members of a team work together to complete a series of challenges.

Question 1

The members of one team are Kristy (*K*), Lyn (*L*), Mike (*M*) and Neil (*N*).

In one of the challenges, these four team members are only allowed to communicate directly with each other as indicated by the edges of the following network.



The adjacency matrix below also shows the allowed lines of communication.

$$\begin{array}{c} \begin{matrix} K & L & M & N \end{matrix} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & \boldsymbol{f} & 0 & 1 \\ 0 & \boldsymbol{g} & 1 & 0 \end{bmatrix} \begin{matrix} K \\ L \\ M \\ N \end{matrix} \end{array}$$

a. Explain the meaning of a **zero** in the adjacency matrix.

____

1 mark

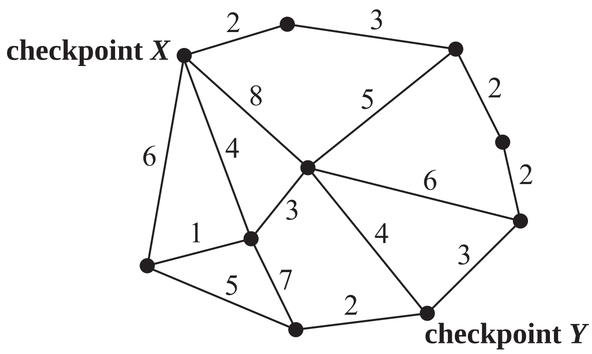
b. Write down the values of *f* and *g* in the adjacency matrix.

f = ____________________ *g* = ____________________

1 mark

Question 2

The diagram below shows a network of tracks (represented by edges) between checkpoints (represented by vertices) in a short-distance running course. The numbers on the edges indicate the time, in minutes, a team would take to run along each track.



Another challenge requires teams to run from checkpoint *X* to checkpoint *Y* using these tracks.

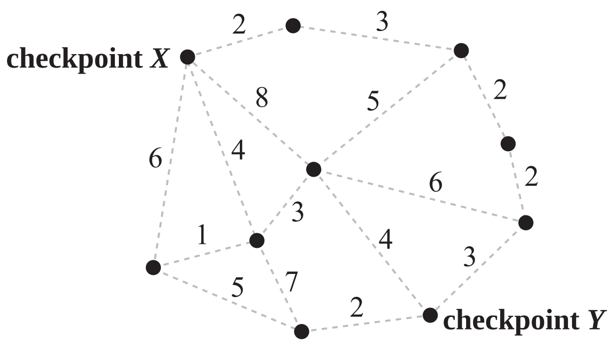
a. What would be the shortest possible time for a team to run from checkpoint *X* to checkpoint *Y*?

1 mark

b. Teams are required to follow a route from checkpoint *X* to checkpoint *Y* that passes through every checkpoint once only.

i. What mathematical term is used to describe such a route?

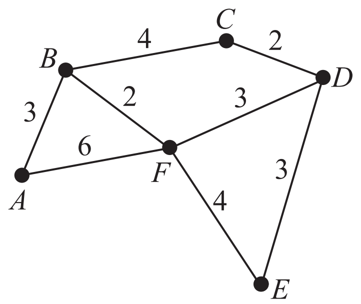
ii. On the network diagram below, draw in the route from checkpoint *X* to checkpoint *Y* that passes through every checkpoint once only.



1 + 1 = 2 marks

Question 3

The following network diagram shows the distances, in kilometres, along the roads that connect six intersections *A*, *B*, *C*, *D*, *E* and *F*.



a. If a cyclist started at intersection *B* and cycled along every road in this network once only, at which intersection would she finish?

1 mark

b. The next challenge involves cycling along every road in this network at least once.

Teams have to start and finish at intersection *A*.

The blue team does this and cycles the shortest possible total distance.

i. Apart from intersection *A*, through which intersections does the blue team pass more than once?

ii. How many kilometres does the blue team cycle?

1 + 1 = 2 marks

c. The red team does not follow the rules and cycles along a bush path that connects two of the intersections. This route allows the red team to ride along every road only once.

Which two intersections does the bush path connect?

1 mark

Question 4

In the final challenge, each team has to complete a construction project that involves activities *A* to *I*.

Table 1

Activity	EST (minutes)	LST (minutes)	Duration (minutes)	Immediate predecessor
A	0	0	5	–
B	5	5	6	*A*
C	5	6	4	*A*
D	11	11	2	*B*
E	5	9	7	*A*
F		10	6	*C*
G	9	13	1	*C*
H	13	13	3	*D*
I	10	14	2	*G*

Table 1 shows the earliest start time (EST), latest start time (LST) and duration, in minutes, for each activity. The immediate predecessor is also shown. The earliest start time for activity *F* is missing.

a. What is the least number of activities that must be completed before activity *F* can commence?

1 mark

b. What is the earliest start time for activity *F*?

1 mark

c. Write down all the activities that must be completed before activity *G* can commence.

1 mark

d. What is the float time, in minutes, for activity *G*?

1 mark

e. What is the shortest time, in minutes, in which this construction project can be completed?

1 mark

f. Write down the critical path for this network.

1 mark

Total 15 marks

Module 6: Matrices

Question 1

In a game of basketball, a successful shot for goal scores one point, two points, or three points, depending on the position from which the shot is thrown.

G is a column matrix that lists the number of points scored for each type of successful shot.

$$G = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

In one game, Oscar was successful with

- 4 one-point shots for goal
- 8 two-point shots for goal
- 2 three-point shots for goal.

a. Write a row matrix, N, that shows the number of each type of successful shot for goal that Oscar had in that game.

$$N = [\qquad\qquad\qquad]$$

1 mark

b. Matrix P is found by multiplying matrix N with matrix G so that

$P = N \times G$

Evaluate matrix P.

1 mark

c. In this context, what does the information in matrix P provide?

1 mark

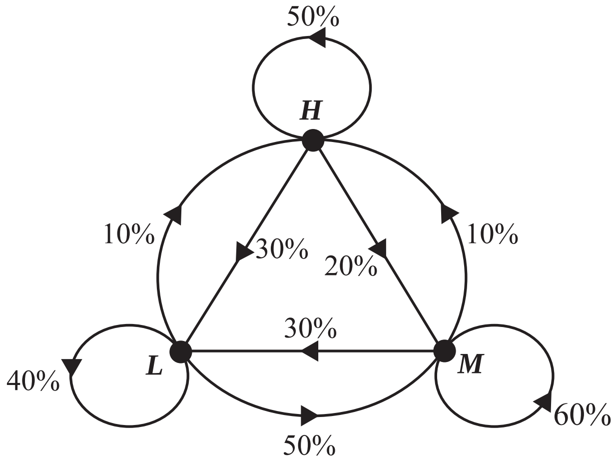
Question 2

The 300 players in Oscar's league are involved in a training program. In week one, 90 players are doing heavy training (H), 150 players are doing moderate training (M) and 60 players are doing light training (L).

The state matrix, S_1, shows the number of players who are undertaking each type of training in the first week.

$$S_1 = \begin{bmatrix} 90 \\ 150 \\ 60 \end{bmatrix} \begin{matrix} H \\ M \\ L \end{matrix}$$

The percentage of players that remain in the same training program, or change their training program from week to week, is shown in the transition diagram below.



a. What information does the 20% in the diagram above provide?

1 mark

The information in the transition diagram above can also be written as the transition matrix T.

$$T = \begin{array}{c} \textit{this week} \\ \begin{array}{ccc} H & M & L \end{array} \\ \begin{bmatrix} 0.5 & 0.1 & 0.1 \\ 0.2 & 0.6 & 0.5 \\ 0.3 & 0.3 & 0.4 \end{bmatrix} \end{array} \begin{array}{l} H \\ M \\ L \end{array} \quad \textit{next week}$$

b. Determine how many players will be doing heavy training in week two.

1 mark

c. Determine how many **fewer** players will be doing moderate training in week three than in week one.

1 mark

d. Show that, after seven weeks, the number of players (correct to the nearest whole number) who are involved in each type of training will not change.

1 mark

Question 3

The basketball coach has written three linear equations which can be used to predict the number of points, p, rebounds, r, and assists, a, that Oscar will have in his next game.

The equations are

$$\begin{aligned} p + r + a &= 33 \\ 2p - r + 3a &= 40 \\ p + 2r + a &= 43 \end{aligned}$$

a. These equations can be written equivalently in matrix form.

Complete the missing information below.

$$\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}\begin{bmatrix} p \\ r \\ a \end{bmatrix} = \begin{bmatrix} 33 \\ 40 \\ 43 \end{bmatrix}$$

1 mark

This matrix equation can be solved in the following way.

$$\begin{bmatrix} p \\ r \\ a \end{bmatrix} = \begin{bmatrix} 7 & -1 & -4 \\ -1 & 0 & 1 \\ x & 1 & 3 \end{bmatrix}\begin{bmatrix} 33 \\ 40 \\ 43 \end{bmatrix}$$

b. Determine the value of x shown in the matrix equation above.

1 mark

c. How many rebounds is Oscar predicted to have in his next game?

1 mark

Question 4

The Dinosaurs (D) and the Scorpions (S) are two basketball teams that play in different leagues in the same city.

The matrix A_1 is the attendance matrix for the first game. This matrix shows the number of people who attended the first Dinosaur game and the number of people who attended the first Scorpion game.

$$A_1 = \begin{bmatrix} 2000 \\ 1000 \end{bmatrix} \begin{matrix} D \\ S \end{matrix}$$

The number of people expected to attend the second game for each team can be determined using the matrix equation

$$A_2 = GA_1$$

where G is the matrix

$$\begin{matrix} & \textit{this game} & \\ & D \quad S & \\ G = & \begin{bmatrix} 1.2 & -0.3 \\ 0.2 & 0.7 \end{bmatrix} & \begin{matrix} D \\ S \end{matrix} \quad \textit{next game} \end{matrix}$$

a. **i.** Determine A_2, the attendance matrix for the second game.

ii. Every person who attends either the second Dinosaur game or the second Scorpion game will be given a free cap.

How many caps, in total, are expected to be given away?

1 + 1 = 2 marks

Assume that the attendance matrices for successive games can be determined as follows.

$A_3 = GA_2$
$A_4 = GA_3$
and so on such that $A_{n+1} = GA_n$

b. Determine the attendance matrix (with the elements written correct to the nearest whole number) for game 10.

1 mark

c. Describe the way in which the number of people attending the Dinosaurs' games is expected to change over the next 80 or so games.

1 mark

The attendance at the first Dinosaur game was 2000 people and the attendance at the first Scorpion game was 1000 people.

Suppose, instead, that 2000 people attend the first Dinosaur game, and 1800 people attend the first Scorpion game.

d. Describe the way in which the number of people attending the Dinosaurs' games is expected to change over the next 80 or so games.

1 mark

Total 15 marks

END OF QUESTION AND ANSWER BOOK

FURTHER MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Further Mathematics Formulas

Core: Data analysis

standardised score: $z = \frac{x - \bar{x}}{s_x}$

least squares line: $y = a + bx$ where $b = r\frac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$

residual value: residual value = actual value – predicted value

seasonal index: seasonal index = $\frac{\text{actual figure}}{\text{deseasonalised figure}}$

Module 1: Number patterns

arithmetic series: $a + (a + d) + \ldots + (a + (n - 1)d) = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}(a + l)$

geometric series: $a + ar + ar^2 + \ldots + ar^{n-1} = \frac{a(1 - r^n)}{1 - r}, r \neq 1$

infinite geometric series: $a + ar + ar^2 + ar^3 + \ldots = \frac{a}{1 - r}, |r| < 1$

Module 2: Geometry and trigonometry

area of a triangle: $\frac{1}{2}bc\sin A$

Heron's formula: $A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{1}{2}(a + b + c)$

circumference of a circle: $2\pi r$

area of a circle: πr^2

volume of a sphere: $\frac{4}{3}\pi r^3$

surface area of a sphere: $4\pi r^2$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a cylinder: $\pi r^2 h$

volume of a prism: area of base × height

volume of a pyramid: $\frac{1}{3}$ area of base × height

Pythagoras' theorem: $c^2 = a^2 + b^2$

sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab \cos C$

Module 3: Graphs and relations

Straight line graphs

gradient (slope): $m = \frac{y_2 - y_1}{x_2 - x_1}$

equation: $y = mx + c$

Module 4: Business-related mathematics

simple interest: $I = \frac{PrT}{100}$

compound interest: $A = PR^n$ where $R = 1 + \frac{r}{100}$

hire purchase: effective rate of interest $\approx \frac{2n}{n+1} \times$ flat rate

Module 5: Networks and decision mathematics

Euler's formula: $v + f = e + 2$

Module 6: Matrices

determinant of a 2 × 2 matrix: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

inverse of a 2 × 2 matrix: $A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $\det A \neq 0$

END OF FORMULA SHEET