2015 VCE Further Mathematics 2 examination report

General comments

The selection of modules by students in 2015 is shown in the table below.

Module	% 2015
1: Number patterns	27
2: Geometry and trigonometry	65
3: Graphs and relations	46
4: Business-related mathematics	31
5: Networks and decision mathematics	49
6: Matrices	83

The 2015 examination presented opportunities for all students to start well with the Core section and each module. Questions then became more challenging as students progressed through each module.

Students are urged to use reading time to identify and comprehend all the questions they are to answer. It was clear that many students did not read some questions carefully and misread or misunderstood what was required.

Most students managed their time well and were able to keep the context of questions in mind when responding.

Many students did not follow the instructions given on the examination paper. For example:

- answers given as a number instead of a percentage (Core, Question 1c.)
- choosing the incorrect variable as the dependent variable (Core, Question 4a.)
- not completing the question by finding the difference between two numbers (Core, Question 5bi.)
- failure to use the cosine rule (Module 2: Geometry and trigonometry, Question 2bii.)
- referring to the vertical axis rather than the horizontal axis (Module 3: Graphs and relations, Question 3bii.)
- shading outside the required region instead of inside (Module 3: Graphs and relations, Question 5b.)
- using the compound formula for a flat rate question (Module 4: Business-related mathematics, Question 2a.)
- not attempting part of a question (Module 4: Business-related mathematics, Question 5c.)
- writing a Hamiltonian circuit when the question asks for a non-Hamiltonian circuit (Module 5: Networks and decision mathematics 2ai.)

Organised presentation of a multi-step calculation was essential for high marks. Many students wrote answers without calculations to two-mark questions. Some working may qualify for a method mark or consequential error mark even if the answer is incorrect.

Students should answer data analysis questions by referring to the statistics provided or obtained. Personal opinion or interpretation of socioeconomic issues is generally not relevant to data analysis.

'Show that' questions give the answer to a basic and appropriate calculation and students are required to write that calculation. The given number in a 'show that' question is sometimes needed in a following question. With this number, students can attempt the following question, even if they cannot complete the 'show that' question.

The given number in a 'show that' question is not to be used in a calculation; it must be the result of a calculation. For example, in Question 2a. in Module 4: Business-related mathematics:

The sound system cost $3800 and was valued at $3150 two years later.

Show that the (flat rate) depreciation was $325 p.a.

'Show that' required students to find the quantity named (annual depreciation) and get the answer provided ($325).

The expected answer could have been obtained as $\frac{3800-3150}{2}=325$

The following calculations use the 325 to show a different result than what was required and were not acceptable for this example question:

- 3800 – 2 × 325 = 3150 – shows how to find the depreciated value after two years
- 3150 + 3 × 325 = 3800 – shows how to find the initial value

Some points that teachers and students could usefully address include:

- proper and effective use of reading time
- rounding numbers
- interpreting the slope of a regression line in terms of the two variables
- dependent and independent variables
- breaking complex questions into small steps
- setting out of a multi-step calculation
- suitable use of technology
- estimating answers where possible to exclude absurd calculation results
- a glossary of relevant terms and formulas.

Specific information

This report provides sample answers or an indication of what answers may have included. Unless otherwise stated, these are not intended to be exemplary or complete responses.

The statistics in this report may be subject to rounding resulting in a total more or less than 100 per cent.

Core

Question 1a.

Marks	0	1	Average
%	44	56	0.6

70–75 years

This question was not well answered. Many students simply wrote ‘5’, while others wrote ‘negatively skewed’.

Question 1b.

Marks	0	1	Average
%	23	77	0.8

14 countries

A common incorrect answer was 9.

Question 1c.

Marks	0	1	Average
%	32	68	0.7

16.4%

$$\frac{30}{183} = 0.1639... \approx 16.4\%$$

An answer rounded to one decimal place was expected. A number of students simply wrote 16% or an unrounded figure. This answer was not accepted. It could not be considered as a rounding error as there was no evidence that rounding had occurred.

Question 2a.

Marks	0	1	Average
%	17	83	0.9

Negatively skewed (with no outliers)

Question 2b.

Marks	0	1	2	Average
%	48	15	37	0.9

Median life expectancy changes with year. For example, in 1953 the median was 52, in 1973 the median was 63 and in 1993 the median was 69.

Observing the change in IQR values between 1953, 1973 and 1993, with appropriate values quoted, was also acceptable.

Some students referred to mean values, which are not discernible from a box plot unless it is perfectly symmetrical.

Question 3a.

Marks	0	1	Average
%	60	40	0.4

On average, *male* life expectancy increased by 0.88 years for each one-year increase in *female* life expectancy.

This question was not well answered, with many students describing the scatterplot instead of interpreting the slope of the line as asked.

The increase in *male* life expectancy needed to be related to one unit (year) increase in *female* life expectancy. Answers such as 'On average, *male* life expectancy increased by 0.88 years for every increase in *female* life expectancy' were not accepted.

Some students incorrectly interpreted slope = 0.88 as $r = 0.88$

Question 3b.

Marks	0	1	Average
%	22	78	0.8

34.4 years

Question 3c.

Marks	0	1	Average
%	48	52	0.5

95% of the variation in *male* life expectancy can be explained by the variation in *female* life expectancy.

A common error was to interpret 'The coefficient of determination is 0.95' as meaning $r = 0.95$ and then using $r^2 = 0.9025 = 90.25\%$ for the interpretation.

Question 4a.

Marks	0	1	Average
%	37	63	0.7

Strong, positive, linear

Many students did not state the form as linear.

Question 4b.

Marks	0	1	Average
%	51	49	0.5

male = 9.69 + 0.81 × *female*

Students needed to correctly enter the data from the table to determine the coefficients for this equation. Some inappropriately used the data from the first column as the independent variable in technology calculation.

Many students did not follow rounding instructions and wrote 9.7, instead of 9.69, in the first box.

Question 5a.

Marks	0	1	Average
%	23	77	0.8

22 years

Question 5bi.

Marks	0	1	2	Average
%	15	27	58	1.5

3 years

Australia
life expectancy = –451.7 + 0.2657 × 2030
= 87.67… years
UK
life expectancy = –350.4 + 0.2143 × 2030
= 84.62… years

Difference = 87.67…– 84.62… = 3.04…

Some students correctly calculated the numbers for Australia and the United Kingdom but then did not find the required difference.

Some students found negative values for life expectancies. Negative values are non-consistent with the context.

Question 5bii.

Marks	0	1	Average
%	55	45	0.5

The regression equation was used to make predictions outside the available range of data.

This question required a response related to the given statistical data rather than any sociological possibilities. Many students wrote about the likely impact of future technological advances, wars, famines, viruses or advances in medicine, or simply stated ‘You don’t know what’s going to happen in the future’. Such responses were not accepted.

Module 1: Number patterns

Questions 1a.–2c.

Marks	0	1	2	3	4	5	6	7	8	Average
%	1	4	5	7	9	12	16	22	25	5.8

1a.

2150 – 2000 = 150

1b.

2600 kg

1c.

20 200 kg

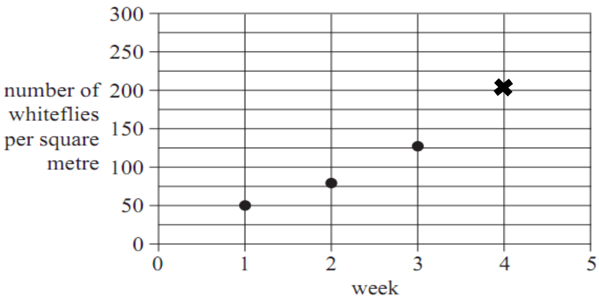
1d.

Week 6

1e.

$C_n + 150$

2a.



2b.

Week 6

2c.

60%

A common incorrect answer was 160%.

Questions 2di.–3b.

Marks	0	1	2	3	4	5	6	7	Average
%	42	19	12	7	6	5	4	4	1.7

2di.

76.8

2dii.

154.8

$W_4 = 1.6 \times 128 - 50 = 154.8$

2diii.

88.3

$W_3 = 128$

$W_4 = 1.6 \times W_3 - k$

$W_4 = 1.6 \times 128 - k$

$W_5 = 1.6 \times W_4 - k$

$98 = 1.6 \times (1.6 \times 128 - k) - k$

Solve: $327.68 - 2.6k = 98$

3a.

Number of ladybirds taken from the greenhouse and also the number of ladybirds released into the field

3b.

857

Constant G means $G_{n+1} = G_n = 1200$

$1200 = 1.25 \times 1200 - m$

$m = 300$

Constant F means $F_{n+1} = F_n = f$ with $m = 300$

$f = 0.65 \times f + 300$

$0.35f = 300$

$f = 857.142\ldots$

This question was not answered well. Most students were unable to see the link between G_n, G_{n+1} and 1200 to find the value of m that was needed to then find f.

Module 2: Geometry and trigonometry

Questions 1a.–1d.

Marks	0	1	2	3	4	Average
%	6	22	27	25	21	2.3

1a.

400 m

1b.

0.6

$$\frac{300}{500} = 0.6$$

1c.

0.008 metres

$$\frac{x}{1} = \frac{400}{50\,000}$$

Many students either misread the question or did not check if their answer was reasonable for a distance on a map. Any answer greater than a metre would not be reasonable in this context. Some students gave answers up to 20 000 kilometres.

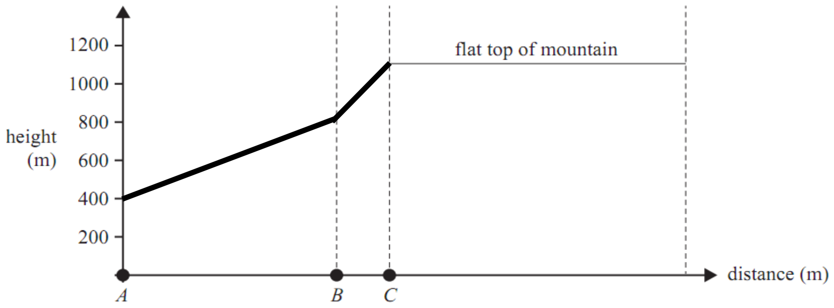
1d.

16°

Questions 1e.–2a.

Marks	0	1	2	3	Average
%	17	13	23	47	2

1e.



2a.

475 m

$$\sin 30^\circ = \frac{x}{950}$$
$$x = 475$$

Students may need to draw triangles as applicable to answer questions. For this question, lines could be drawn east and north from *C* to *T*.

Questions 2bi.–3a.

Marks	0	1	2	3	4	Average
%	21	19	20	17	23	2

2bi.

908 m

$$\sqrt{\left(950^2 + 1400^2 - 2 \times 950 \times 1400 \times \cos\left(40^\circ\right)\right)}$$
$$= 908.196$$

2bii.

142° (after a demonstrated substitution into the cosine rule to find $\angle CET$ or $\angle CTE$)

$$1400^2 = 950^2 + 908^2 - 2 \times 950 \times 908 \times \cos(\angle CTE)$$
$$\angle CTE = 97.76...^\circ$$
or
$$950^2 = 1400^2 + 908^2 - 2 \times 1400 \times 908 \times \cos(\angle CET)$$
$$\angle CET = 42.24...^\circ$$

Hence bearing of *E* from *T* = 360 – 120 – 97.76 = 142.24º

or

360 – 120 – 140 + 42.24 = 142.24º

To qualify for full marks, responses required evidence of use of the cosine rule.

A method mark was available in this two-mark question if the final answer was incorrect. However, some calculations were poorly set out and this method mark could not always be allocated.

3a.

$$h = 2.4 + \sqrt{2.4^2 - 2^2}$$
$$= 2.4 + 1.326...$$
$$\approx 3.73 \text{ m}$$

Questions 3b.–4

Marks	0	1	2	3	4	Average
%	35	19	19	13	14	1.5

Question 3b.

76 m^2

Front and back: $2 \times \left(2.4 \times 4 + \frac{1}{2} \times 4 \times 1.33\right) \approx 24.52$

Walls and roof: $4 \times 2.4 \times 5.4 \approx 51.84$

Total = $24.52 + 51.84 = 76.36$

Few students answered this question correctly. Many did not make use of the answer to Question 3a.

Students often used the rule for total surface area of a rectangular prism, not recognising that there was no floor or single rectangular roof section.

Many calculations were poorly set out and a method mark could not be awarded. Students are encouraged to clearly label each step, such as 'Wall:' or 'Front:' in an extended calculation and to draw supporting diagrams where applicable.

Question 4

7.3 m

Obtuse angle in the thin triangle at the top is: 180 – 38 = 142°

Let required height = x

Solve ($37^2 = 31^2 + x^2 - 2 \times 31 \times x \times \cos(142°)$)

$\therefore x = 7.269\ldots \approx 7.3$

Other valid methods used the sine rule or applications of Pythagoras' theorem.

The most common error was to consider the two right triangles as being similar; however, they were not. Many students used cos(38°) to find 24.42 m as the low height but then inappropriately applied a scale factor 37:31 to find the incorrect high height.

A number of other students incorrectly assumed that 38° also applied to the top angle of the larger right triangle.

Module 3: Graphs and relations

Questions 1a.–2b.

Marks	0	1	2	3	4	Average
%	0	1	7	26	65	3.6

1a.

$1200

1b.

8 days

2a.

18 000 yen

2b.

90 yen

The slope of the line will provide the answer. The two points to help determine this should be read off the graph and as far apart as possible, such as (0, 0) and (200, 18 000).

Questions 3a.–4d.

Marks	0	1	2	3	4	5	6	7	8	Average
%	4	8	18	15	12	12	12	12	8	4.1

3a.

$$8075 = 95 \times 100 - k$$
$$8075 = 9500 - k$$
$$k = 9500 - 8075$$
$$k = 1425$$

Algebraic solution of equations is a required skill. A transposition step in this 'show that' question was expected, and *k* = 1425 needed to be the result of an appropriate calculation.

3bi.

$15

$$0 = 95 \times dollars - 1425$$

$$1425 = 95 \times dollars$$

$$dollars = \frac{1425}{95} = 15$$

3bii.

There is a $15 commission or fixed charge for any conversion into yen.

Other acceptable answers included '$15 base fee is charged before the agency will convert dollars to yen' or 'You need more than $15 to buy any yen'.

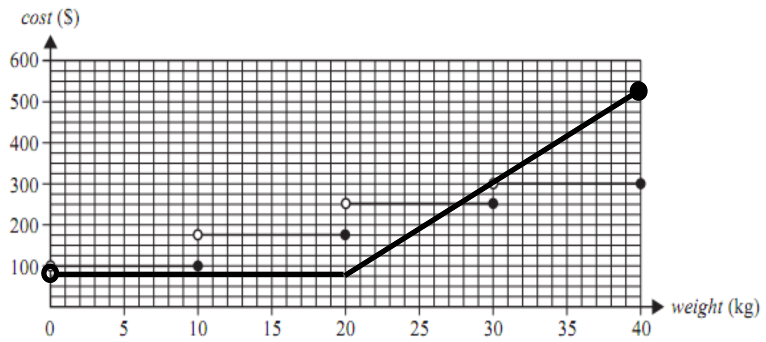
4a.

$250

4b.

$300

4c.



Many students found this question challenging. Many were able to correctly locate the horizontal line, but often without the open circle required at (0, 75). A much smaller number of students were able to correctly draw in the inclined line but most of these did not include the filled dot at *weight* = 40.

The connection between the horizontal and inclined lines at *weight* = 20 was sometimes marked with a dot inside an open circle. As the graph does not 'break' at this point, there was no need for either of these.

4d.

27.8 kg

$$22.5 \times weight - 375 = 250$$
$$22.5 \times weight = 625$$
$$weight = 27.77...$$

The most common incorrect answer was 30 kg. However, the open circle at this point, for luggage paid at the airport, means there cannot be a point of intersection here.

The inclined line cuts through the step graph at around *weight* = 28 where *cost* = $250 and a calculation is needed to find the answer correct to one decimal place as required.

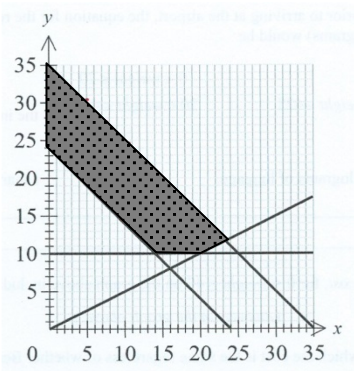
Questions 5a.–5c.

Marks	0	1	2	3	Average
%	23	29	19	30	1.6

5a.

Ben must attend at least 10 lessons in Japanese.

5b.



The question required the feasible region to be shaded. Some students used shading to highlight **excluded** sections, leaving their feasible region as unshaded. These students could still qualify for marks as long as they clearly labelled or identified their (unshaded) answer with a legend.

5c.

23 English lessons

Module 4: Business-related mathematics

Question 1a.–1c.

Marks	0	1	2	3	Average
%	18	16	24	42	1.9

1a.

$8.00

Let x = the amount before 10% GST is added

$$x + \text{GST} = 88$$
$$x + 10\% \text{ of } x = 88$$
$$x + 0.1x = 88$$
$$1.1x = 88$$
$$x = 88 \div 1.1$$
$$x = 80$$

Then

80 + GST = 88

GST = 88 – 80 = 8

A common incorrect answer was 10% of 88 = $8.80

1b.

$352

1c.

$99

Questions 2a.–3b.

Marks	0	1	2	3	4	5	Average
%	13	17	18	18	19	16	2.6

2a.

$$\frac{3800 - 3150}{2} = 325$$

The 325 should not have been substituted into a calculation. It needed to be the result of a calculation.

2b.

10 years

Let t = *number of years*

Solve: 3800 – 325t = 550

t = 10

2c.

13.11%

Solve: $1040 = 2100(1-r)^5$

r = 0.13114…

3a.

$12 500

$$P = \frac{460 \times 100}{3.68} = 12\ 500$$

3b.

It will last forever.

Many students did not know that perpetuities pay out only the interest earned, while the principal remains unchanged.

The most common incorrect answer was $\frac{12\,500}{460} \approx 27 \text{ years}$

Questions 4a.–5b.

Marks	0	1	2	3	4	5	Average
%	41	14	12	12	13	7	1.7

4a.

$4072.54

$$A = 4000\left(1 + \frac{3.6}{1200}\right)^6 = 4072.542...$$

Answers rounded to $4072.55 or $4072.50 were not accepted.

4b.

4 × 0.8% = 3.2%

4c.

$6664.63

N = 20
I = 3.2
PV = –2000
PMT = –200
FV = 6664.629…
P/Y = 4
C/Y = 4

Answers of $6664.60 or $6664.65 were not accepted.

5a.

5.91%

N = 144
I = 5.9100…
PV = 50 000
PMT = –485.60
FV = 0
P/Y = 12
C/Y = 12

5b.

$2951

N = 12
I = 5.91
PV = 50 000
PMT = –485.60
FV = –47 048.7077…
P/Y = 12
C/Y = 12
Paid off = 50 000 – 47 048.71
= 2951.29

Students who gave an incorrect answer to Question 5a. may have qualified for a consequential error mark for this question if they included their technology finance solver input.

A number of students found the future value after one year but did not go on to find the amount paid off.

Question 5c.

Marks	0	1	2	Average
%	76	7	17	0.4

10 months

After six years:

N = 72
I = 5.91
PV = 50 000
PMT = –485.60
FV = –29 376.045…
P/Y = 12
C/Y = 12

Deduct $3500 payment from FV and use result as new PV.

New PV = 29 376.05 – 3500.00 = 25 876.05

Next find N needed to get FV = 0

N = 61.960…

I = 5.91
PV = 25 876.05
PMT = –485.60
FV = 0
P/Y = 12
C/Y = 12

Few students attempted this question. Students who partially completed this question may have qualified for a method mark if they had included their technology finance solver input.

Module 5: Networks and decision mathematics

Questions 1a.–1c.

Marks	0	1	2	3	Average
%	0	2	23	74	2.7

1a.

$300

1b.

$920

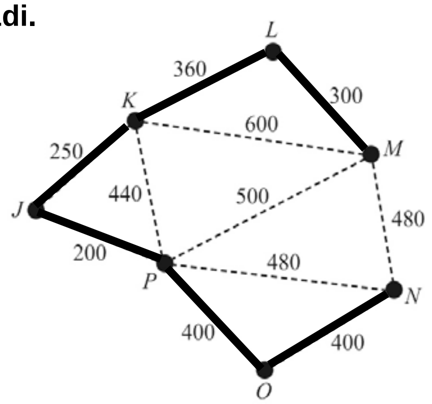
1c.

N and *P* (or *P* and *N*)

Questions 1di.–2b.

Marks	0	1	2	3	4	5	Average
%	10	15	17	20	18	19	2.8

1di.



Many students were unable to find this minimum spanning tree.

Common incorrect trees excluded *PO* or *KL* instead of *MN*.

Some students drew complete circuits.

1dii.

$120

Disconnecting *J-P* and *O-P* will reduce cost by $600 but must then add in *N-M* to add $480 back in.

480 – 600 = –120

2ai.

S - *Q* - *R* - *S* - *U*

2aii.

Town S is passed through twice.

2b.

162 km

Sum all the distances for the Hamiltonian circuit: Factory – *T* – *S* – *R* – *Q* – *U* – Factory

Question 3a.–3fii.

Marks	0	1	2	3	4	5	6	7	Average
%	10	15	12	12	18	14	13	6	3.3

3a.

3 minutes

3b.

4 minutes

Common incorrect answers were 12 and 13 minutes.

3c.

3 minutes

3d.

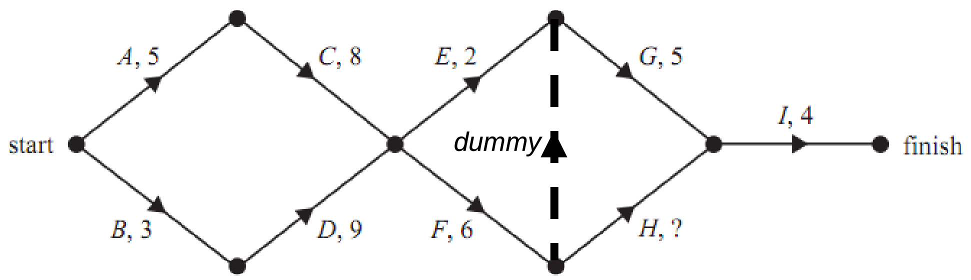
A - *C* - *F* - *H* - *I*

3e.

The critical path is increased by 7 minutes to become 33 minutes.

It was insufficient to simply write an answer of ‘7 minutes’ or ‘33 minutes’. Reference to an ‘increase’ or ‘new critical path’ was expected.

3fi.



The dummy activity needed to indicate that *F* has become a prerequisite for *G*. This required an arrow on the line. Then, this connection needed to be identified as having a duration of zero or be labelled as *dummy*.

3fii.

The critical path is increased by 2 minutes to become 28 minutes.

A number of students incorrectly stated that 'there is no effect since a dummy takes zero time', while others stated that 'there is no effect since *G* is not on the critical path'.

Module 6: Matrices

Questions 1a.–1cii.

Marks	0	1	2	3	4	5	Average
%	0	4	4	9	19	63	4.3

1a.

120 students

1bi.

$$Q = S_0P = \begin{bmatrix} 5 & 10 & 3 & 2 \\ 15 & 30 & 9 & 6 \\ 10 & 20 & 6 & 4 \end{bmatrix}$$

1bii.

30 intermediate-level students

A common incorrect answer was 60 students.

1ci.

$C \times Q$

Also accepted was the matrix calculation written as:

$$\begin{bmatrix} 15 & 25 & 40 \end{bmatrix} \times \begin{bmatrix} 5 & 10 & 3 & 2 \\ 15 & 30 & 9 & 6 \\ 10 & 20 & 6 & 4 \end{bmatrix}$$

Some students incorrectly gave $Q \times C$.

1cii.

$340

Questions 2a.–3a.

Marks	0	1	2	3	4	5	Average
%	9	7	10	16	25	33	3.4

2a.

All of the advanced-level students stay as advanced-level students.

2bi.

$$S_1 = \begin{bmatrix} 10 \\ 58 \\ 52 \end{bmatrix}$$

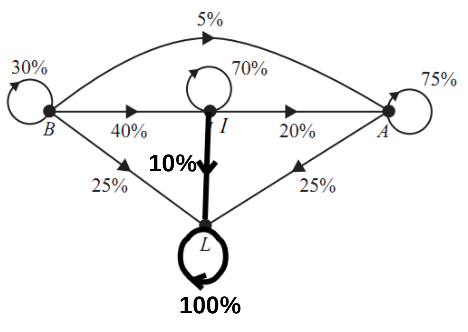
2bii.

12 intermediate-level students

0.20 × 60 = 12

A common incorrect answer was 2.

3a.



One mark was available for a transition from *I* to *L*, including the arrow and the 10% label. Another mark was available for the loop at *L*, including the 100% label.

A number of students missed the loop entirely, while others incorrectly labelled it as 1%. Some students incorrectly added a second 5% directed transition from *B* to *A.* Others unnecessarily added a lot of 0% transitions in the reverse directions for all shown transitions.

Questions 3b.–3e.

Marks	0	1	2	3	4	5	Average
%	36	17	18	14	7	8	1.6

3b.

17.5%

0.25 × 20 + 0.1 × 60+ 0.25 × 40 = 21 students left

$\frac{21}{120} = 0.175 = 17.5\%$

Many students did not convert 21 students into the required percentage.

3c.

43 advanced-level students

$$S_2 = \begin{bmatrix} 0.30 & 0 & 0 & 0 \\ 0.40 & 0.70 & 0 & 0 \\ 0.05 & 0.20 & 0.75 & 0 \\ 0.25 & 0.10 & 0.25 & 1 \end{bmatrix}^2 \begin{bmatrix} 20 \\ 60 \\ 40 \\ 0 \end{bmatrix} = \begin{bmatrix} 1.8 \\ 37.4 \\ 42.55 \\ 38.25 \end{bmatrix}$$

Some students rounded 42.55 to 42 despite the question instruction, ‘Write your answer correct to the nearest whole number’.

3d.

5 assessments

$$S_4 = (T_2)^4 \times R_0 \approx 54 > 50$$

$$S_5 = (T_2)^5 \times R_0 \approx 43 < 50$$

3e.

7 intermediate-level students

$$R_1 = \begin{bmatrix} 0.30 & 0 & 0 & 0 \\ 0.40 & 0.70 & 0 & 0 \\ 0.05 & 0.20 & 0.75 & 0 \\ 0.25 & 0.10 & 0.25 & 1 \end{bmatrix} \times \begin{bmatrix} 20 \\ 60 \\ 40 \\ 0 \end{bmatrix} + \begin{bmatrix} 4 \\ 2 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 10 \\ 52 \\ 46 \\ 21 \end{bmatrix}$$

$$R_2 = \begin{bmatrix} 0.30 & 0 & 0 & 0 \\ 0.40 & 0.70 & 0 & 0 \\ 0.05 & 0.20 & 0.75 & 0 \\ 0.25 & 0.10 & 0.25 & 1 \end{bmatrix} \times \begin{bmatrix} 10 \\ 52 \\ 46 \\ 21 \end{bmatrix} + \begin{bmatrix} 4 \\ 2 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 7 \\ 42.4 \\ 48.4 \\ 40.2 \end{bmatrix}$$

$$R_3 = \begin{bmatrix} 0.30 & 0 & 0 & 0 \\ 0.40 & 0.70 & 0 & 0 \\ 0.05 & 0.20 & 0.75 & 0 \\ 0.25 & 0.10 & 0.25 & 1 \end{bmatrix} \times \begin{bmatrix} 7 \\ 42.4 \\ 48.4 \\ 40.2 \end{bmatrix} + \begin{bmatrix} 4 \\ 2 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 6.1 \\ 34.48 \\ 48.13 \\ 58.29 \end{bmatrix}$$

20% of 34.48 intermediate students = 6.896

Students were not expected to write out all of these matrix calculations to qualify for a method mark if their final answer was incorrect. The progression in these state matrices could be shown by using the correct equation to write the matrices R_1, R_2 and R_3.

Of those who attempted this question, many inappropriately used $R_3 = (T_2)^3 R_0 + V$, whereas $R_3 = TR_2 + V$ needed to be used instead.

Some students found and interpreted R_4 instead of R_3.