VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education 2017

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER | | | | | | | | | Letter | |

MATHEMATICAL METHODS

Written examination 1

Wednesday 8 November 2017

Reading time: 9.00 am to 9.15 am (15 minutes)
Writing time: 9.15 am to 10.15 am (1 hour)

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 12 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination
- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (4 marks)

a. Let $f:(-2, \infty) \rightarrow R, f(x)=\dfrac{x}{x+2}$.

Differentiate f with respect to x. 2 marks

b. Let $g(x)=(2-x^3)^3$.

Evaluate $g'(1)$. 2 marks

Question 2 (4 marks)

Let $y = x \log_e(3x)$.

a. Find $\dfrac{dy}{dx}$. 2 marks

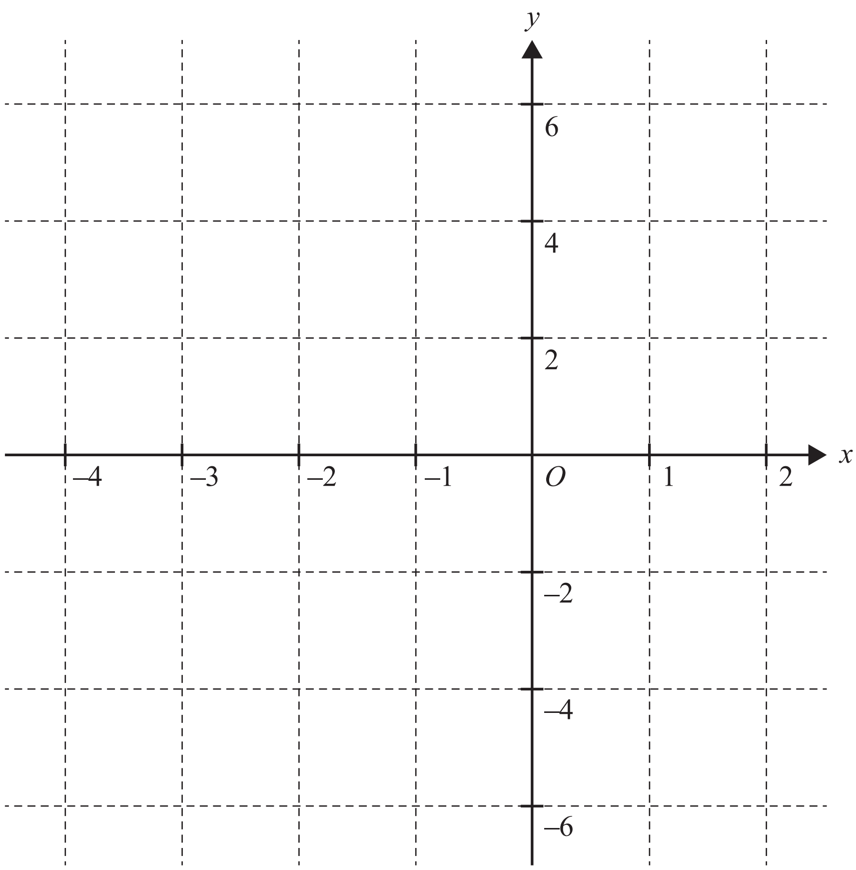
b. Hence, calculate $\int_1^2 (\log_e(3x) + 1)dx$. Express your answer in the form $\log_e(a)$, where a is a positive integer. 2 marks

Question 3 (4 marks)

Let $f : [-3, 0] \rightarrow R$, $f(x) = (x + 2)^2(x - 1)$.

a. Show that $(x + 2)^2(x - 1) = x^3 + 3x^2 - 4$. 1 mark

b. Sketch the graph of f on the axes below. Label the axis intercepts and any stationary points with their coordinates. 3 marks



Question 4 (2 marks)

In a large population of fish, the proportion of angel fish is $\frac{1}{4}$.

Let $\hat{P}$ be the random variable that represents the sample proportion of angel fish for samples of size n drawn from the population.

Find the smallest integer value of n such that the standard deviation of $\hat{P}$ is less than or equal to $\frac{1}{100}$.

Question 5 (4 marks)

For Jac to log on to a computer successfully, Jac must type the correct password. Unfortunately, Jac has forgotten the password. If Jac types the wrong password, Jac can make another attempt. The probability of success on any attempt is $\frac{2}{5}$. Assume that the result of each attempt is independent of the result of any other attempt. A maximum of three attempts can be made.

a. What is the probability that Jac does not log on to the computer successfully? 1 mark

b. Calculate the probability that Jac logs on to the computer successfully. Express your answer in the form $\frac{a}{b}$, where a and b are positive integers. 1 mark

c. Calculate the probability that Jac logs on to the computer successfully on the second or on the third attempt. Express your answer in the form $\frac{c}{d}$, where c and d are positive integers. 2 marks

Question 6 (3 marks)

Let $\left(\tan(\theta)-1\right)\left(\sin(\theta)-\sqrt{3}\cos(\theta)\right)\left(\sin(\theta)+\sqrt{3}\cos(\theta)\right)=0.$

a. State all possible values of $\tan(\theta)$. 1 mark

b. Hence, find all possible solutions for $\left(\tan(\theta)-1\right)\left(\sin^2(\theta)-3\cos^2(\theta)\right)=0,$ where $0\leq\theta\leq\pi.$ 2 marks

Question 7 (5 marks)

Let $f:[0,\infty)\to R, f(x)=\sqrt{x+1}$.

a. State the range of f. 1 mark

b. Let $g:(-\infty, c]\to R$, $g(x)=x^2+4x+3$, where $c<0$.

i. Find the largest possible value of c such that the range of g is a subset of the domain of f. 2 marks

ii. For the value of c found in **part b.i.**, state the range of $f(g(x))$. 1 mark

c. Let $h:R\to R$, $h(x)=x^2+3$.

State the range of $f(h(x))$. 1 mark

Question 8 (5 marks)

For events A and B from a sample space, $\Pr(A \mid B) = \frac{1}{5}$ and $\Pr(B \mid A) = \frac{1}{4}$. Let $\Pr(A \cap B) = p$.

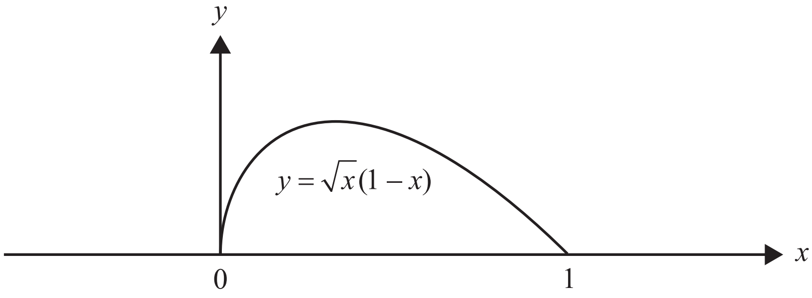
a. Find Pr(A) in terms of p. 1 mark

b. Find $\Pr(A' \cap B')$ in terms of p. 2 marks

c. Given that $\Pr(A \cup B) \leq \frac{1}{5}$, state the largest possible interval for p. 2 marks

Question 9 (9 marks)

The graph of $f:[0,\ 1] \rightarrow R, f(x) = \sqrt{x}(1-x)$ is shown below.



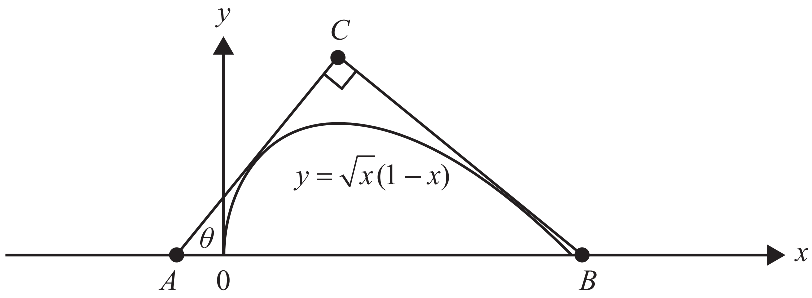
a. Calculate the area between the graph of f and the x-axis. 2 marks

b. For x in the interval (0, 1), show that the gradient of the tangent to the graph of f is $\dfrac{1-3x}{2\sqrt{x}}$. 1 mark

Question 9 – continued
TURN OVER

The edges of the **right-angled** triangle ABC are the line segments AC and BC, which are tangent to the graph of f, and the line segment AB, which is part of the horizontal axis, as shown below.

Let θ be the angle that AC makes with the positive direction of the horizontal axis, where $45° \le \theta < 90°$.



c. Find the equation of the line through B and C in the form $y = mx + c$, for $\theta = 45°$. 2 marks

d. Find the coordinates of C when $\theta = 45°$. 4 marks

END OF QUESTION AND ANSWER BOOK

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MATHEMATICAL METHODS

Written examination 1

FORMULA SHEET

Instructions

This formula sheet is provided for your reference. A question and answer book is provided with this formula sheet.

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Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$		$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, \ n \neq -1$	
$\frac{d}{dx}\left((ax+b)^n\right) = an(ax+b)^{n-1}$		$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, n \neq -1$	
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$		$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$	
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$		$\int \frac{1}{x} dx = \log_e(x) + c, \ x > 0$	
$\frac{d}{dx}\left(\sin(ax)\right) = a \ \cos(ax)$		$\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$	
$\frac{d}{dx}\left(\cos(ax)\right) = -a \ \sin(ax)$		$\int \cos(ax)dx = \frac{1}{a}\sin(ax) + c$	
$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a \ \sec^2(ax)$			
product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$		

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A \mid B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = \mathrm{E}(X)$	variance	$\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

Probability distribution		**Mean**	**Variance**
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x-\mu)^2\, p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$\mathrm{E}(\hat{P}) = p$
standard deviation	$\mathrm{sd}(\hat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$

END OF FORMULA SHEET