2016 VCE Further Maths 2 examination report

General comments

The 2016 Further Mathematics 2 written examination was the first for the revised Further Mathematics study design. Students were required to complete:

- a compulsory Core section of Data Analysis (worth 24 marks)
- a compulsory Core section of Recursion and financial modelling (worth 12 marks)
- two modules (worth 12 marks each).

The selection of modules by students in 2016 is shown in the table below.

Module	% 2016
Matrices	87
Networks and decision mathematics	51
Geometry and measurement	31
Graphs and relations	30

Most students were able to start Data analysis and Financial modelling in the Core section and their two chosen modules well. Throughout each section, questions became progressively more challenging.

Students are expected to be familiar with the formula sheet included with the examination.

There was evidence that some students ran out of time. Students should ensure that they plan their work in order to complete the entire examination in the allotted time.

Students were asked to write their solutions and answers in blue or black pen. Scanned images are used for assessing and students should ensure their responses can be clearly read.

Some responses were unreasonable in the context of the question. For example, in Question 6c. of the Core section, it was unreasonable for the value of a caravan to decrease by $5000 for every kilometre that it travelled; yet this response was very occasionally given. Students are urged to consider their answers within the context of the question where possible. If the answer seems unreasonable, then an error has been made and this error may be found and corrected if time permits.

Where two or more marks were available for a question, a response that showed only a single, incorrect answer gained no marks. If the student had demonstrated an understanding of the mathematics required by the question, a method mark may have been available.

Some questions required the further application of a previous answer. If the previous answer was incorrect, the student may have been eligible for consequential error consideration. For this to apply, working out needed to show a correct substitution of the previous reasonable, but incorrect, answer into a relevant calculation. The resulting answer then needed to match that substitution and be a reasonable answer.

Rounding was required in a number of questions, including rounding to a specified number of significant figures. Students were expected to follow all rounding instructions given on the examination.

Points that teachers and students could usefully address include:

- effective use of reading time
- rounding
- breaking complex questions into small steps
- setting out the solution of a multi-step calculation
- appropriate use of technology, including the financial solver
- estimating answers where possible to exclude calculation results that are not reasonable
- a glossary of relevant terms in the student's bound reference.

Specific information

This report provides sample answers or an indication of what answers may have included. Unless otherwise stated, these are not intended to be exemplary or complete responses.

The statistics in this report may be subject to rounding resulting in a total more or less than 100 per cent.

Core – Data analysis

Question 1ai.

Marks	0	1	Average
%	37	63	0.7

17.8 mm

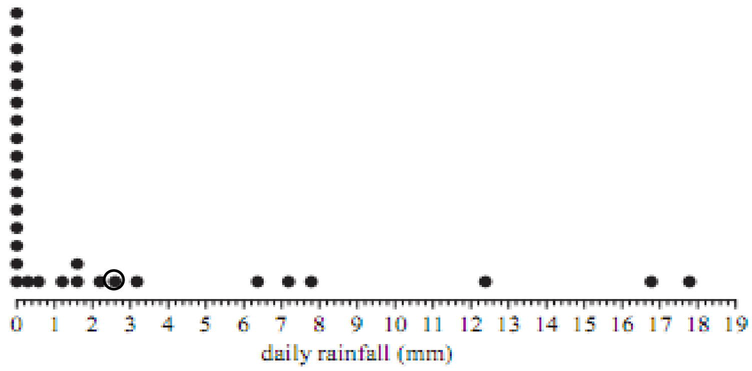
Question 1aii.

Marks	0	1	Average
%	10	90	0.9

0 mm

Question 1b.

Marks	0	1	Average
%	36	64	0.7



The correct point is circled at daily rainfall = 2.6 mm

Some students did not answer this question.

Question 1ci.

Marks	0	1	Average
%	13	87	0.9

16 days

Question 1cii.

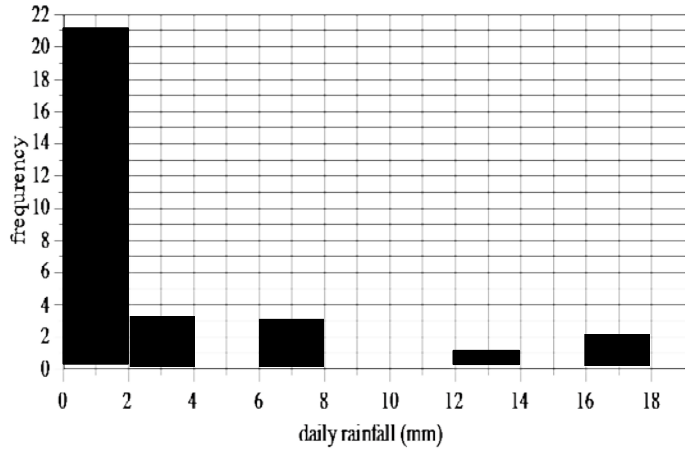
Marks	0	1	Average
%	14	86	0.9

10%

$$\frac{3}{30} = 0.1 = 10\%$$

Question 1d.

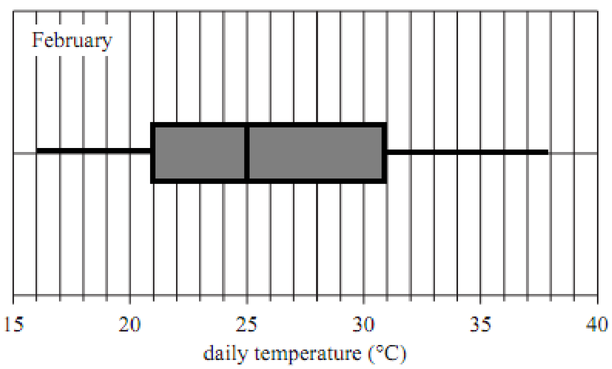
Marks	0	1	2	Average
%	41	7	52	1.1



Many students inappropriately drew columns with interval widths of only one.

Question 2ai.

Marks	0	1	Average
%	4	96	1



Question 2aii.

Marks	0	1	Average
%	29	71	0.7

75%

A common incorrect answer was 25%.

Question 2bi.

Marks	0	1	Average
%	44	56	0.6

July: positively skewed with an outlier

May: symmetric

Common unacceptable answers for July included symmetrically skewed, evenly distributed, bell shaped and normally distributed.

Question 2bii.

Marks	0	1	Average
%	40	60	0.6

15.5 °C

$11 + 1.5 \times 3 = 15.5$

Question 2biii.

Marks	0	1	Average
%	70	30	0.3

The medians for the two months differ. In May, the median maximum temperature is about 14.5 °C, while in July, the median maximum temperature is about 9 °C.

The answer needed to refer to the difference between the two median temperatures. Simply quoting the two median values was not sufficient. Accuracy in reading the scales was an issue for some students.

Alternatively, comparing the two interquartile range (IQR) values could have been used as the difference in the IQRs also indicates the presence of an association.

It appeared that some students confused ‘maximum daily temperature’ with the maximum of the boxplot.

Some students referred to average or mean temperatures; however, this cannot be accurately determined from a boxplot unless the distributions are clearly symmetric.

Question 3a.

Marks	0	1	Average
%	22	78	0.8

Strong, linear, positive

A relatively common incorrect response was ‘Strong, linear and positively skewed’.

Question 3bi.

Marks	0	1	2	3	Average
%	27	15	30	28	1.6

apparent temperature = –1.7 + 0.94 × *actual temperature*

Regression analysis by technology, and not the use of formulas, was required to answer this question.

The first column contained the response variable rather than the explanatory variable. Students who did not notice this gave their answer as (2.4, 1.0).

There was much evidence of confusion between decimal point rounding and significant figure rounding. Many students rounded to two decimal places rather than two significant figures as required. The number 0.9 has only one significant figure, whereas 0.90 has two significant figures.

Question 3bii.

Marks	0	1	Average
%	72	28	0.3

On average, when the actual temperature is 0 °C, the apparent temperature is –1.7 °C.

Another accepted answer was: When the actual temperature is 0 °C, the predicted apparent temperature is –1.7 °C.

A common incorrect answer confused intercept and slope. Another common incorrect answer mixed up the response variable (apparent temperature) and the explanatory variable (actual temperature).

Question 3c.

Marks	0	1	Average
%	51	49	0.5

97% of the variation in apparent temperature can be explained by the variation in actual temperature.

Some students mixed up the response variable and the explanatory variable in this statement and wrote, ‘97% of the variation in actual temperature can be explained by the variation in apparent temperature’.

Other common unacceptable answers used terms that suggested variation in actual temperature caused the variation in apparent temperature. An example was, ‘97% of the variation in actual temperature was due to the variation in apparent temperature’.

Question 3di.

Marks	0	1	Average
%	54	46	0.5

That the association is linear

Question 3dii.

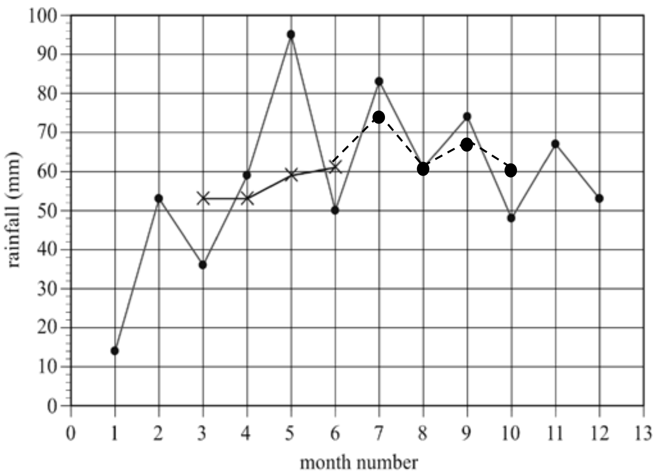
Marks	0	1	Average
%	54	46	0.5

Yes, since there is no clear pattern in the residual plot.

Reference to a lack of pattern or the randomness of the plot was required. An unacceptable answer was that the '… points are all scattered evenly above and below …' with no mention of randomness.

Question 4a.

Marks	0	1	2	Average
%	37	28	35	1



The correct five-median points are shown as dots connected by dotted lines above.

Median smoothing is a graphical technique and requires some accuracy in the correct placement of crosses or dots. Reading the values from points on the graph is unlikely to produce accurate enough placement of points and should be discouraged.

Many students did not answer this question.

Question 4b.

Marks	0	1	2	Average
%	49	3	48	1

$$M_1 = \frac{124+140}{2} = 132 \text{ and } M_2 = \frac{140+225}{2} = 182.5$$

$$\therefore \frac{132+182.5}{2} = 157.25 \text{ as required}$$

The data needed for these calculations should have been taken from the table rather than from the graph.

Many students did not answer this question.

Core – Recursion and financial modelling

Question 5a.

Marks	0	1	Average
%	8	92	0.9

$15 000

Question 5b.

Marks	0	1	Average
%	27	73	0.8

$V_0 = 15\ 000$
$V_1 = 1.04 \times 15\ 000 = 15\ 600$
$\therefore \quad V_2 = 1.04 \times 15\ 600 = 16\ 224$

'Using recursion' begins with writing the initial value (i.e. $V_0 = 15\ 000$)

Then the first calculation using the recurrence rule must be shown and the answer labelled.
($V_1 = 1.04 \times 15\ 000 = 15\ 600$ in this case)

Subsequent calculation(s) must then show the use of the prior answer to calculate the next value.
($V_2 = 1.04 \times 15\ 600 = 16\ 224$ in this case)

Some students used only the explicit rule $V_n = R^n V_0$ to write $V_2 = 1.04^2 \times 15\ 600 = 16\ 224$, but this was not using recursion as required.

Question 5c.

Marks	0	1	Average
%	35	65	0.7

4%

Common incorrect answers were 1.04 or 0.04%.

Question 5di.

Marks	0	1	Average
%	39	61	0.6

$V_n = 1.04^n \times 15\ 000$

Question 5dii.

Marks	0	1	Average
%	53	47	0.5

$22.203.66

$1.04^{10} \times 15\,000 = 22\,203.664\ldots$

Currency answers rounded to the nearest cent needed to show two decimal places.

Many students only wrote an answer of $22 203.7, which might arise from a calculator setting that restricts the number of displayed digits.

Some students appeared to round all currency answers to the nearest five or ten cents. A common incorrect answer was $22 203.70, which had clearly been rounded from a written $22 203.664... This was assessed as a rounding error.

Question 6a.

Marks	0	1	Average
%	61	39	0.4

$$\frac{38000-16000}{8}=2750$$

'Show that' questions give the answer to a basic and appropriate calculation, and students must write that calculation. The given number in a 'show that' question is sometimes needed in a following question. With this number, students can attempt the following question, even if they cannot complete the 'show that' question.

The given number in a 'show that' question is not to be used in a calculation; it must be the result of a calculation. The following calculations use the 2750 to show a different result than what is required and were not acceptable for this question:

- 38 000 – 8 × 2750 = 16 000 – shows how to find the depreciated value after eight years
- 16 000 + 8 × 2750 = 38 000 – shows how to find the initial value

Question 6b.

Marks	0	1	Average
%	67	33	0.4

$C_0 = 38\,000$, $C_{n+1} = C_n - 2750$

A recurrence relation has the initial value written first, followed by the recurrence rule.

Common errors included:

- failure to include the initial value, C_0
- writing the initial value as C_n, not C_0
- using different symbols for different parts of the recurrence relation, e.g. $V_0 = 38\,000$, $C_{n+1} = C_n - 2750$
- using a rule for the *n*th term $C_n = 38000 - 2750n$ instead of the recurrence rule.

Question 6c.

Marks	0	1	Average
%	71	29	0.3

$0.55 per km

$$\frac{38000-16000}{5000\times 8}=0.55$$

A very common incorrect answer of $4.40 was found by ignoring the eight years over which the depreciation occurred.

Question 7ai.

Marks	0	1	Average
%	54	46	0.5

$65 076.22

N = 12
I% = 6.9
PV = 70 000
PMT = –800
FV = –65 076.219…
P/Y = C/Y= 12

Rather than use a financial solver to answer the question above, a number of students adopted a formulaic approach, almost always unsuccessfully, based on the compound interest formula.

Question 7aii.

Marks	0	1	Average
%	74	26	0.3

$4676.22

Repayments – reduction in the principle
= 12 × 800 – (70 000 – 65 076.22)
= 9600 – 4923.78
= 4676.22

A common incorrect answer was $4923.78, which is the reduction in the principal over the year. This failed to take into account the $9600 total of repayments made in the year.

Question 7b.

Marks	0	1	2	Average
%	86	6	8	**0.2**

$28 204

N = 36
I% = 6.9
PV = 70 000
PMT = –800
FV = –54 151.599…
P/Y = C/Y = 12

$\therefore$ Balance after 3 years = $54 151.60
Then find the PV needed to repay a "new" loan over 3 years at 6.9% per annum and $800 per month.

N = 36
I% = 6.9
PV = 25 947.576…
PMT = –800
FV = 0
P/Y = C/Y = 12

Finally, lump sum payment needed

= 3yr balance − PV needed
= 54 151.60 – 25 947.58

= 28 204.02

The stages to solving this multi-step question were:
1. Find the actual balance, or FV, after the first 3 years.
2. Find the balance needed now (PV) so that FV = 0 in 3 years from now at 6.9% per annum.
3. Lump sum needed after first 3 years
= Balance needed (stage 2) – Actual balance achieved (stage 1)

Many students stopped after stage 1, finding only the actual balance of $54 151.60 after 3 years.

Correct tables of input values for the financial solver may have illustrated working out to qualify for a method mark even if the final answer was incorrect.

Module 1 – Matrices

Question 1a.

Marks	0	1	Average
%	9	91	**0.9**

4×1

A common incorrect answer was 1×4.

An answer of (4, 1) was not accepted.

Question 1bi.

Marks	0	1	Average
%	21	79	**0.8**

[6000]

The product of two matrices produces a matrix. The brackets needed to be included.

Question 1bii.

Marks	0	1	Average
%	58	42	**0.4**

Total booking fees collected for the month

Question 2a.

Marks	0	1	Average
%	11	89	**0.9**

Ben and Elka

Question 2b.

Marks	0	1	Average
%	29	71	**0.7**

Amara and Dana

Question 3a.

Marks	0	1	Average
%	15	85	**0.9**

$d = 298$, $e = 94$, $f = 130$

Question 3b.

Marks	0	1	Average
%	65	35	**0.4**

$0.65 \times 520 + 0.25 \times 320 + 0.25 \times 80 + 0.5 \times 80 = 478$ customers

A matrix product such as TS_0 does not show an understanding of how 478 is calculated.

Question 3c.

Marks	0	1	Average
%	55	45	**0.5**

20 customers

80 chose sea transport in 2014
For 2015, the sea transport number is $25\% \times 80 = 20$

A common incorrect answer was 94.

Question 3d.

Marks	0	1	Average
%	83	17	0.2

71%

478 customers were expected to choose air travel in 2015.

65% of 520 who chose air travel in 2014 also chose air travel in 2015

i.e. 338 customers.

Required percentage $= \frac{338}{478} \times \frac{100}{1} = 70.71...$

Common incorrect answers were 65% and 94%.

Question 3ei.

Marks	0	1	Average
%	86	14	0.2

80 customers who have no bookings/no travel in that year will be removed from the study

Many students were unable to explain the –80 in matrix *B*.

The answer needed to refer to the removal from the study of 80 people selected from the no travel group (*N*) in 2017 and in 2018.

A common unacceptable response was, ‘80 people chose not to travel each year’.

Question 3eii.

Marks	0	1	2	Average
%	46	9	46	1

190

The values in R_{2017} and R_{2018}, below, have been rounded to 2dp.

$$R_{2017} = \begin{bmatrix} 0.65 & 0.25 & 0.25 & 0.50 \\ 0.15 & 0.60 & 0.20 & 0.15 \\ 0.05 & 0.10 & 0.25 & 0.20 \\ 0.15 & 0.05 & 0.30 & 0.15 \end{bmatrix} \begin{bmatrix} 646 \\ 465 \\ 164 \\ 85 \end{bmatrix} + \begin{bmatrix} 80 \\ 80 \\ 40 \\ -80 \end{bmatrix} = \begin{bmatrix} 699.65 \\ 501.45 \\ 176.80 \\ 102.10 \end{bmatrix}$$

Therefore:

$$R_{2018} = \begin{bmatrix} 0.65 & 0.25 & 0.25 & 0.50 \\ 0.15 & 0.60 & 0.20 & 0.15 \\ 0.05 & 0.10 & 0.25 & 0.20 \\ 0.15 & 0.05 & 0.30 & 0.15 \end{bmatrix} \begin{bmatrix} 699.65 \\ 501.45 \\ 176.80 \\ 102.10 \end{bmatrix} + \begin{bmatrix} 80 \\ 80 \\ 40 \\ -80 \end{bmatrix} = \begin{bmatrix} 755.39 \\ 536.49 \\ \boxed{189.75} \\ 118.38 \end{bmatrix}$$

Many students found the correct matrix R_{2018} but did not extract the answer 190 customers who chose sea travel.

Some students only found R_{2017}, and some of these students simply called it R_{2018} instead.

A method mark for this two-mark question may have been earned for a correct and labelled matrix R_{2017}, even if the final answer was incorrect.

Module 2 – Networks and decision mathematics

Question 1a.

Marks	0	1	Average
%	17	83	0.9

Alooma and Easyside

Question 1bi.

Marks	0	1	Average
%	47	53	0.6

Draw a third edge joining *E* and *D*

Many students did not answer question.

Question 1bii.

Marks	0	1	Average
%	70	30	0.3

The driver can return to Dovenest without going through any other suburb.

Question 2a.

Marks	0	1	Average
%	28	72	0.7

Ramp *V*

Question 2b.

Marks	0	1	Average
%	23	77	0.8

Either:

- *X – Y – T – U – Z – V – W*
- *X – Y – T – U – Z – W – V*.

Question 2c.

Marks	0	1	Average
%	69	31	0.3

Four ways:

- *X – Y – T – U – Z – V – W – X*
- *X – W – V – Z – U – T – Y – X*
- *X – Y – T – U – V – Z – W – X*
- *X – W – Z – V – U – T – Y – X*

Each Hamiltonian cycle can be reversed.

Question 3a.

Marks	0	1	Average
%	39	61	0.6

11 days

Activity *M* cannot be started until path *C – G – J* is completed.

Question 3b.

Marks	0	1	Average
%	24	76	0.8

A – E – I – K

Question 3c.

Marks	0	1	Average
%	63	37	0.4

Activity *H*

A common incorrect answer was activity *M*.

Question 3d.

Marks	0	1	Average
%	79	21	0.2

$2000

Can reduce either activity *E* or *I* by one day, but crashing *I* is cheapest at $2000.

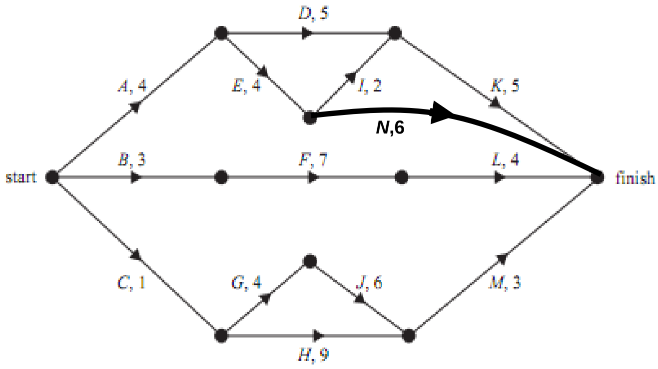
This makes the duration of *A – E – I – K* = 14 days.

This is same as the duration of *A – D – K*, which cannot be crashed. Therefore, there is no point in crashing activity *I* any further.

The cheapest and shortest duration possible is 14 days at a cost of $2000 for crashing activity *I* by one day.

Question 3ei.

Marks	0	1	Average
%	79	21	0.2



This was a directed graph and an arrow needed to be included on the line for activity *N*.

Some students did not answer this question.

Question 3eii.

Marks	0	1	Average
%	73	27	0.3

9 days

Module 3 – Geometry and measurement

Question 1a.

Marks	0	1	Average
%	26	74	0.8

5755 mm^2

$4\pi \times 21.4^2 = 5754.895\ldots$

Question 1b.

Marks	0	1	Average
%	18	82	0.8

214 mm

$5 \times 2 \times 21.4 = 214$

Some students did not check whether their answer was reasonable. Given that there were five golf balls, any length greater than two metres could not be correct.

Question 2a.

Marks	0	1	Average
%	22	78	0.8

25 m

$50 \times \sin(30°) = 25$

Question 2b.

Marks	0	1	Average
%	34	66	0.7

5°

$$\tan^{-1}\left(\frac{16.8}{200}\right) = 4.8016...$$

Question 3a.

Marks	0	1	Average
%	58	42	0.4

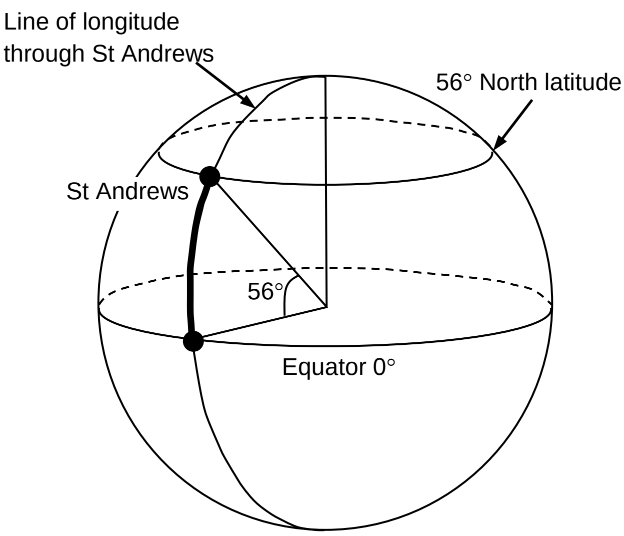
6255 km

The angle between the equator and St Andrews is 56°.

Distance to equator

$$= \frac{\theta}{360} \times 2\pi r$$
$$= \frac{56}{360} \times 2\pi \times 6400$$
$$= 6255.2...$$

This question was not answered well.



The distance to the equator from any point on Earth is found by using the angle between the line of latitude through the point and the line of latitude 0° called the equator. In this question, the line through St Andrews was given as 56° north of the equator.

The distance from St Andrews to the equator is the same distance as for every point on the same line of latitude (56° north). The distance to be calculated is along an arc with an angle of 56° and radius of 6400 km.

The figures for longitude (east or west) were not relevant to this question.

Question 3b.

Marks	0	1	Average
%	63	37	0.4

4.32 pm on Thursday

06:32 + 10 hours = 16:32 Thursday

or

6.32 am + 10 hours = 4.32 pm on the same day

Many students ignored the given time difference of 10 hours between these two cities. They should have ignored the longitudes given for the two cities instead.

The 10-hour ‘time of day’ difference is based on the time zones of the two cities. There is also the same 10-hour time difference between St Andrews and each of Geelong, Mallacoota, Mildura, Sydney, etc. since these are all in the same time zone as Melbourne.

The 10-hour ‘time of day’ difference in this question meant that the time zone of the eastern city of this pair (Melbourne in this case) is 10 hours **ahead** of the time zone of the other city (St Andrews). Hence, 10 hours had to be added to the time in St Andrews to get the simultaneous ‘time of day’ in Melbourne.

Because Earth rotates 360° approximately every 24 hours, it takes about one hour for Earth to turn by 15° of longitude. Time zones are based on this 15° difference in longitude and all locations within the same time zone have the same simultaneous ‘time of day’.

Melbourne and Mallacoota are within the same time zone and are about 5° of longitude apart. While there is no ‘time of day’ difference, there is a ‘solar time’ difference of about a third of one hour (20 minutes) between these two places.

The ‘solar time’ difference between two places on Earth is based on the sun’s relative position in the sky at those two places. With Mallacoota located about 5° **east** of Melbourne, and on about the same latitude, Mallacoota will see sunrise about 20 minutes **ahead** of Melbourne.

Question 4a.

Marks	0	1	2	Average
%	34	10	56	1.2

$a^2 = 80^2 + 100^2 - 2 \times 80 \times 100 \times \cos\left(104^\circ\right)$

$\therefore a = 142.3...$

$\therefore a \approx 142m$

This question required the use of the cosine rule to find the required length. A correct substitution into the cosine rule was required.

Question 4b.

Marks	0	1	Average
%	79	21	0.2

087°

$$\frac{SinT}{100} = \frac{Sin(104)^{\circ}}{142}$$

$\therefore T = 43.1...^{\circ}$

$\therefore$ Bearing of *R* from *P* = 130 – 43 = 087°

A three-figure bearing was required as shown.
Some students found the angle *PRQ*, which did not readily help to give the required bearing.

Question 5a.

Marks	0	1	Average
%	52	48	0.5

13 metres

$$147.5 = \frac{100}{360} \times \pi \times r^2$$

$$\therefore r = \sqrt{\frac{360 \times 147.5}{100 \times \pi}} = 13.0...$$

Common incorrect answers came from finding the correct fraction of an incorrect circumference of a circle. This was usually due to an incorrect formula for the circumference.

Question 5b.

Marks	0	1	2	Average
%	75	2	22	0.5

199 m^2

The shaded section is a large sector minus a smaller sector (both with angles of 260° at the centre)

$$\text{Large sector area} = \frac{260}{360} \times \pi \times 12^2 = 326.7256...$$

$$\text{Small sector area} = \frac{260}{360} \times \pi \times 7.5^2 = 127.6272...$$

$$\therefore \quad 326.7256... - 127.6272... = 199.09...$$

This question was not answered well.

To be considered for a possible method mark even if the answer was incorrect, setting out a multi-step solution should have clearly shown the several steps and calculations.

Common incorrect answers were found by students who calculated the difference between the areas of:

- two sectors, each with an angle at the centre of 100°
- a circle of radius 12 m and a sector or radius of 12 m and with an angle of 100° at the centre
- two circles of radii 12 m and 7.5 m.

Module 4 – Graphs and relations

Question 1a.

Marks	0	1	Average
%	9	91	0.9

$1500

Question 1b.

Marks	0	1	Average
%	10	90	0.9

15 goals

Question 1c.

Marks	0	1	Average
%	14	86	0.9

$1000

$8 \times 125 = 1000$

Question 1d.

Marks	0	1	Average
%	43	57	0.6

28 goals

Question 2a.

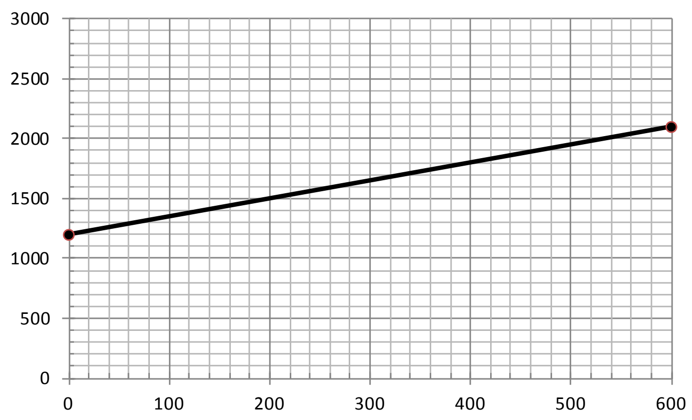
Marks	0	1	Average
%	9	91	0.9

300 balls

$1650 = 1200 + 1.5n$
$\therefore\ n = 300$

Question 2b.

Marks	0	1	Average
%	42	58	0.6



Question 2c.

Marks	0	1	Average
%	47	53	0.6

$7.50 selling price

Cost of 200 balls = $1500

$$\therefore \text{sell price} = \frac{1500}{200} = 7.5$$

Students should consider their answers in the context of the question. This might avoid unreasonable answers, such as $1000 to buy a hockey ball.

Question 3a.

Marks	0	1	Average
%	75	25	0.3

The number of Jink sticks produced each month must be no more than twice the number of Flick sticks produced in that month.

Reference to an inequality was expected, such as ‘no more than’.

This question was not well answered, often with Flick and Jinks reversed in the sentences.

Question 3b.

Marks	0	1	Average
%	42	58	0.6

$y \leq 300$

The expected answer is an inequality.

Some students wrote the relationship as the equality $y = x$.

Question 3c.

Marks	0	1	Average
%	42	58	0.6

$38 200 maximum profit

Maximum profit at (200, 300)

$P = 200 \times 62 + 300 \times 86 = 38\,200$

Question 3d.

Marks	0	1	2	Average
%	93	0	7	0.2

$m = n = 84$

This question assessed the sliding line concept but most students were unable to find a solution. Many relied on trial and error methods, with very little success.

The profit equation is now $Q = mx + ny$

Maximum profit occurs at $x = 400$, $y = 100$

This point lies on the line $x + y = 500$

Hence the maximum profit equation must have the same gradient as $x + y = 500$

This gradient is –1 and so $m = n$

Therefore $Q = 400m + 100n$

can become $Q = 400m + 100m$

i.e. $Q = 500m$

Next substitute the maximum profit = 42 000

Hence $42\,000 = 500m$

$$\therefore m = \frac{42\,000}{500} = 84$$

and $m = 84$ and hence $n = 84$ also.