VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education 2017

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

Letter

STUDENT NUMBER

SPECIALIST MATHEMATICS

Written examination 2

Friday 9 June 2017

Reading time: 10.00 am to 10.15 am (15 minutes)
Writing time: 10.15 am to 12.15 pm (2 hours)

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	6	6	60
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 23 pages.
- Formula sheet.
- Answer sheet for multiple-choice questions.

Instructions
- Write your **student number** in the space provided above on this page.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination
- Place the answer sheet for multiple-choice questions inside the front cover of this book.
- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION A – Multiple-choice questions

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1; an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$

Question 1
The number of asymptotes of the graph of the function with rule $f(x) = \dfrac{x^3 - 7x + 5}{x^2 + 3x - 4}$ is

A. 0
B. 1
C. 2
D. 3
E. 4

Question 2
The equation $x^2 + y^2 + 2ky + 4 = 0$, where k is a real constant, will represent a circle only if

A. $k > 2$
B. $k < -2$
C. $k \neq \pm 2$
D. $k < -2$ or $k > 2$
E. $-2 < k < 2$

Question 3
For the function $f: R \to R, f(x) = k \arctan(ax - b) + c$, where $k > 0$, $c > 0$ and $a, b \in R$, $f(x) > 0$ if

A. $c < \dfrac{k\pi}{2}$
B. $c \geq \dfrac{k\pi}{2}$
C. $x > \dfrac{b}{a}$
D. $c + k > \dfrac{\pi}{2}$
E. $c \geq \dfrac{\pi}{2}$

Question 4

If $\sin(\theta + \phi) = a$ and $\sin(\theta - \phi) = b$, then $\sin(\theta)\cos(\phi)$ is equal to

A. ab

B. $\sqrt{a^2 + b^2}$

C. $\sqrt{ab}$

D. $\sqrt{a^2 - b^2}$

E. $\dfrac{a + b}{2}$

Question 5

Given that A, B, C and D are non-zero rational numbers, the expression $\dfrac{3x+1}{x(x-2)^2}$ can be represented in partial fraction form as

A. $\dfrac{A}{x} + \dfrac{B}{(x-2)}$

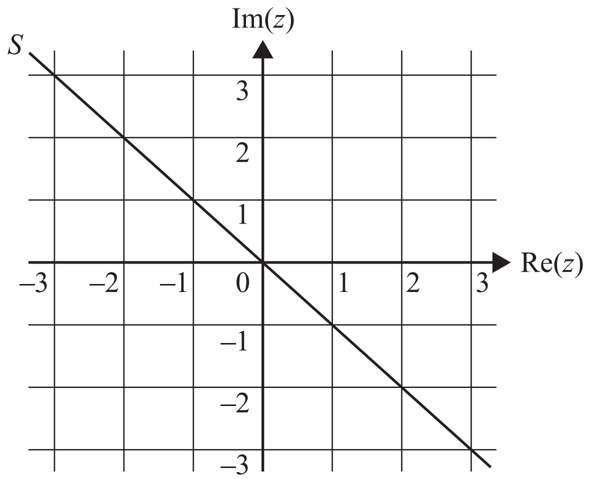
B. $\dfrac{A}{x} + \dfrac{B}{(x-2)^2}$

C. $\dfrac{A}{x} + \dfrac{B}{(x-2)} + \dfrac{C}{(x-2)^2}$

D. $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{(x-2)}$

E. $\dfrac{A}{x} + \dfrac{Bx}{(x-2)} + \dfrac{Cx + D}{(x-2)^2}$

Question 6



The relation that defines the line *S* above is

A. $|z+2|=|z+2i|$

B. $\text{Arg}(z)=\frac{3\pi}{4}$

C. $|z-2|=|z+2i|$

D. $\text{Im}(z)=\text{Arg}\left(\frac{3\pi}{4}\right)+\text{Arg}\left(-\frac{\pi}{4}\right)$

E. $|z-2|=|z-2i|$

Question 7

$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \left(\sin^3(x)\cos^2(x)\right)dx$ is equivalent to

A. $\int_{\frac{1}{2}}^{\frac{1}{\sqrt{2}}} \left(u^4 - u^2\right)du$ where $u = \cos(x)$

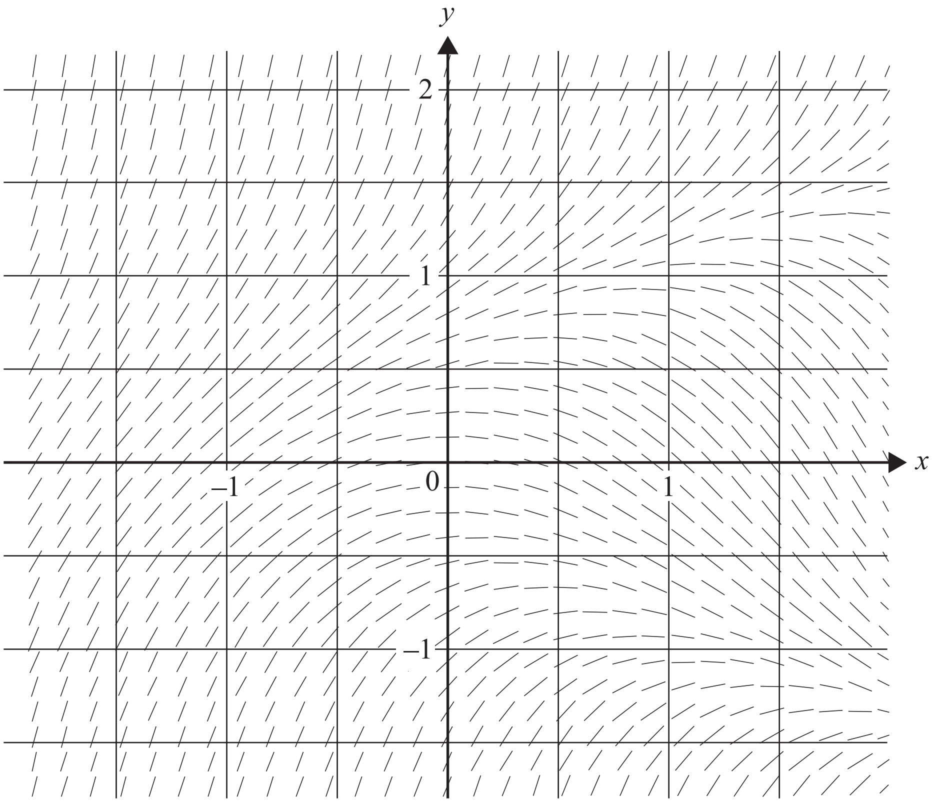
B. $-\int_{\frac{1}{\sqrt{2}}}^{\frac{1}{2}} \left(u^2 - u^4\right)du$ where $u = \cos(x)$

C. $-\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \left(u^2 - u^4\right)du$ where $u = \sin(x)$

D. $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \left(u^2 - u^4\right)du$ where $u = \sin(x)$

E. $-\int_{\frac{1}{\sqrt{2}}}^{\frac{1}{2}} \left(u^2 - u^4\right)du$ where $u = \sin(x)$

Question 8



The differential equation that best represents the direction field above is

A. $\dfrac{dy}{dx} = x - y^2$

B. $\dfrac{dy}{dx} = y - x$

C. $\dfrac{dy}{dx} = y^2 - x^2$

D. $\dfrac{dy}{dx} = y^2 - x$

E. $\dfrac{dy}{dx} = y + x$

Question 9

The gradient of the tangent to a curve at any point $P(x, y)$ is half the gradient of the line segment joining P and the point $Q(-1, 1)$.

The coordinates of points on the curve satisfy the differential equation

A. $\frac{dy}{dx} = \frac{y+1}{2(x-1)}$

B. $\frac{dy}{dx} = \frac{2(y-1)}{x+1}$

C. $\frac{dy}{dx} = \frac{x-1}{2(y+1)}$

D. $\frac{dy}{dx} = \frac{2(x-1)}{y+1}$

E. $\frac{dy}{dx} = \frac{y-1}{2(x+1)}$

Question 10

A solution to the differential equation $\frac{dy}{dx} = \frac{\cos(x+y) - \cos(x-y)}{e^{x+y}}$ can be obtained from

A. $\int \frac{e^y}{\sin(y)}\,dy = -\int \frac{2\sin(x)}{e^x}\,dx$

B. $\int \frac{e^y}{\cos(y)}\,dy = \int \frac{2}{e^x}\,dx$

C. $\int \frac{e^y}{\cos(y)}\,dy = -\int \frac{2\cos(x)}{e^x}\,dx$

D. $\int \frac{e^{-y}}{\sin(y)}\,dy = \int 2e^{-x}\sin(x)\,dx$

E. $\int \frac{e^y}{\cos(y)}\,dy = \int \frac{2\sin(x)}{e^x}\,dx$

Question 11

Two particles have positions given by $\underset{\sim}{r}_1 = \left(3 - 4t^2\right)\underset{\sim}{i} + (t + b)\underset{\sim}{j}$ and $\underset{\sim}{r}_2 = 5t^2\underset{\sim}{i} + \left(t^2 - 1\right)\underset{\sim}{j}$, where $t \geq 0$ and b is a real constant.

The particles will collide if the value of b is

A. $\frac{2-\sqrt{3}}{3}$

B. $\sqrt{3}-1$

C. $\frac{2+\sqrt{3}}{3}$

D. $\frac{-2-\sqrt{3}}{3}$

E. $-\sqrt{3}-1$

Question 12

If $\underset{\sim}{u} = 3\underset{\sim}{i} + 6\underset{\sim}{j} - 2\underset{\sim}{k}$ and $\underset{\sim}{v} = 2\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$, then the vector resolute of $\underset{\sim}{u}$ in the direction of $\underset{\sim}{v}$ is

A. $\frac{20}{7}\left(3\underset{\sim}{i} + 6\underset{\sim}{j} - 2\underset{\sim}{k}\right)$

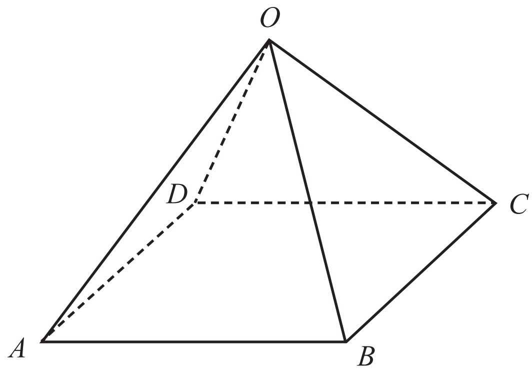
B. $\frac{20}{9}\left(2\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}\right)$

C. $\frac{20}{49}\left(3\underset{\sim}{i} + 6\underset{\sim}{j} - 2\underset{\sim}{k}\right)$

D. $\frac{20}{3}\left(2\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}\right)$

E. $\frac{3}{7}\left(3\underset{\sim}{i} + 6\underset{\sim}{j} - 2\underset{\sim}{k}\right)$

Question 13



Let *OABCD* be a right square pyramid where $\underset{\sim}{a} = \overrightarrow{OA}$, $\underset{\sim}{b} = \overrightarrow{OB}$, $\underset{\sim}{c} = \overrightarrow{OC}$ and $\underset{\sim}{d} = \overrightarrow{OD}$.

An equation correctly relating these vectors is

A. $\underset{\sim}{a} + \underset{\sim}{c} = \underset{\sim}{b} + \underset{\sim}{d}$

B. $\left(\underset{\sim}{a} - \underset{\sim}{c}\right).\left(\underset{\sim}{d} - \underset{\sim}{c}\right) = 0$

C. $\underset{\sim}{a} + \underset{\sim}{d} = \underset{\sim}{b} + \underset{\sim}{c}$

D. $\left(\underset{\sim}{a} - \underset{\sim}{d}\right).\left(\underset{\sim}{c} - \underset{\sim}{b}\right) = 0$

E. $\underset{\sim}{a} + \underset{\sim}{b} = \underset{\sim}{c} + \underset{\sim}{d}$

Question 14

Given that the vectors $\underset{\sim}{a} = \underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k}$, $\underset{\sim}{b} = 2\underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k}$ and $\underset{\sim}{c} = \lambda\underset{\sim}{i} - \underset{\sim}{j} + 4\underset{\sim}{k}$ are **linearly dependent**, the value of λ is

A. –10

B. –8

C. 2

D. 4

E. 8

Question 15

A particle of mass 2 kg has an initial velocity of $\underset{\sim}{i} - 6\underset{\sim}{j}$ ms^{-1}.

After a change of momentum of $6\underset{\sim}{i} - 2\underset{\sim}{j}$ kg ms^{-1}, the particle's velocity, in ms^{-1}, is

A. $3\underset{\sim}{i} - \underset{\sim}{j}$

B. $2\underset{\sim}{i} - 12\underset{\sim}{j}$

C. $4\underset{\sim}{i} - 7\underset{\sim}{j}$

D. $2\underset{\sim}{i} + 5\underset{\sim}{j}$

E. $11\underset{\sim}{i} + 2\underset{\sim}{j}$

Question 16

A person of mass M kg carrying a bag of mass m kg is standing in a lift that is accelerating downwards at a ms^{-2}.

The force of the lift floor acting on the person has magnitude

A. $Mg + mg$

B. $Mg + (M + m)a$

C. $Mg - (M + m)a$

D. $(M + m)(g + a)$

E. $(M + m)(g - a)$

Question 17

The acceleration, a ms^{-2}, of a particle moving in a straight line is given by $a = v^2 + 1$, where v is the velocity of the particle at any time t. The initial velocity of the particle when at origin O is 2 ms^{-1}.

The displacement of the particle from O when its velocity is 3 ms^{-1} is

A. $\log_e(2)$

B. $\frac{1}{2}\log_e\left(\frac{10}{3}\right)$

C. $\frac{1}{2}\log_e(2)$

D. $\frac{1}{2}\log_e\left(\frac{5}{2}\right)$

E. $\log_e\left(\frac{4}{5}\right)$

Question 18

X is a random variable with a mean of 5 and a standard deviation of 4, and Y is a random variable with a mean of 3 and a standard deviation of 2.

If X and Y are independent random variables and $Z = X - 2Y$, then Z will have mean μ and standard deviation σ given by

A. $\mu = -1, \sigma = 0$

B. $\mu = -1, \sigma = 4\sqrt{2}$

C. $\mu = 2, \sigma = 8$

D. $\mu = 2, \sigma = 4\sqrt{2}$

E. $\mu = -1, \sigma = 2\sqrt{6}$

Question 19

The petrol consumption of a particular model of car is normally distributed with a mean of 12 L/100 km and a standard deviation of 2 L/100 km.

The probability that the average petrol consumption of 16 such cars exceeds 13 L/100 km is closest to

A. 0.0104

B. 0.0193

C. 0.0228

D. 0.3085

E. 0.3648

Question 20

The mass of suspended matter in the air in a particular locality is normally distributed with a mean of μ micrograms per cubic metre and a standard deviation of $\sigma = 8$ micrograms per cubic metre. The mean of 100 randomly selected air samples was found to be 40 micrograms per cubic metre.

Based on this, a 90% confidence interval for μ, correct to two decimal places, is

A. (38.68, 41.32)

B. (26.84, 53.16)

C. (38.43, 41.57)

D. (24.32, 55.68)

E. (37.93, 42.06)

END OF SECTION A

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$

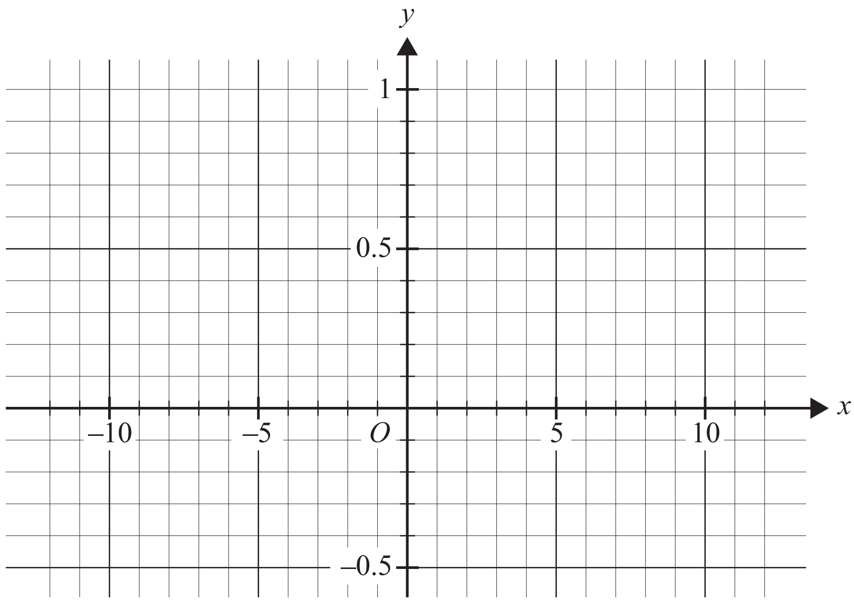
Question 1 (12 marks)

a. **i.** Use an appropriate double angle formula with $t = \tan\left(\frac{5\pi}{12}\right)$ to deduce a quadratic equation of the form $t^2 + bt + c = 0$, where b and c are real values. 2 marks

ii. Hence show that $\tan\left(\frac{5\pi}{12}\right) = 2 + \sqrt{3}$. 1 mark

Consider $f:\left[\sqrt{3}, 6+3\sqrt{3}\right] \to R,\ f(x) = \arctan\left(\frac{x}{3}\right) - \frac{\pi}{6}$.

b. Sketch the graph of f on the axes below, labelling the end points with their coordinates. 3 marks



c. The region between the graph of f and the y-axis is rotated about the y-axis to form a solid of revolution.

i. Write down a definite integral in terms of y that gives the volume of the solid formed. 2 marks

ii. Find the volume of the solid, correct to the nearest integer. 1 mark

d. A fish pond that has a shape approximately like that of the solid of revolution in **part c.** is being filled with water. When the depth is h metres, the volume, V m^3, of water in the pond is given by

$$V = \tan\left(h + \frac{\pi}{6}\right) - h - \frac{\sqrt{3}}{3}$$

If water is flowing into the pond at a rate of 0.03 m^3 per minute, find the rate at which the depth is increasing when the depth is 0.6 m. Give your answer in metres per minute, correct to three decimal places. 3 marks

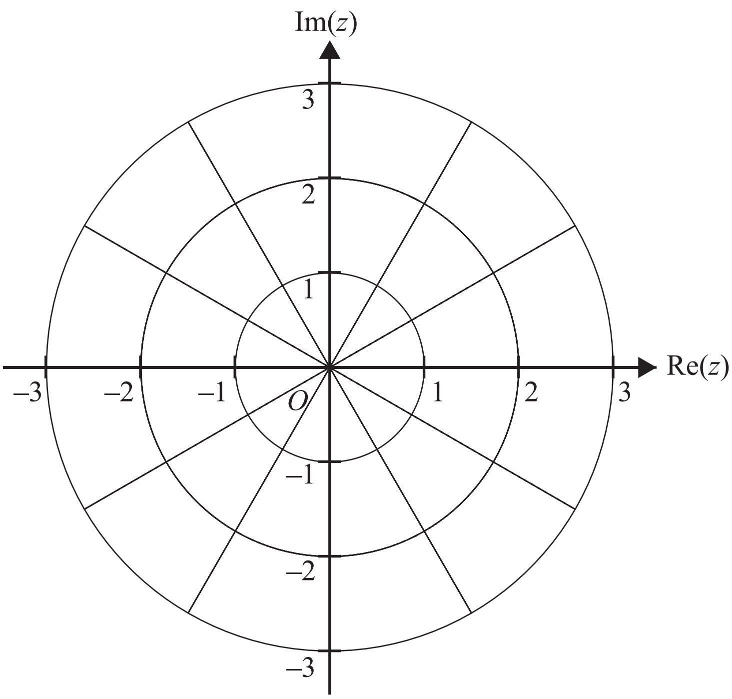
Question 2 (11 marks)

One root of a quadratic equation with real coefficients is $\sqrt{3}+i$.

a. **i.** Write down the other root of the quadratic equation. 1 mark

ii. Hence determine the quadratic equation, writing it in the form $z^2 + bz + c = 0$. 2 marks

b. Plot and label the roots of $z^3 - 2\sqrt{3}z^2 + 4z = 0$ on the Argand diagram below. 3 marks



c. Find the equation of the line that is the perpendicular bisector of the line segment joining the origin and the point $\sqrt{3}+i$. Express your answer in the form $y = mx + c$. 2 marks

d. The three roots plotted in **part b.** lie on a circle.

Find the equation of this circle, expressing it in the form $|z-\alpha| = \beta$, where $\alpha, \beta \in R$. 3 marks

Question 3 (12 marks)

Bacteria are spreading over a Petri dish at a rate modelled by the differential equation

$$\frac{dP}{dt} = \frac{P}{2}(1-P),\ 0 < P < 1$$

where P is the **proportion** of the dish covered after t hours.

a. **i.** Express $\frac{2}{P(1-P)}$ in partial fraction form. 1 mark

ii. Hence show by integration that $\frac{t-c}{2} = \log_e\left(\frac{P}{1-P}\right)$, where c is a constant of integration. 2 marks

iii. If half of the Petri dish is covered by the bacteria at $t = 0$, express P in terms of t. 2 marks

After one hour, a toxin is added to the Petri dish, which harms the bacteria and reduces their rate of growth. The differential equation that models the rate of growth is now

$$\frac{dP}{dt} = \frac{P}{2}(1-P) - \frac{\sqrt{P}}{20} \quad \text{for } t \geq 1$$

b. Find the limiting value of P, which is the maximum possible proportion of the Petri dish that can now be covered by the bacteria. Give your answer correct to three decimal places. 2 marks

c. The total time, T hours, measured from time $t = 0$, needed for the bacteria to cover 80% of the Petri dish is given by

$$T = \int_q^r \left(\frac{1}{\frac{P}{2}(1-P) - \frac{\sqrt{P}}{20}} \right) dP + s$$

where q, r and $s \in R$.

Find the values of q, r and s, giving the value of q correct to two decimal places. 2 marks

d. Given that $P = 0.75$ when $t = 3$, use Euler's method with a step size of 0.5 to estimate the value of P when $t = 3.5$. Give your answer correct to three decimal places. 3 marks

Question 4 (8 marks)

A cricketer hits a ball at time $t = 0$ seconds from an origin O at ground level across a level playing field.

The position vector $\underset{\sim}{r}(t)$, from O, of the ball after t seconds is given by

$\underset{\sim}{r}(t) = 15t\,\underset{\sim}{i} + \left(15\sqrt{3}\,t - 4.9t^2\right)\underset{\sim}{j}$, where $\underset{\sim}{i}$ is a unit vector in the forward direction, $\underset{\sim}{j}$ is a unit vector vertically up and displacement components are measured in metres.

a. Find the initial velocity of the ball and the initial angle, in degrees, of its trajectory to the horizontal. 2 marks

b. Find the maximum height reached by the ball, giving your answer in metres, correct to two decimal places. 2 marks

c. Find the time of flight of the ball. Give your answer in seconds, correct to three decimal places. 1 mark

d. Find the range of the ball in metres, correct to one decimal place. 1 mark

e. A fielder, more than 40 m from O, catches the ball at a height of 2 m above the ground.

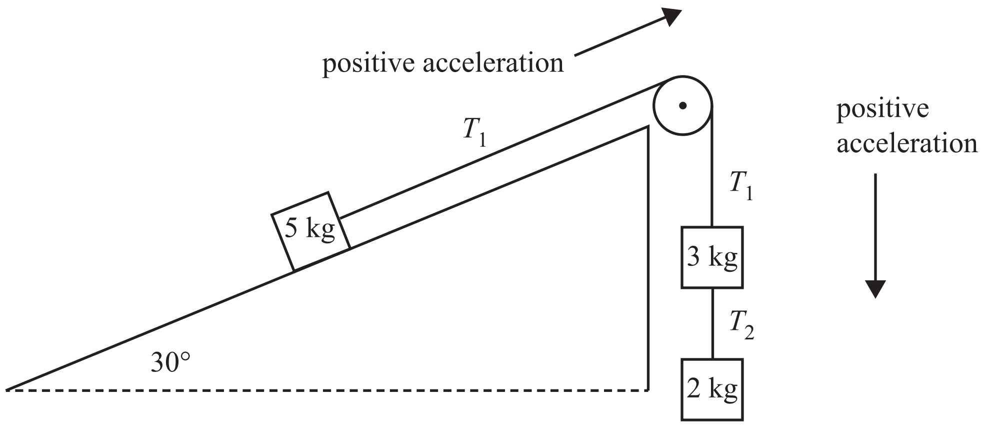
How far horizontally from O is the fielder when the ball is caught? Give your answer in metres, correct to one decimal place. 2 marks

Question 5 (10 marks)

A 5 kg mass is initially held at rest on a smooth plane that is inclined at 30° to the horizontal. The mass is connected by a light inextensible string passing over a smooth pulley to a 3 kg mass, which in turn is connected to a 2 kg mass.

The 5 kg mass is released from rest and allowed to accelerate up the plane.

Take acceleration to be positive in the directions indicated.



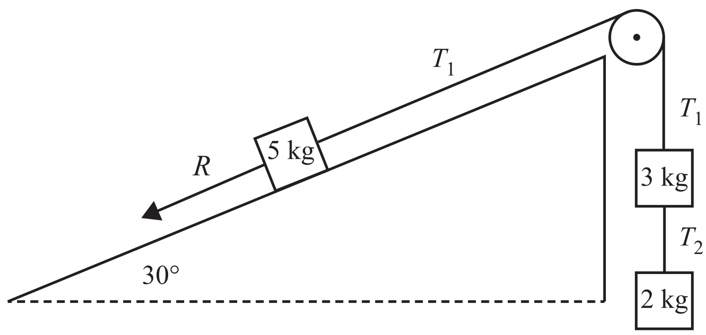
a. Write down an equation of motion, in the direction of motion, for each mass. 3 marks

b. Show that the acceleration of the 5 kg mass is $\frac{g}{4}$ ms^{-2}. 1 mark

c. Find the tensions T_1 and T_2 in the string in terms of g. 2 marks

d. Find the momentum of the 5 kg mass, in kg ms^{-1}, after it has moved 2 m up the plane, giving your answer in terms of g. 2 marks

e. A resistance force R acting parallel to the inclined plane is added to hold the system in equilibrium, as shown in the diagram below.



Find the magnitude of R in terms of g. 2 marks

Question 6 (7 marks)

A bank claims that the amount it lends for housing is normally distributed with a mean of \$400 000 and a standard deviation of \$30 000.

A consumer organisation believes that the average loan amount is higher than the bank claims.

To check this, the consumer organisation examines a random sample of 25 loans and finds the sample mean to be \$412 000.

a. Write down the two hypotheses that would be used to undertake a one-sided test. 1 mark

b. Write down an expression for the p value for this test and evaluate it to four decimal places. 2 marks

c. State with a reason whether the bank's claim should be rejected at the 5% level of significance. 1 mark

d. What is the largest value of the sample mean that could be observed before the bank's claim was rejected at the 5% level of significance? Give your answer correct to the nearest 10 dollars. 1 mark

e. If the average loan made by the bank is actually \$415 000 and not \$400 000 as originally claimed, what is the probability that a random selection of 25 loans has a sample mean that is at most \$410 000? Give your answer correct to three decimal places. 2 marks

END OF QUESTION AND ANSWER BOOK

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2017

SPECIALIST MATHEMATICS

Written examination 2

FORMULA SHEET

Instructions

This formula sheet is provided for your reference. A question and answer book is provided with this formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Specialist Mathematics formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder	$2\pi rh$
volume of a cylinder	$\pi r^2 h$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a pyramid	$\frac{1}{3}Ah$
volume of a sphere	$\frac{4}{3}\pi r^3$
area of a triangle	$\frac{1}{2}bc\sin(A)$
sine rule	$\frac{a}{\sin(A)}=\frac{b}{\sin(B)}=\frac{c}{\sin(C)}$
cosine rule	$c^2 = a^2 + b^2 - 2ab\cos(C)$

Circular functions

$\cos^2(x) + \sin^2(x) = 1$	
$1 + \tan^2(x) = \sec^2(x)$	$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$
$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$	$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$
$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$	$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
$\tan(x+y) = \frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$	$\tan(x-y) = \frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$
$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$	
$\sin(2x) = 2\sin(x)\cos(x)$	$\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$

Circular functions – continued

Function	$\sin^{-1}$ or arcsin	$\cos^{-1}$ or arccos	$\tan^{-1}$ or arctan
Domain	$[-1, 1]$	$[-1, 1]$	R
Range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + iy = r\left(\cos(\theta) + i\sin(\theta)\right) = r\,\text{cis}(\theta)$	
$\lvert z\rvert = \sqrt{x^2 + y^2} = r$	$-\pi < \text{Arg}(z) \le \pi$
$z_1 z_2 = r_1 r_2\,\text{cis}(\theta_1 + \theta_2)$	$\frac{z_1}{z_2} = \frac{r_1}{r_2}\text{cis}\left(\theta_1 - \theta_2\right)$
$z^n = r^n\,\text{cis}(n\theta)$ (de Moivre's theorem)	

Probability and statistics

for random variables X and Y	$\text{E}(aX + b) = a\text{E}(X) + b$ $\text{E}(aX + bY) = a\text{E}(X) + b\text{E}(Y)$ $\text{var}(aX + b) = a^2\text{var}(X)$
for independent random variables X and Y	$\text{var}(aX + bY) = a^2\text{var}(X) + b^2\text{var}(Y)$
approximate confidence interval for μ	$\left(\bar{x} - z\frac{s}{\sqrt{n}},\ \bar{x} + z\frac{s}{\sqrt{n}}\right)$
distribution of sample mean $\overline{X}$	mean $\text{E}\left(\overline{X}\right) = \mu$ variance $\text{var}\left(\overline{X}\right) = \frac{\sigma^2}{n}$

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e \lvert x \rvert + c$
$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$	$\int \sin(ax)\, dx = -\frac{1}{a}\cos(ax) + c$
$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$	$\int \cos(ax)\, dx = \frac{1}{a}\sin(ax) + c$
$\frac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$	$\int \sec^2(ax)\, dx = \frac{1}{a}\tan(ax) + c$
$\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0$
$\frac{d}{dx}\left(\cos^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}}$	$\int \frac{-1}{\sqrt{a^2-x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0$
$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$	$\int \frac{a}{a^2+x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$
	$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, \ n \neq -1$
	$\int (ax+b)^{-1} dx = \frac{1}{a}\log_e \lvert ax+b \rvert + c$
product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$
Euler's method	If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$
acceleration	$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$
arc length	$\int_{x_1}^{x_2} \sqrt{1 + \left(f'(x)\right)^2}\, dx$ or $\int_{t_1}^{t_2} \sqrt{\left(x'(t)\right)^2 + \left(y'(t)\right)^2}\, dt$

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$
$\lvert \underset{\sim}{r} \rvert = \sqrt{x^2 + y^2 + z^2} = r$
$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$
$\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos(\theta) = x_1x_2 + y_1y_2 + z_1z_2$

Mechanics

momentum	$\underset{\sim}{p} = m\underset{\sim}{v}$
equation of motion	$\underset{\sim}{R} = m\underset{\sim}{a}$

END OF FORMULA SHEET