VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2013

FURTHER MATHEMATICS
Written examination 1

Friday 1 November 2013
Reading time: 3.00 pm to 3.15 pm (15 minutes)
Writing time: 3.15 pm to 4.45 pm (1 hour 30 minutes)

MULTIPLE-CHOICE QUESTION BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of modules*	*Number of modules to be answered*	*Number of marks*
A	13	13			13
B	54	27	6	3	27
					Total 40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question book of 43 pages with a detachable sheet of miscellaneous formulas in the centrefold.
- Answer sheet for multiple-choice questions.
- Working space is provided throughout the book.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

At the end of the examination
- You may keep this question book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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SECTION A

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Core: Data analysis

Use the following information to answer Questions 1 and 2.

The following ordered stem plot shows the percentage of homes connected to broadband internet for 24 countries in 2007.

key 1|6 = 16%

1	
1	6 7
2	0 1 1 3 4 4
2	5 7 8 9
3	0 0 1 1 1 2 2 3
3	5 7 8 8
4	

Question 1

The number of these countries with more than 22% of homes connected to broadband internet in 2007 is

A. 4
B. 5
C. 19
D. 20
E. 22

Question 2

Which one of the following statements relating to the data in the ordered stem plot is **not** true?

A. The minimum is 16%.
B. The median is 30%.
C. The first quartile is 23.5%.
D. The third quartile is 32%.
E. The maximum is 38%.

Use the following information to answer Questions 3 and 4.

The heights of 82 mothers and their eldest daughters are classified as 'short', 'medium' or 'tall'. The results are displayed in the frequency table below.

		Mother		
		Short	**Medium**	**Tall**
Eldest daughter	**Short**	16	10	3
	Medium	8	14	11
	Tall	5	7	8

Question 3

The number of mothers whose height is classified as 'medium' is

A. 7
B. 10
C. 14
D. 31
E. 33

Question 4

Of the mothers whose height is classified as 'tall', the percentage who have eldest daughters whose height is classified as 'short' is closest to

A. 3%
B. 4%
C. 14%
D. 17%
E. 27%

Use the following information to answer Questions 5 and 6.

The time, in hours, that each student spent sleeping on a school night was recorded for 1550 secondary-school students. The distribution of these times was found to be approximately normal with a mean of 7.4 hours and a standard deviation of 0.7 hours.

Question 5

The time that 95% of these students spent sleeping on a school night could be

A. less than 6.0 hours.

B. between 6.0 and 8.8 hours.

C. between 6.7 and 8.8 hours.

D. less than 6.0 hours or greater than 8.8 hours.

E. less than 6.7 hours or greater than 9.5 hours.

Question 6

The number of these students who spent more than 8.1 hours sleeping on a school night was closest to

A. 16

B. 248

C. 1302

D. 1510

E. 1545

Question 7

For a city, the correlation coefficient between

- population density and distance from the centre of the city is $r = -0.563$
- house size and distance from the centre of the city is $r = 0.357$.

Given this information, which one of the following statements is true?

A. Around 31.7% of the variation observed in house size in the city can be explained by the variation in distance from the centre of the city.

B. Population density tends to increase as the distance from the centre of the city increases.

C. House sizes tend to be larger as the distance from the centre of the city decreases.

D. The slope of a least squares regression line relating population density to distance from the centre of the city is positive.

E. Population density is more strongly associated with distance from the centre of the city than is house size.

Question 8

The table below shows the hourly rate of pay earned by 10 employees in a company in 1990 and in 2010.

Employee	Hourly rate of pay ($)	
	1990	2010
Ben	9.53	17.02
Lani	9.15	16.71
Freya	8.88	15.10
Jill	8.60	15.93
David	7.67	14.40
Hong	7.96	13.32
Stuart	6.42	15.40
Mei Lien	11.86	19.79
Tim	14.64	23.38
Simon	15.31	25.11

The value of the correlation coefficient, r, for this set of data is closest to

A. 0.74

B. 0.86

C. 0.92

D. 0.93

E. 0.96

Question 9

The following data was recorded in an investigation of the relationship between *age* and reaction *time*. In this investigation, *age* is the independent variable.

***Age* (years)**	***Time* (seconds)**
34	0.31
47	0.40
48	0.39
48	0.38
50	0.45
56	0.43
57	0.35
57	0.38
57	0.43
57	0.39
61	0.45
96	0.48

Several statistics were calculated for this data.

When the data was checked, a recording error was found; the age of a 69-year-old had been incorrectly entered as 96. The recording error was corrected and the statistics were recalculated.

The statistic that will remain unchanged when recalculated is the

A. slope of the three median line.

B. intercept of the least squares regression line.

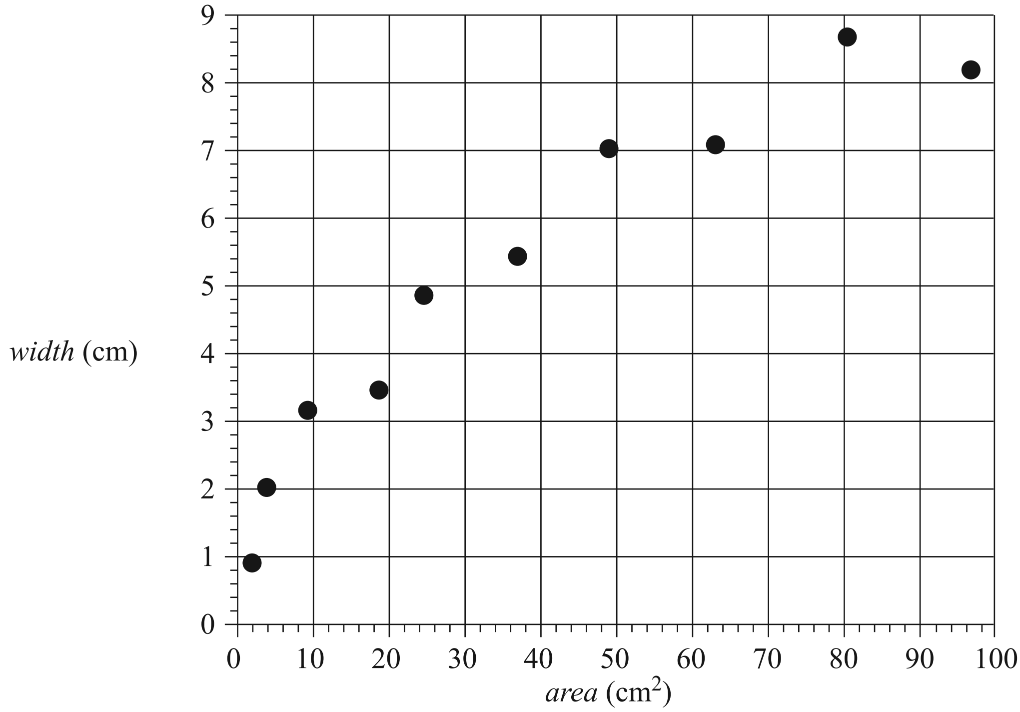
C. correlation coefficient, *r*.

D. range of *age*.

E. standard deviation of *age*.

Question 10

The data in the scatterplot below shows the *width*, in cm, and the surface *area*, in cm^2, of leaves sampled from 10 different trees. The scatterplot is non-linear.



To linearise the scatterplot, $(width)^2$ is plotted against *area* and a least squares regression line is then fitted to the linearised plot.

The equation of this least squares regression line is

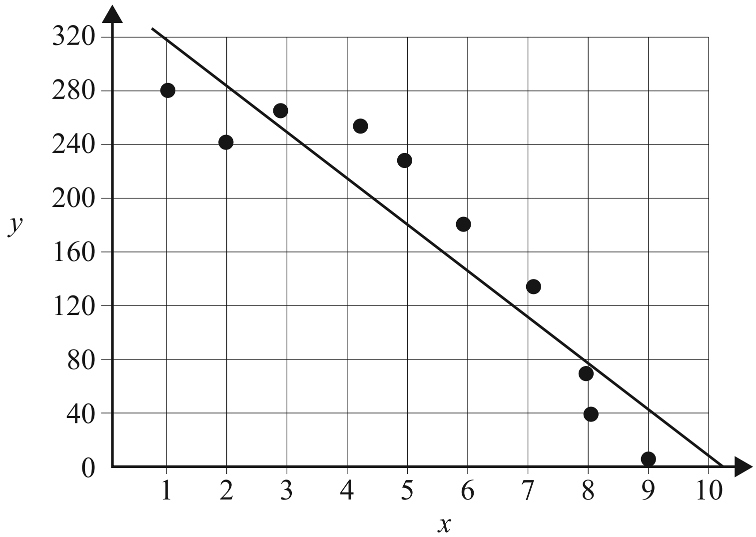
$$(width)^2 = 1.8 + 0.8 \times area$$

Using this equation, a leaf with a surface area of 120 cm^2 is predicted to have a width, in cm, closest to

A. 9.2
B. 9.9
C. 10.6
D. 84.6
E. 97.8

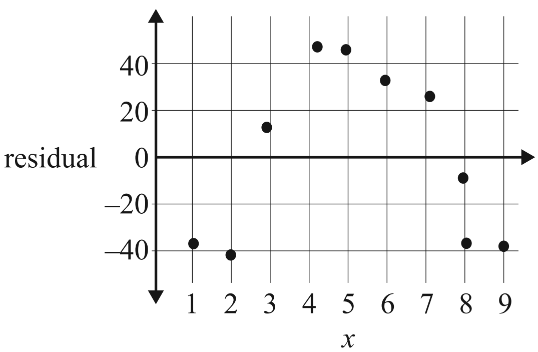
Question 11

A least squares regression line is fitted to data in a scatterplot, as shown below.



The corresponding residual plot is closest to

A.



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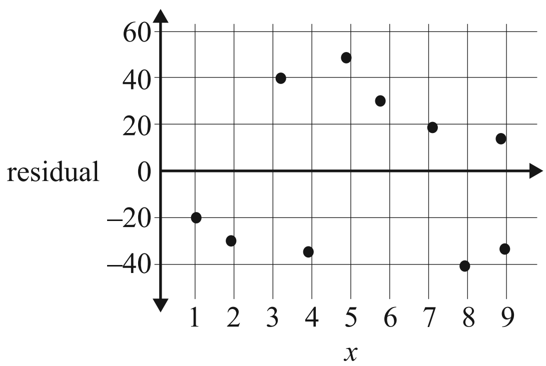

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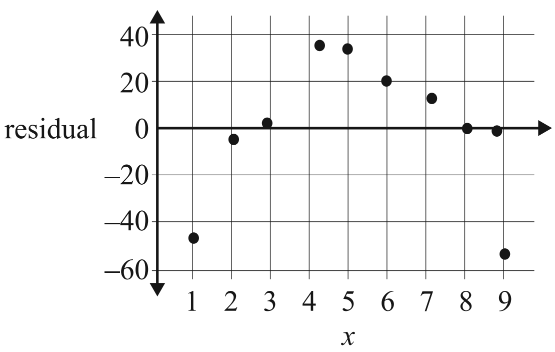

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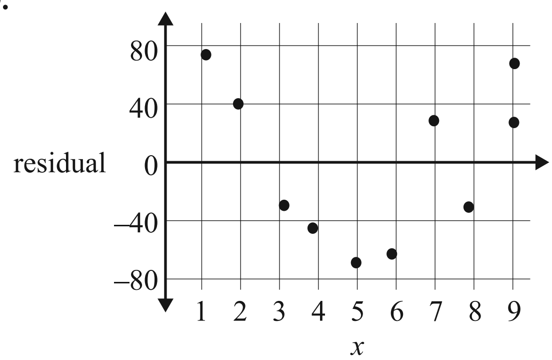
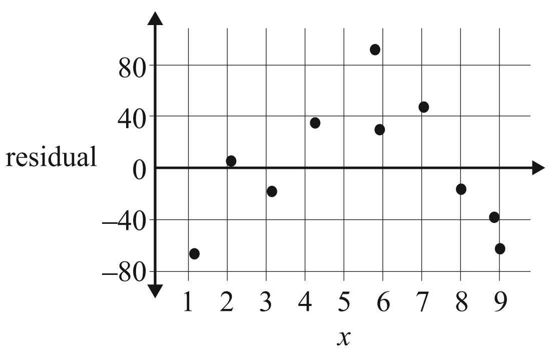
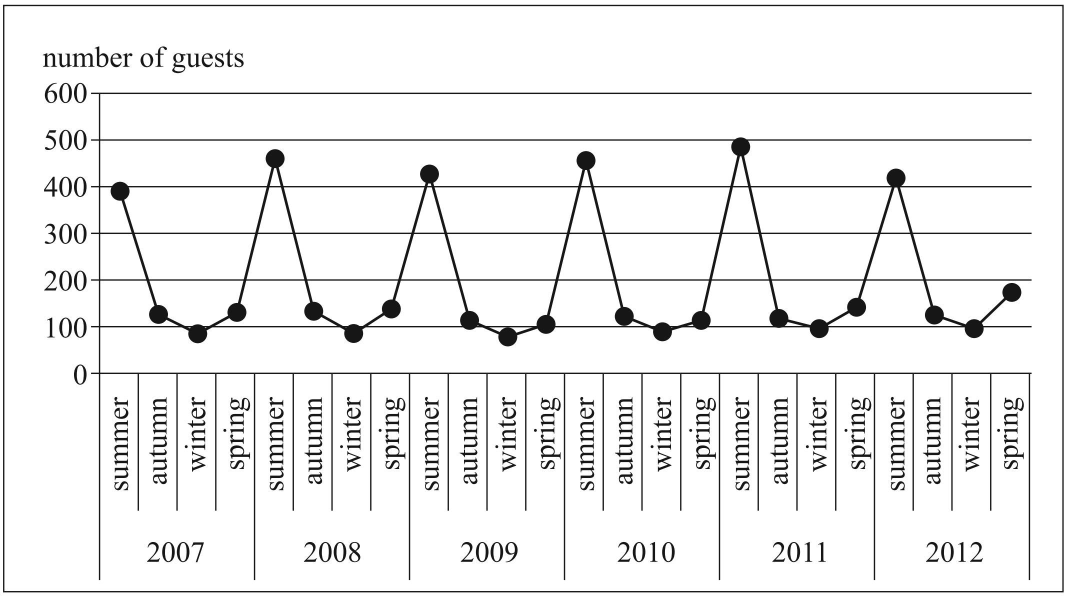


E.



Use the following information to answer Questions 12 and 13.

The time series plot below displays the number of guests staying at a holiday resort during summer, autumn, winter and spring for the years 2007 to 2012 inclusive.



Question 12

Which one of the following best describes the pattern in the time series?

A. random variation only
B. decreasing trend with seasonality
C. seasonality only
D. increasing trend only
E. increasing trend with seasonality

Question 13

The table below shows the data from the times series plot for the years 2007 and 2008.

Year	Season	Number of guests
2007	summer	390
	autumn	126
	winter	85
	spring	130
2008	summer	460
	autumn	136
	winter	86
	spring	142

Using four-mean smoothing with centring, the smoothed number of guests for winter 2007 is closest to

A. 85
B. 107
C. 183
D. 192
E. 200

END OF SECTION A
TURN OVER

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SECTION B

Instructions for Section B

Select **three** modules and answer **all** questions within the modules selected in pencil on the answer sheet provided for multiple-choice questions.

Show the modules you are answering by shading the matching boxes on your multiple-choice answer sheet **and** writing the name of the module in the box provided.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Module 1: Number patterns

Before answering these questions you must **shade** the Number patterns box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

The first three terms of an arithmetic sequence are 3, 5 and 7.

The ninth term of this sequence is

A. 9

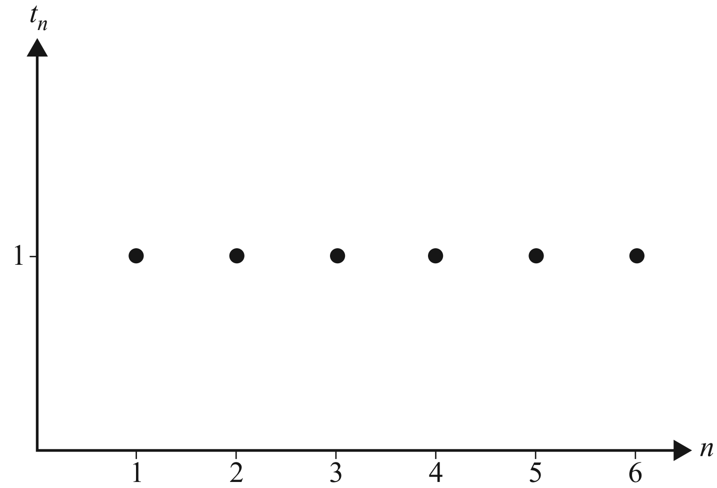
B. 17

C. 19

D. 21

E. 768

Question 2



The graph above shows the first six terms of a sequence.

This sequence could be

A. an arithmetic sequence that sums to one.

B. an arithmetic sequence with a common difference of one.

C. a Fibonacci-related sequence whose first term is one.

D. a geometric sequence with an infinite sum of one.

E. a geometric sequence with a common ratio of one.

Question 3

The first time a student played an online game, he played for 18 minutes.

Each time he played the game after that, he played for 12 minutes longer than the previous time.

After completing his 15th game, the **total** time he had spent playing these 15 games was

A. 186 minutes
B. 691 minutes
C. 1206 minutes
D. 1395 minutes
E. 1530 minutes

Question 4

The vertical distance, in m, that a hot air balloon rises in each successive minute of its flight is given by the geometric sequence

64.0, 60.8, 57.76 …

The total vertical distance, in m, that the balloon rises in the first 10 minutes of its flight is closest to

A. 38
B. 40
C. 473
D. 514
E. 1280

Question 5

A sequence is generated by the difference equation

$$t_{n+1} = 2 \times t_n \qquad t_1 = 1$$

The nth term of this sequence is

A. $t_n = 1 \times 2^{n-1}$
B. $t_n = 1 + 2^{n-1}$
C. $t_n = 1 + 2 \times (n - 1)$
D. $t_n = 2 + (n - 1)$
E. $t_n = 2 \times 1^{n-1}$

Question 6

There are 3000 tickets available for a concert.

On the first day of ticket sales, 200 tickets are sold.

On the second day, 250 tickets are sold.

On the third day, 300 tickets are sold.

This pattern of ticket sales continues until all 3000 tickets are sold.

How many days does it take for all of the tickets to be sold?

A. 5
B. 6
C. 8
D. 34
E. 57

Question 7

The following are either three consecutive terms of an arithmetic sequence or three consecutive terms of a geometric sequence.

Which one of these sequences could **not** include 2 as a term?

A. –1, 0.5, –0.25

B. –1, –3, –5

C. 5, 12.5, 31.25

D. 6, 8, 10

E. 8, 16, 32

Question 8

The initial rate of pay for a job is $10 per hour.

A worker's skill increases the longer she works on this job. As a result, the hourly rate of pay increases each month.

The hourly rate of pay in the *n*th month of working on this job is given by the difference equation

$$S_{n+1} = 0.2 \times S_n + 15 \qquad S_1 = 10$$

The maximum hourly rate of pay that the worker can earn in this job is closest to

A. $3.00

B. $12.00

C. $12.50

D. $18.75

E. $75.00

Question 9

Eleven speed bumps are placed on a road.

The speed bumps are placed so that the distance between consecutive speed bumps decreases according to an arithmetic sequence.

The distance between the first and last speed bumps is exactly 100 m.

The smallest distance between consecutive speed bumps is 2 m.

The largest distance, in m, between two consecutive speed bumps is

A. 16

B. 18

C. 20

D. 22

E. 24

Module 2: Geometry and trigonometry

Before answering these questions you must **shade** the Geometry and trigonometry box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

The perimeter of a regular pentagon is 100 cm.

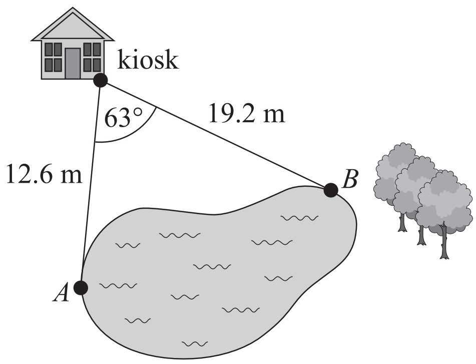
The side length of this pentagon, in cm, is

A. 5
B. 10
C. 20
D. 25
E. 50

Question 2

The distances from a kiosk to points *A* and *B* on opposite sides of a pond are found to be 12.6 m and 19.2 m respectively.

The angle between the lines joining these points to the kiosk is 63°.



The distance, in m, across the pond between points *A* and *B* can be found by evaluating

A. $\frac{1}{2} \times 12.6 \times 19.2 \times \sin(63°)$

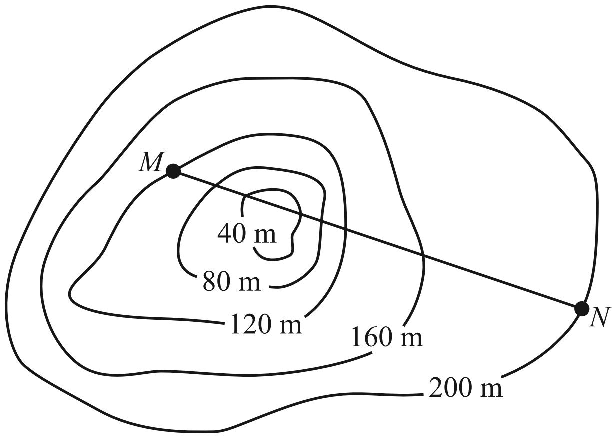
B. $\frac{19.2 \times \sin(63°)}{12.6}$

C. $\sqrt{12.6^2 + 19.2^2}$

D. $\sqrt{12.6^2 + 19.2^2 - 2 \times 12.6 \times 19.2 \times \cos(63°)}$

E. $\sqrt{s(s-12.6)(s-19.2)(s-63)}$, where $s = \frac{1}{2}(12.6 + 10.2 + 63)$

Question 3

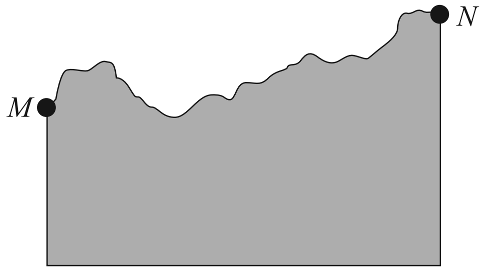


The contour map shown above has contours at 40 m intervals.

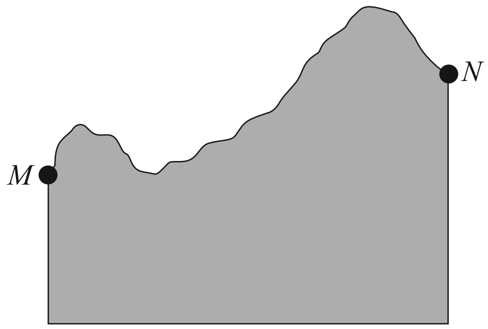
The cross-section along the line segment MN could be

A.

B.



C.



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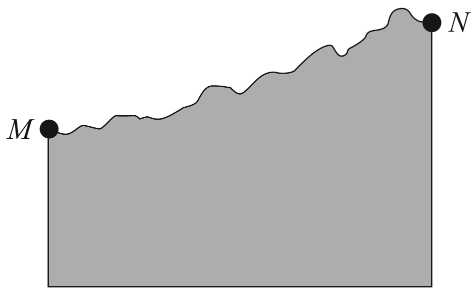
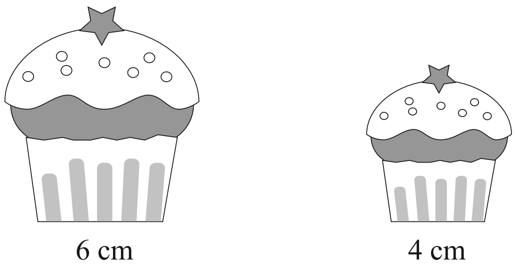


E.

Question 4

A cafe sells two sizes of cupcakes with a similar shape.

The large cupcake is 6 cm wide at the base and the small cupcake is 4 cm wide at the base.



The price of a cupcake is proportional to its volume.

If the large cupcake costs $5.40, then the small cupcake will cost

A. $1.60
B. $2.32
C. $2.40
D. $3.40
E. $3.60

Question 5

The scale used on a map is 1:50 000.

On this map, a distance of 4 km would be represented by

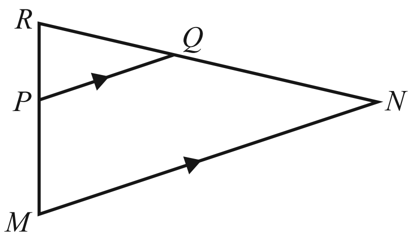
A. 2.0 cm
B. 5.0 cm
C. 8.0 cm
D. 12.5 cm
E. 20.0 cm

Question 6

In triangle *MNR*, point *P* lies on side *MR* and point *Q* lies on side *NR*.

The lines *PQ* and *MN* are parallel.

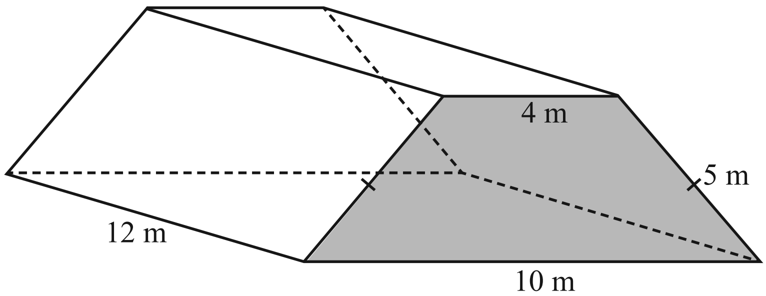
The length of *RQ* is 4 cm, the length of *QN* is 6 cm and the length of *PQ* is 5 cm.



The length of *MN*, in cm, is equal to

A. 7.5
B. 8.3
C. 12.0
D. 12.5
E. 15.0

Question 7



A greenhouse is built in the shape of a trapezoidal prism, as shown in the diagram above.

The cross-section of the greenhouse (shaded) is an isosceles trapezium. The parallel sides of this trapezium are 4 m and 10 m respectively. The two equal sides are each 5 m.

The length of the greenhouse is 12 m.

The five exterior surfaces of the greenhouse, **not** including the base, are made of glass.

The total area, in m^2, of the glass surfaces of the greenhouse is

A. 196
B. 212
C. 224
D. 344
E. 672

Question 8

There are four telecommunications towers in a city. The towers are called Grey Tower, Black Tower, Silver Tower and White Tower.

Grey Tower is 10 km due west of Black Tower.

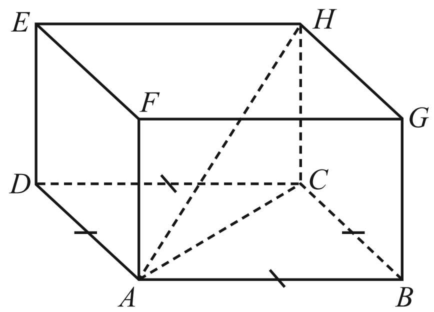
Silver Tower is 10 km from Grey Tower on a bearing of 300°.

White Tower is 10 km due north of Silver Tower.

Correct to the nearest degree, the bearing of Black Tower from White Tower is

A. 051°
B. 129°
C. 141°
D. 309°
E. 321°

Question 9



A rectangular prism with a square base, *ABCD*, is shown above.

The diagonal of the prism, *AH*, is 8 cm.

The height of the prism, *HC*, is 4 cm.

The volume of this rectangular prism is

A. 64 cm^3

B. 96 cm^3

C. 128 cm^3

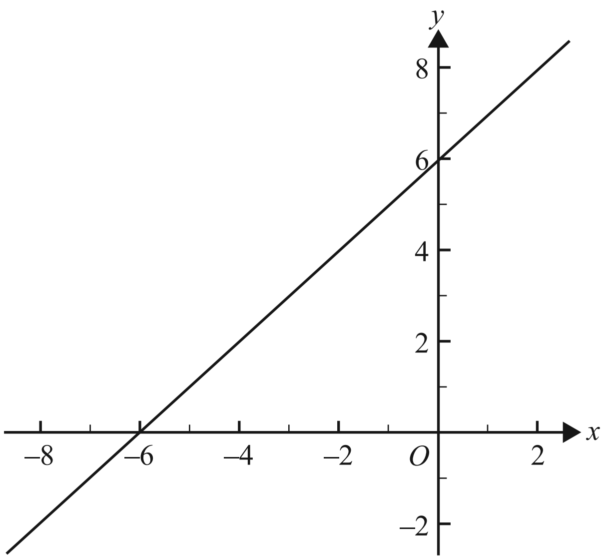
D. 192 cm^3

E. 256 cm^3

Module 3: Graphs and relations

Before answering these questions you must **shade** the Graphs and relations box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1



The equation of the line shown on the graph is

A. $y = x - 6$
B. $y = x + 6$
C. $y = 6 - x$
D. $y = -6$
E. $y = 6$

Question 2

A point that lies on the graph of $3x - 2y = -5$ is

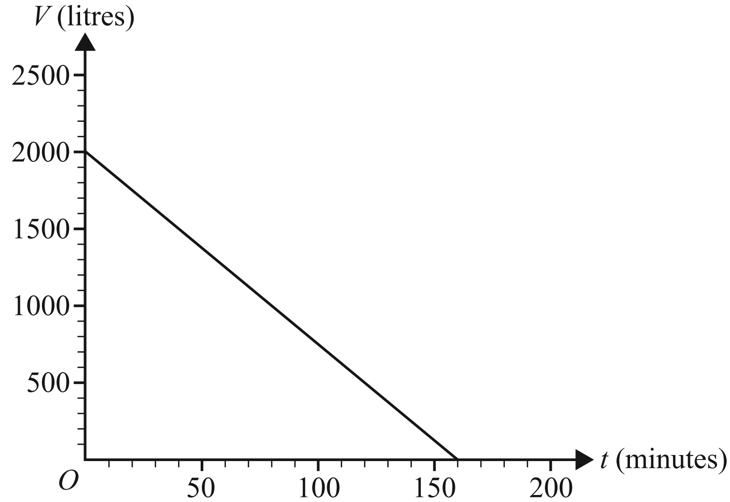
A. (3, –2)
B. (1, 1)
C. (1, –1)
D. (2, –3)
E. (–1, 1)

Question 3

A full tank holds 2000 litres of water.

Water is pumped out of the tank at a constant rate.

The graph below shows how the volume of water in the tank, V, changes with time, t.



The constant rate, in litres per minute, at which the water is being pumped out of the tank is

A. 0.8
B. 2.0
C. 12.5
D. 80.0
E. 160.0

Question 4

The Blue Caps cricket club has different prices for its junior and senior subscriptions.

The total cost for two junior subscriptions and one senior subscription is $225.

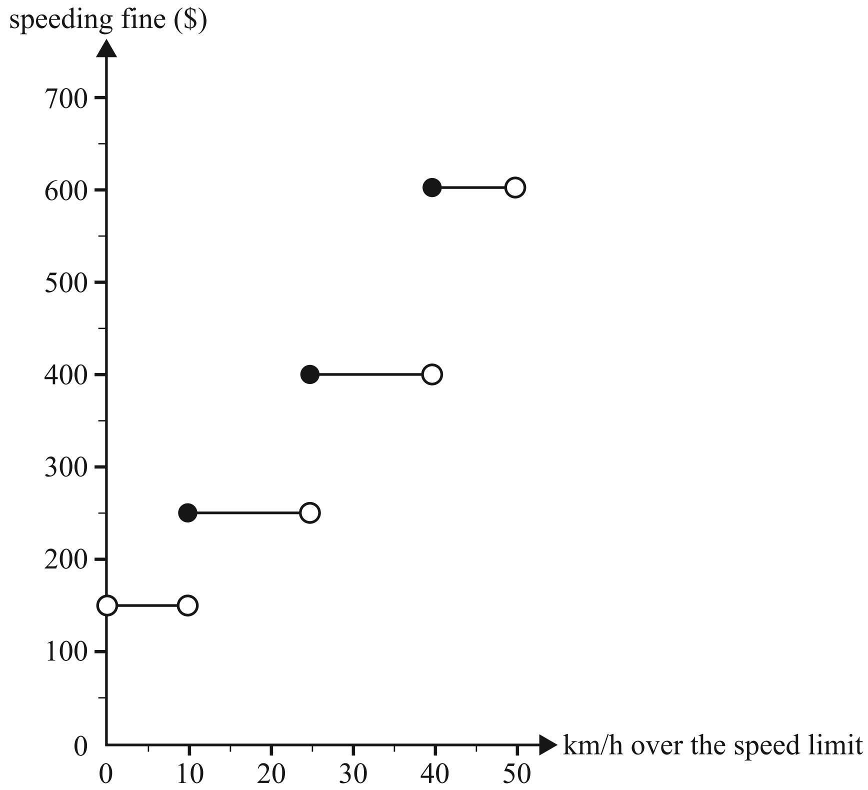
The cost of a senior subscription is three times the cost of a junior subscription.

The cost of a senior subscription is

A. $45
B. $75
C. $90
D. $135
E. $180

Question 5

The step graph below shows the speeding fines that are given for exceeding the speed limit by different amounts.



A driver was fined for driving at a speed of 65 km/h in a zone with a speed limit of 40 km/h.

The fine given was

A. $65

B. $150

C. $250

D. $400

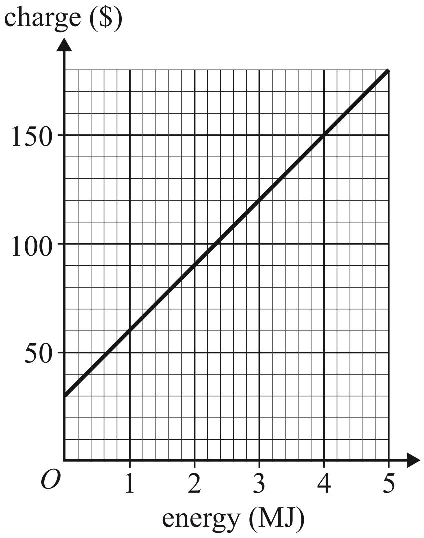
E. $600

Question 6

In one month, an energy company charges a $30 service fee plus a supply charge of two cents per megajoule (MJ) of energy used.

The graph that best models this situation is

A.



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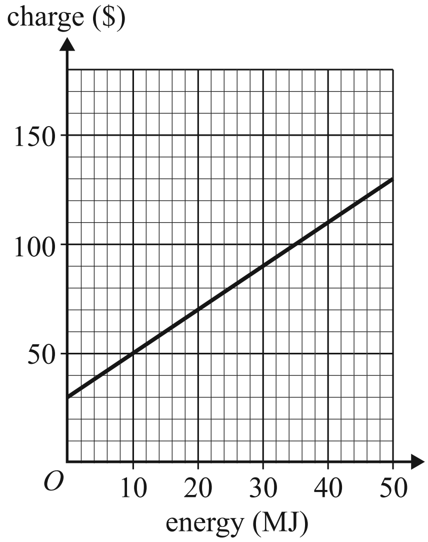
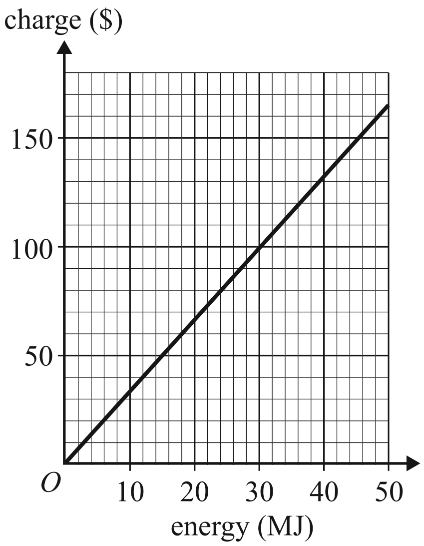
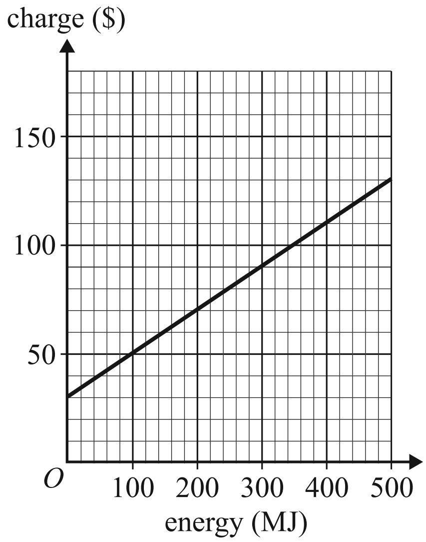


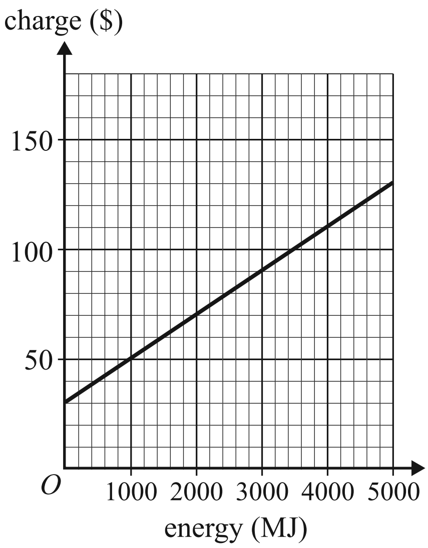
C.



D.



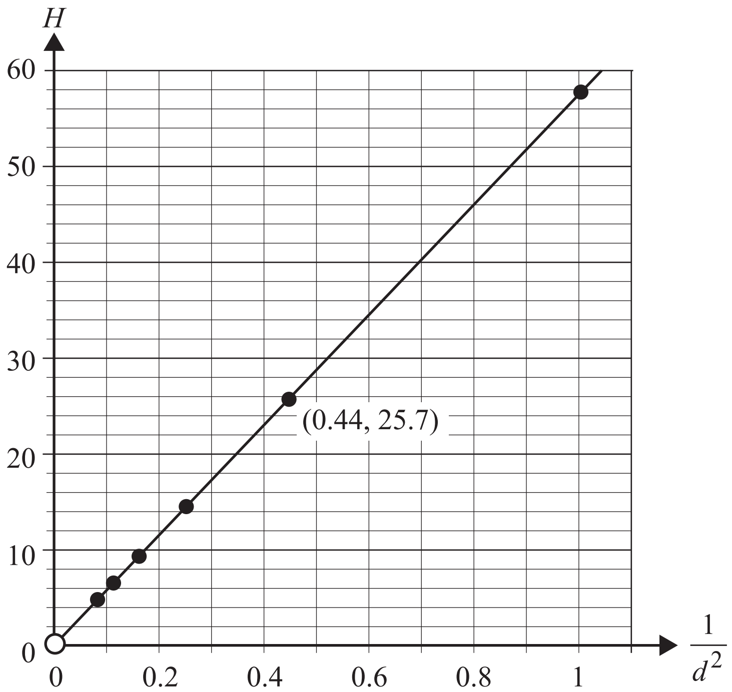
E.



Question 7

The heat intensity of a fire, H, is recorded at different distances, d, from the fire.

When H is plotted against $\frac{1}{d^2}$, the data points lie on a straight line, as shown below.



The point (0.44, 25.7) lies on the line.

Given this information, the rule that relates the intensity of the fire, H, to the distance, d, from the fire is closest to

A. $H = \frac{58.4}{d^2}$

B. $H = \frac{38.7}{d^2}$

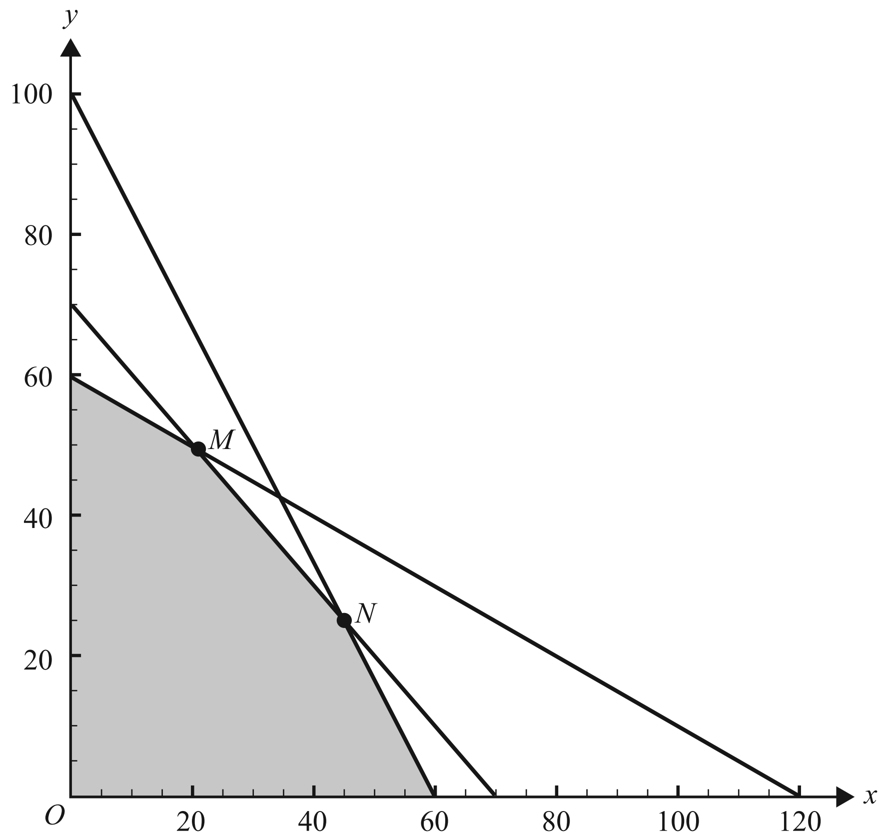
C. $H = \frac{4.98}{d^2}$

D. $H = 38.7d$

E. $H = 58.4d$

Question 8

The shaded region in the graph below represents the feasible region for a linear programming problem.



An objective function $Z = ax + by$ has its value maximised at both vertex M and vertex N.

The values of a and b could be

A. $a = 15$ and $b = -15$

B. $a = 15$ and $b = 15$

C. $a = 15$ and $b = 25$

D. $a = 25$ and $b = 50$

E. $a = 50$ and $b = -25$

Question 9

A cafe sells the first 200 cups of hot chocolate each day at a special price of \$3.00 per cup. After that, a cup of hot chocolate will be sold for \$4.50.

The revenue, R, in dollars, made from selling n cups of hot chocolate each day is given by the rule

$$R = \begin{cases} 3n & 0 \le n \le 200 \\ 4.50n - 300 & n > 200 \end{cases}$$

The cost, C, in dollars, of making n cups of hot chocolate each day is

$$C = 500 + 1.30n$$

To break even, the number of cups of hot chocolate that must be sold each day is

A. 63
B. 200
C. 250
D. 295
E. 300

Module 4: Business-related mathematics

> Before answering these questions you must **shade** the Business-related mathematics box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

A phone that normally retails for \$200 is discounted to \$170.

The percentage discount is

A. 10%
B. 15%
C. 20%
D. 25%
E. 30%

Question 2

The closing price of a share on Monday was \$30.

The closing price of the share on Tuesday was 5% more than its closing price on Monday.

The closing price of the share on Wednesday was 5% less than its closing price on Tuesday.

Which one of the following calculations will give the closing price of the share, in dollars, on Wednesday?

A. $30 \times 1.05 \times 0.95$
B. $30 \times 1.05 \times -0.05$
C. $30 + 1.05 \times 0.95$
D. $30 + 0.05 \times 30 - 0.05 \times 30$
E. $30 + 1.05 - 0.95$

Question 3

\$10 000 is invested for five years. Interest is earned at a rate of 8% per annum, compounding quarterly.

Which one of the following calculations will give the total interest earned, in dollars, by this investment?

A. $10\,000 \times 1.02^5 - 10\,000$
B. $10\,000 \times 1.02^{20} - 10\,000$
C. $10\,000 \times 1.08^5 - 10\,000$
D. $10\,000 \times 1.08^{20} - 10\,000$
E. $10\,000 \times 1.02^{20}$

Question 4

The purchase price of a car is \$15 000.

A deposit of \$3000 is paid and the balance will be repaid with 36 monthly payments of \$400.

The annual flat rate of interest charged is closest to

A. 1.3%
B. 4.0%
C. 5.3%
D. 6.7%
E. 20.0%

Question 5

$100 000 is invested in a perpetuity at an interest rate of 6% per annum.

After 10 quarterly payments have been made, the amount of money that remains invested in the perpetuity is

A. $15 000
B. $40 000
C. $85 000
D. $94 000
E. $100 000

Question 6

A worker has received an annual salary increase of 3% for the past two years.

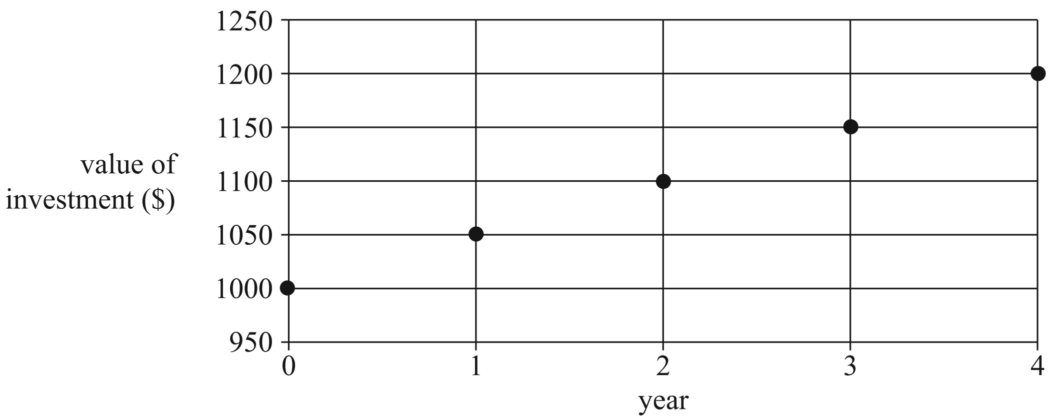
This year, the worker's annual salary is $46 500.

Two years ago, her salary was closest to

A. $42 315
B. $43 750
C. $43 830
D. $45 140
E. $49 330

Question 7

The graph below shows the growth in value of a $1000 investment over a period of four years.



A different amount of money is invested under the same investment conditions for eight years.

In total, the amount of interest earned on this investment is $600.

The amount of money invested is

A. $500
B. $600
C. $1500
D. $2000
E. $2400

Question 8

A car is purchased for $25 000. The value of the car is to be depreciated each year by 20% using the reducing balance method.

In the fourth year, the car will depreciate in value by

A. $2048

B. $2560

C. $5000

D. $10 240

E. $14 760

Question 9

The following information relates to the repayment of a home loan of $300 000.

- The loan is to be repaid fully with monthly payments of $2500.
- Interest compounds monthly.
- After the first monthly payment has been made, the amount owing on the loan is $299 000.

Which one of the following statements is true?

A. After two months, $297 995 is still owing on the loan.

B. $1000 of interest has been paid in the first month.

C. The loan will be fully repaid in less than 15 years.

D. Halfway through the term of the loan, the amount still owing will be $150 000.

E. Payments of $2750 rather than $2500 per month will reduce the time to repay the loan fully by more than three years.

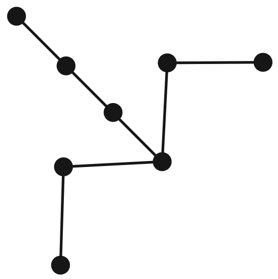
Module 5: Networks and decision mathematics

Before answering these questions you must **shade** the Networks and decision mathematics box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

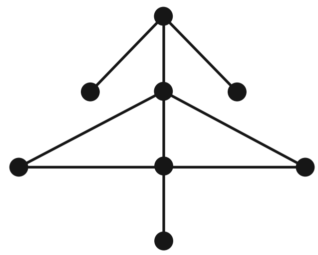
Question 1

Which one of the following graphs is a tree?

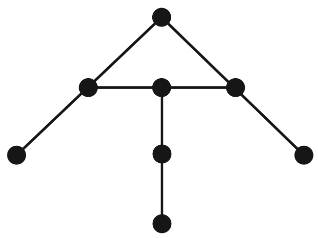
A.



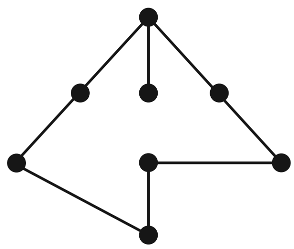
B.



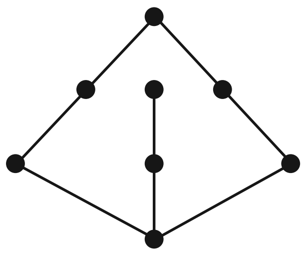
C.



D.



E.

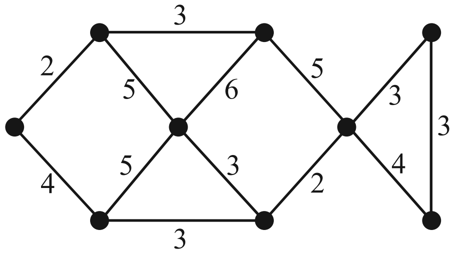


Question 2

The number of edges needed to make a complete graph with four vertices is

A. 2
B. 3
C. 4
D. 5
E. 6

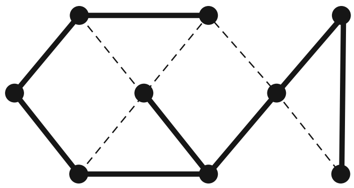
Question 3



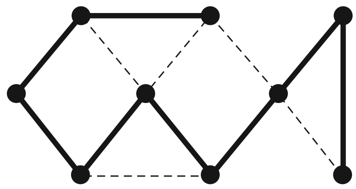
The vertices of the graph above represent nine computers in a building. The computers are to be connected with optical fibre cables, which are represented by edges. The numbers on the edges show the costs, in hundreds of dollars, of linking these computers with optical fibre cables.

Based on the same set of vertices and edges, which one of the following graphs shows the cable layout (in bold) that would link all the computers with optical fibre cables for the minimum cost?

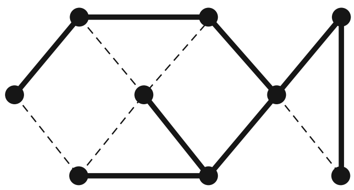
A.



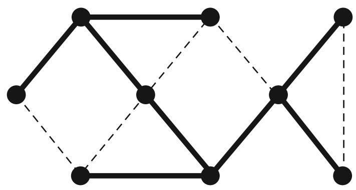
B.



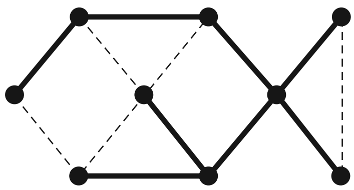
C.



D.



E.



Question 4

Kate, Lexie, Mei and Nasim enter a competition as a team. In this competition, the team must complete four tasks, *W*, *X*, *Y* and *Z*, as quickly as possible.

The table shows the time, in minutes, that each person would take to complete each of the four tasks.

	Kate	Lexie	Mei	Nasim
W	6	3	4	6
X	4	3	5	5
Y	5	7	9	6
Z	3	2	3	2

If each team member is allocated one task only, the minimum time in which this team would complete the four tasks is

A. 10 minutes

B. 12 minutes

C. 13 minutes

D. 14 minutes

E. 15 minutes

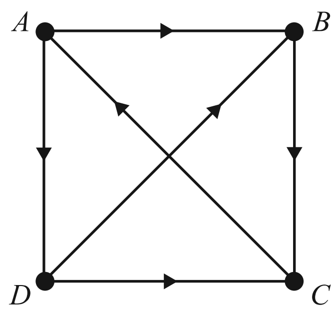
Question 5

Four people, Ash (*A*), Binh (*B*), Con (*C*) and Dan (*D*), competed in a table tennis tournament.

In this tournament, each competitor played each of the other competitors once.

The results of the tournament are summarised in the directed graph below.

Each arrow shows the winner of a game played in the tournament. For example, the arrow from *C* to *A* shows that Con defeated Ash.



In the tournament, each competitor was given a ranking that was determined by calculating the sum of their one-step and two-step dominances. The competitor with the highest sum is ranked number one (1). The competitor with the second-highest sum was ranked number two (2), and so on.

Using this method, the rankings of the competitors in this tournament were

A. Dan (1), Ash (2), Con (3), Binh (4)

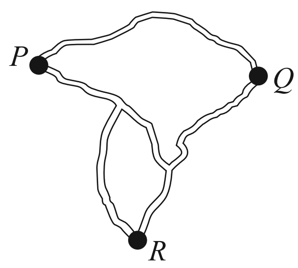
B. Dan (1), Ash (2), Binh (3), Con (4)

C. Con (1), Dan (2), Ash (3), Binh (4)

D. Ash (1), Dan (2), Binh (3), Con (4)

E. Ash (1), Dan (2), Con (3), Binh (4)

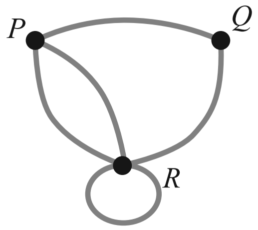
Question 6



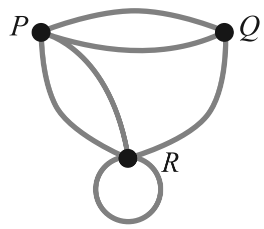
The map above shows the road connections between three towns, *P*, *Q* and *R*.

The graph that could be used to model these road connections is

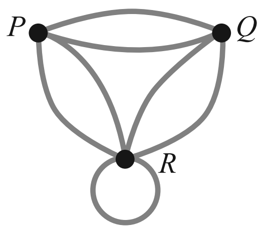
A.



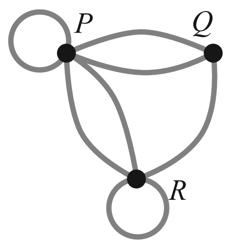
B.



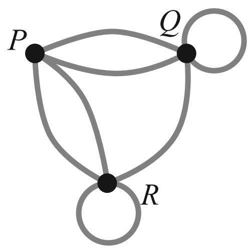
C.



D.



E.



Question 7

A connected graph consists of five vertices and four edges.

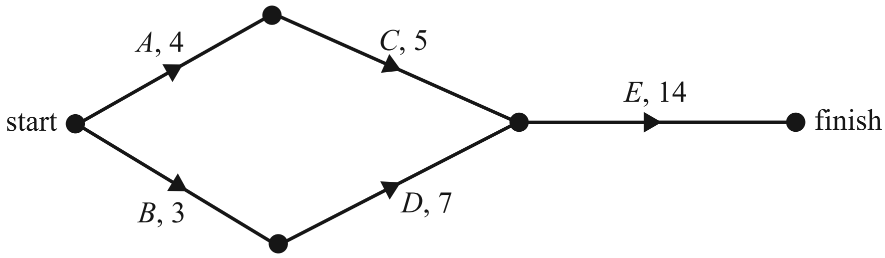
Consider the following five statements.

- The graph is planar.
- The graph has more than one face.
- All vertices are of even degree.
- The sum of the degrees of the vertices is eight.
- The graph cannot have a loop.

How many of these statements are always true for such a graph?

A. 1
B. 2
C. 3
D. 4
E. 5

Question 8



The graph above shows five activities, *A*, *B*, *C*, *D* and *E*, that must be completed to finish a project.

The number next to each letter shows the completion time, in hours, for the activity.

Each of the five activities can have its completion time reduced by a maximum of one hour at a cost of $100 per hour.

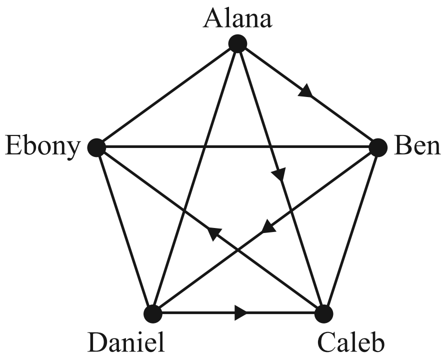
The least cost to achieve the greatest reduction in the time taken to finish the project is

A. $100
B. $200
C. $300
D. $400
E. $500

Question 9

Alana, Ben, Ebony, Daniel and Caleb are friends. Each friend has a different age.

The arrows in the graph below show the relative ages of some, but not all, of the friends. For example, the arrow in the graph from Alana to Caleb shows that Alana is older than Caleb.



Using the information in the graph, it can be deduced that the **second-oldest** person in this group of friends is

A. Alana

B. Ben

C. Caleb

D. Daniel

E. Ebony

Module 6: Matrices

Before answering these questions you must **shade** the Matrices box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 0 \\ 0 \\ 2 \end{bmatrix} - 2 \times \begin{bmatrix} 0 \\ -1 \\ -1 \\ 0 \end{bmatrix} \text{ equals}$$

A. $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

B. $\begin{bmatrix} 2 \\ 2 \\ 2 \\ 2 \end{bmatrix}$

C. $\begin{bmatrix} 2 \\ -2 \\ -2 \\ 2 \end{bmatrix}$

D. $\begin{bmatrix} 4 \\ 0 \\ 0 \\ 4 \end{bmatrix}$

E. $\begin{bmatrix} 0 \\ 2 \\ 2 \\ 0 \end{bmatrix}$

Question 2

Matrix A has three rows and two columns.

Matrix B has four rows and three columns.

Matrix $C = B \times A$ has

A. two rows and three columns.

B. three rows and two columns.

C. three rows and three columns.

D. four rows and two columns.

E. four rows and three columns.

Question 3

A coffee shop sells three types of coffee, Brazilian (B), Italian (I) and Kenyan (K). The regular customers buy one cup of coffee each per day and choose the type of coffee they buy according to the following transition matrix, T.

$$T = \begin{array}{c} \text{choose today} \\ \begin{array}{ccc} B & I & K \end{array} \\ \begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0 & 0.8 & 0.1 \\ 0.2 & 0.1 & 0.8 \end{bmatrix} \end{array} \begin{array}{c} B \\ I \\ K \end{array} \text{ choose tomorrow}$$

On a particular day, 84 customers bought Brazilian coffee, 96 bought Italian coffee and 81 bought Kenyan coffee.

If these same customers continue to buy one cup of coffee each per day, the number of these customers who are expected to buy each of the three types of coffee in the long term is

A.

Brazilian	85
Italian	85
Kenyan	91

B.

Brazilian	87
Italian	58
Kenyan	116

C.

Brazilian	88
Italian	86
Kenyan	87

D.

Brazilian	89
Italian	89
Kenyan	83

E.

Brazilian	116
Italian	89
Kenyan	58

Question 4

$$2.8x + 0.7y = 10$$
$$1.4x + ky = 6$$

The set of simultaneous linear equations above does **not** have a solution if k equals

A. –0.35

B. –0.250

C. 0

D. 0.25

E. 0.35

Question 5

Five students, Richard (*R*), Brendon (*B*), Lee (*L*), Arif (*A*) and Karl (*K*), were asked whether they played each of the following sports, football (*F*), golf (*G*), soccer (*S*) or tennis (*T*). Their responses are displayed in the table below.

Student	**Sport played**			
	Football (*F*)	**Golf (*G*)**	**Soccer (*S*)**	**Tennis (*T*)**
R	yes	no	no	yes
B	yes	yes	yes	no
L	no	no	no	yes
A	no	yes	no	yes
K	yes	no	no	yes

If 1 is used to indicate that the student plays a particular sport and 0 is used to indicate that the student does not play a particular sport, which one of the following matrices could be used to represent the information in the table?

A.

$$\begin{array}{c} \begin{array}{ccccc} R & B & L & A & K \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{array}{c} F \\ G \\ S \\ T \end{array} \end{array}$$

B.

$$\begin{array}{c} \begin{array}{cccc} F & G & S & T \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} \begin{array}{c} R \\ B \\ L \\ A \\ K \end{array} \end{array}$$

C.

$$\begin{array}{c} \begin{array}{ccccc} R & B & L & A & K \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \end{bmatrix} \begin{array}{c} F \\ G \\ S \\ T \end{array} \end{array}$$

D.

$$\begin{array}{c} \begin{array}{cccc} F & G & S & T \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \begin{array}{c} R \\ B \\ L \\ A \\ K \end{array} \end{array}$$

E.

$$\begin{array}{c} \begin{array}{cccc} F & G & S & T \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \begin{array}{c} R \\ B \\ L \\ A \\ K \end{array} \end{array}$$

Question 6

A worker can assemble 10 bookcases and four desks in 360 minutes, and eight bookcases and three desks in 280 minutes.

If each bookcase takes *b* minutes to assemble and each desk takes *d* minutes to assemble, the matrix $\begin{bmatrix} b \\ d \end{bmatrix}$ will be given by

A. $\begin{bmatrix} -1.5 & 2 \\ 4 & -5 \end{bmatrix} \begin{bmatrix} 360 \\ 280 \end{bmatrix}$

B. $\begin{bmatrix} 10 & 4 \\ 8 & 3 \end{bmatrix} \begin{bmatrix} 360 \\ 280 \end{bmatrix}$

C. $\begin{bmatrix} 3 & -4 \\ -8 & 10 \end{bmatrix} \begin{bmatrix} 360 \\ 280 \end{bmatrix}$

D. $\begin{bmatrix} 5 & -2 \\ -4 & 1.5 \end{bmatrix} \begin{bmatrix} 360 \\ 280 \end{bmatrix}$

E. $\begin{bmatrix} 10 \\ 4 \end{bmatrix} \begin{bmatrix} 360 \end{bmatrix} + \begin{bmatrix} 8 \\ 3 \end{bmatrix} \begin{bmatrix} 280 \end{bmatrix}$

Question 7

A school has three computer classes, *A*, *B* and *C*. There are 15 students in each class.

Each student is given a mark out of 100 based on their performance in a test.

Matrix *M* below displays the marks obtained by these 45 students, listed by class.

$$M = \begin{bmatrix} 56 & 78 & 79 & 43 & 67 & 56 & 80 & 85 & 75 & 89 & 55 & 64 & 95 & 34 & 63 \\ 90 & 45 & 56 & 65 & 76 & 79 & 27 & 45 & 69 & 73 & 70 & 63 & 65 & 34 & 59 \\ 76 & 76 & 89 & 47 & 50 & 66 & 68 & 89 & 88 & 90 & 45 & 67 & 78 & 45 & 87 \end{bmatrix} \begin{matrix} A \\ B \\ C \end{matrix} \quad \text{class}$$

Two other matrices, *S* and *R*, are defined below.

$$S = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad \text{and} \quad R = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

Which one of the following matrix expressions can be used to generate a matrix that displays the mean mark obtained for each class?

A. $\frac{1}{45}M$

B. $\frac{1}{3}R \times M$

C. $\frac{1}{3}R \times M \times S$

D. $\frac{1}{15}M \times S$

E. $\frac{1}{15}S \times R \times M$

Question 8

The matrix S_{n+1} is determined from the matrix S_n using the rule $S_{n+1} = T S_n - C$ where T, S_0 and C are defined as follows.

$$T = \begin{bmatrix} 0.5 & 0.6 \\ 0.5 & 0.4 \end{bmatrix}, \quad S_0 = \begin{bmatrix} 100 \\ 250 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} 20 \\ 20 \end{bmatrix}$$

Given this information, the matrix S_2 equals

A. $\begin{bmatrix} 100 \\ 250 \end{bmatrix}$

B. $\begin{bmatrix} 148 \\ 122 \end{bmatrix}$

C. $\begin{bmatrix} 170 \\ 140 \end{bmatrix}$

D. $\begin{bmatrix} 180 \\ 130 \end{bmatrix}$

E. $\begin{bmatrix} 190 \\ 160 \end{bmatrix}$

Question 9

P, Q, R and S are matrices such that the matrix product $P = QRS$ is defined.

Matrix Q and matrix S are square, non-zero matrices for which $Q + S$ is **not defined**.

Which one of the following matrix expressions **is defined**?

A. $R - S$

B. $Q + R$

C. P^2

D. R^{-1}

E. $P \times S$

END OF MULTIPLE-CHOICE QUESTION BOOK

FURTHER MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Further Mathematics formulas

Core: Data analysis

standardised score:	$z = \frac{x - \bar{x}}{s_x}$
least squares line:	$y = a + bx$ where $b = r\frac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$
residual value:	residual value = actual value – predicted value
seasonal index:	seasonal index = $\frac{\text{actual figure}}{\text{deseasonalised figure}}$

Module 1: Number patterns

arithmetic series:	$a + (a + d) + \ldots + (a + (n-1)d) = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}(a + l)$
geometric series:	$a + ar + ar^2 + \ldots + ar^{n-1} = \frac{a(1-r^n)}{1-r}, r \neq 1$
infinite geometric series:	$a + ar + ar^2 + ar^3 + \ldots = \frac{a}{1-r}, \lvert r \rvert < 1$

Module 2: Geometry and trigonometry

area of a triangle:	$\frac{1}{2}bc\sin A$
Heron's formula:	$A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{1}{2}(a + b + c)$
circumference of a circle:	$2\pi r$
area of a circle:	πr^2
volume of a sphere:	$\frac{4}{3}\pi r^3$
surface area of a sphere:	$4\pi r^2$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a cylinder:	$\pi r^2 h$
volume of a prism:	area of base × height
volume of a pyramid:	$\frac{1}{3}$ area of base × height

Pythagoras' theorem: $c^2 = a^2 + b^2$

sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab \cos C$

Module 3: Graphs and relations

Straight line graphs

gradient (slope): $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

equation: $y = mx + c$

Module 4: Business-related mathematics

simple interest: $I = \dfrac{PrT}{100}$

compound interest: $A = PR^n$ where $R = 1 + \dfrac{r}{100}$

hire purchase: effective rate of interest $\approx \dfrac{2n}{n+1} \times$ flat rate

Module 5: Networks and decision mathematics

Euler's formula: $v + f = e + 2$

Module 6: Matrices

determinant of a 2 × 2 matrix: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

inverse of a 2 × 2 matrix: $A^{-1} = \dfrac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $\det A \neq 0$

END OF FORMULA SHEET