VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2017

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

Letter

STUDENT NUMBER

MATHEMATICAL METHODS

Written examination 1

Tuesday 6 June 2017

Reading time: 2.00 pm to 2.15 pm (15 minutes)
Writing time: 2.15 pm to 3.15 pm (1 hour)

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
8	8	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 13 pages.
- Formula sheet.
- Working space is provided throughout the book.

Instructions
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination
- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (4 marks)

a. Let $y = e^{2x}\cos\left(\frac{x}{2}\right)$.

Find $\frac{dy}{dx}$. 2 marks

b. Let $f:(0, \pi) \to R$, where $f(x) = \log_e(\sin(x))$.

Evaluate $f'\left(\frac{\pi}{3}\right)$. 2 marks

Question 2 (5 marks)

a. Find an antiderivative of $\cos(1 - x)$ with respect to x. 1 mark

b. Evaluate $\int_1^2 \left(3x^2 + \frac{4}{x^2}\right) dx$. 2 marks

c. Find $f(x)$ given that $f(4) = 25$ and $f'(x) = \frac{3}{8}x^2 - 10x^{-\frac{1}{2}} + 1, x > 0$. 2 marks

Question 3 (3 marks)

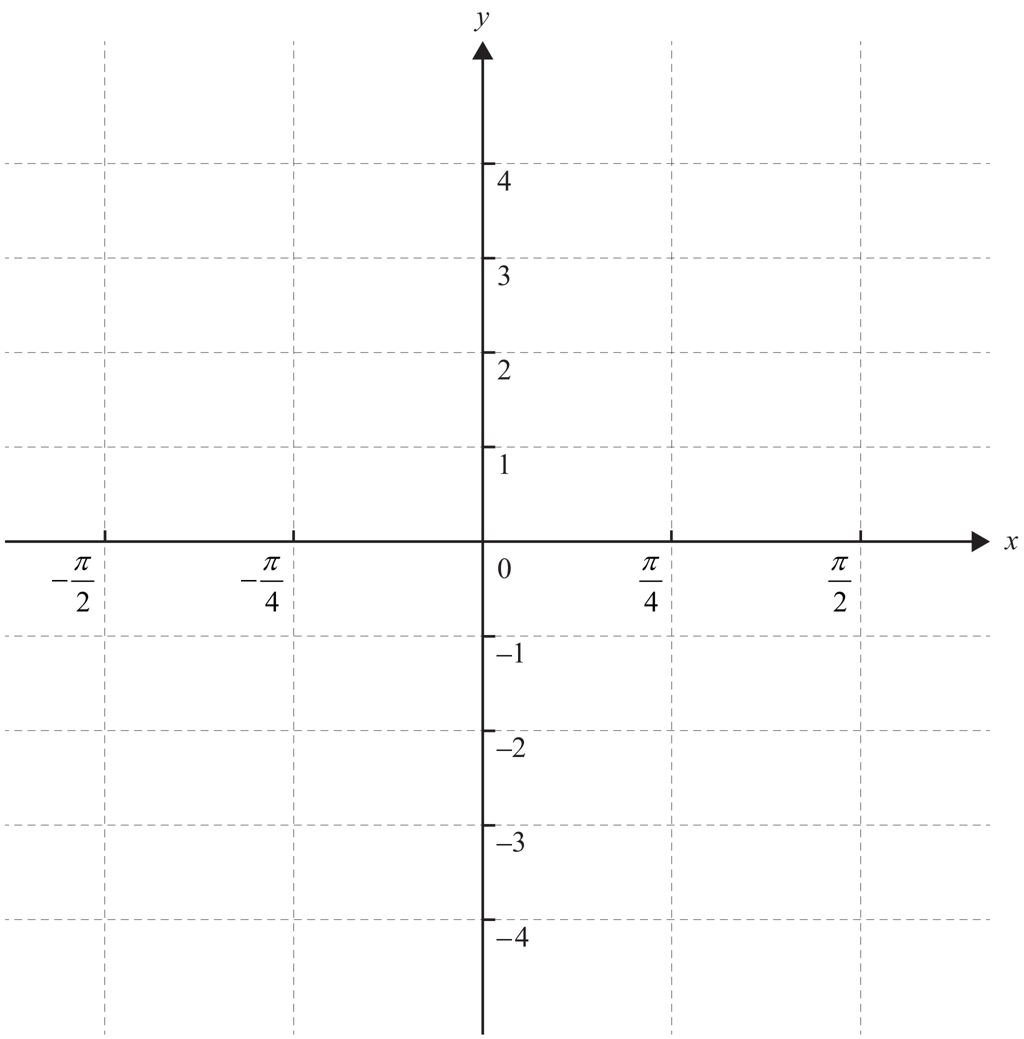
a. State the smallest positive value of k such that $x = \frac{3\pi}{4}$ is a solution of $\tan(x) = \cos(kx)$. 1 mark

b. Solve $2\sin^2(x) + 3\sin(x) - 2 = 0$, where $0 \leq x \leq 2\pi$. 2 marks

Question 4 (5 marks)

Let $f:\left[-\frac{\pi}{2},\frac{\pi}{2}\right]\to R$, where $f(x)=\tan(2x)+1$.

a. Sketch the graph of f on the axes below. Label any asymptotes with the appropriate equation, and label the end points and the axis intercepts with their coordinates. 4 marks



b. Use features of the graph in **part a.** to find the average value of f between $x = -\frac{\pi}{8}$ and $x = \frac{\pi}{8}$. 1 mark

Question 5 (6 marks)

Records of the arrival times of trains at a busy station have been kept for a long period. The random variable X represents the number of minutes **after** the scheduled time that a train arrives at this station, that is, the lateness of the train. Assume that the lateness of one train arriving at this station is independent of the lateness of any other train.

The distribution of X is given in the table below.

x	-1	0	1	2
$\Pr(X = x)$	0.1	0.4	0.3	p

a. Find the value of p. 1 mark

b. Find $E(X)$. 1 mark

c. Find $\text{var}(X)$. 2 marks

d. A passenger catches a train at this station on five separate occasions.

What is the probability that the train arrives **before** the scheduled time on exactly four of these occasions? 2 marks

Question 6 (3 marks)

At a large sporting arena there are a number of food outlets, including a cafe.

a. The cafe employs five men and four women. Four of these people are rostered at random to work each day. Let $\hat{P}$ represent the sample proportion of men rostered to work on a particular day.

i. List the possible values that $\hat{P}$ can take. 1 mark

ii. Find $\Pr(\hat{P} = 0)$. 1 mark

b. There are over 80 000 spectators at a sporting match at the arena. Five in nine of these spectators support the Goannas team. A simple random sample of 2000 spectators is selected.

What is the standard deviation of the distribution of $\hat{P}$, the sample proportion of spectators who support the Goannas team? 1 mark

Question 7 (6 marks)

Let $f: R \to R$, where $f(x) = 2x^3 + 1$, and let $g: R \to R$, where $g(x) = 4 - 2x$.

a. **i.** Find $g(f(x))$. 1 mark

ii. Find $f(g(x))$ and express it in the form $k - m(x - d)^3$, where m, k and d are integers. 2 marks

b. The transformation $T: R^2 \to R^2$ with rule $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & a \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} b \\ c \end{bmatrix}$, where a, b and c are integers, maps the graph of $y = g(f(x))$ onto the graph of $y = f(g(x))$.

Find the values of a, b and c. 3 marks

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Question 8 (8 marks)

The rule for a function f is given by $f(x) = \sqrt{2x+3} - 1$, where f is defined on its maximal domain.

a. Find the domain and rule of the inverse function f^{-1}. 2 marks

b. Solve $f(x) = f^{-1}(x)$. 2 marks

Question 8 – continued

c. Let $g: D \to R,\ g(x) = \sqrt{2x + c} - 1,$ where D is the maximal domain of g and c is a real number.

i. For what value(s) of c does $g(x) = g^{-1}(x)$ have no real solutions? 2 marks

ii. For what value(s) of c does $g(x) = g^{-1}(x)$ have exactly one real solution? 2 marks

END OF QUESTION AND ANSWER BOOK

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MATHEMATICAL METHODS

Written examination 1

FORMULA SHEET

Instructions

This formula sheet is provided for your reference.
A question and answer book is provided with this formula sheet.

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Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$		$\int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$	
$\frac{d}{dx}\left((ax+b)^n\right) = an\left(ax+b\right)^{n-1}$		$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, n \neq -1$	
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$		$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$	
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$		$\int \frac{1}{x} dx = \log_e(x) + c,\ x > 0$	
$\frac{d}{dx}\left(\sin(ax)\right) = a\ \cos(ax)$		$\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$	
$\frac{d}{dx}\left(\cos(ax)\right) = -a\ \sin(ax)$		$\int \cos(ax)dx = \frac{1}{a}\sin(ax) + c$	
$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\ \sec^2(ax)$			
product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$		

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A\|B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = \mathrm{E}(X)$	variance	$\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X - \mu)^2) = \mathrm{E}(X^2) - \mu^2$

Probability distribution		**Mean**	**Variance**
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x - \mu)^2\, p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x)dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$\mathrm{E}(\hat{P}) = p$
standard deviation	$\mathrm{sd}(\hat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$

END OF FORMULA SHEET

Probability

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END OF FORMULA SHEET