2018 VCE Mathematical Methods 2 examination report

General comments

There were some excellent responses to the 2018 Mathematical Methods 2 examination, and most students were able to attempt the five questions in Section B. Many students found the last parts of Questions 1, 2, 4 and 5 challenging.

Advice to students

- Some questions specified the form for the answer, for example, Question 1e. Students should be familiar with how to obtain particular forms using a suitable combination of technology and by-hand computation.
- Re-read questions to make sure all parts are of the question are being answered or the answer makes sense. For example, Question 1b. and 1hi. required values of *b* and *a*, which meant that there was more than one value for these. Question 2dii. required two answers.
- Be familiar with the order properties of the real number system, for example, in Question 1b. $b > 32$ did not mean $b \geq 33$ and in Question 4f. $\Pr(X < 15)$ did not mean $\Pr(X \leq 14)$.
- When transcribing answers from technology be careful with negative signs. Sign errors occurred in Question 1d., Question 1g. and Question 3b.
- Be careful that you enter expressions into your technology correctly. Problems occurred in Question 1g., Question 3c., Question 4f. and Question 5.
- Sometimes coordinates were required and only the *x* value was given. This occurred in Question 1a., Question 3e. and Question 5a.
- If the functions have been defined at the start of the question, it is acceptable to use the function name, such as $f(x)$, throughout the question rather than writing out the entire expression. This avoids transcription errors. This occurred in Question 3. Also, ensure the correct function name is used as applicable.
- Show a method for questions worth more than one mark. A small number of students did not show their method in the probability question, Question 4.
- Take time when sketching graphs. Question 2d. required the graph to be continuous and one curve. Addition of ordinates is a suitable technique for sketching graphs.
- Students should know which formulation to use for the average rate of change and average value of a function (Question 2). In Question 4e., some students found the median rather than the expected value.
- Students need to be familiar with $\hat{P}$ and $\hat{p}$ notation for the probability and statistics Area of Study.
- Care needs to be taken when writing definite integrals. Check the terminals and the functions are in the correct order and that the correct function names are used. Problems occurred in Question 1e., Question 3c., Question 5c. and Question 5e.
- Mathematical notation, not technology syntax, is to be used at all times. Problems occurred in Question 1b. and Question 4.

Specific information

This report provides sample answers or an indication of what answers may have included. Unless otherwise stated, these are not intended to be exemplary or complete responses.

The statistics in this report may be subject to rounding, resulting in a total more or less than 100 per cent.

Section A

The table below indicates the percentage of students who chose each option. The correct answer is indicated by the shading.

Question	% A	% B	% C	% D	% E	% No answer	Comments
1	2	2	**95**	2	0	0	
2	**88**	3	2	2	5	0	
3	34	6	7	**48**	4	0	$f:[a,\ b) \to R,\ f(x)=\frac{1}{x}$, $f(a)=\frac{1}{a},\ f(b)=\frac{1}{b},\ f(a)>f(b)$, Range $\left(\frac{1}{b},\ \frac{1}{a}\right]$
4	18	11	**48**	13	10	0	*A* (3, 2), $g(x)=\frac{1}{2}f(x-1)$, Dilate by a factor of a $\frac{1}{2}$ from the *x*-axis: $(3,\ 1)$ Translate 1 unit to the right: $(4,\ 1)$
5	**67**	7	3	9	15	0	
6	6	10	6	**58**	20	0	
7	2	**83**	8	6	2	0	
8	5	**41**	11	7	36	0	$\int_1^{12} g(x)dx=5,\ \int_{12}^{5} g(x)dx=-6$ $\int_1^{12} g(x)dx=\int_1^5 g(x)dx+\int_5^{12} g(x)dx$ so $5=\int_1^5 g(x)dx+\ 6$ $\int_1^5 g(x)dx=-1$
9	15	9	**57**	11	7	0	
10	4	9	**74**	8	5	0	
11	38	29	**26**	4	3	0	$y=\tan(ax)$ $y=\tan\left(\frac{x}{2}\right),\ \text{Period}=2\pi$ Asymptotes are at $x=\pi,\ x=3\pi$ *x*-intercept is 2π
12	13	8	13	9	**58**	0	
13	5	7	15	14	**59**	1	
14	6	**60**	16	12	6	1	

Question	% A	% B	% C	% D	% E	% No answer	Comments
15	11	9	12	19	49	1	$\int_0^m f(x)dx=\frac{1}{2}$ $-\frac{m^4}{48}+\frac{m^2}{3}-\frac{1}{2}=0$ $m^4-16m^2+24=0$
16	7	49	18	9	15	1	Area of the rectangles = $\frac{\pi}{6}\left(f\left(\frac{\pi}{6}\right)+f\left(\frac{\pi}{3}\right)+f\left(\frac{\pi}{2}\right)\right)=\frac{7\pi}{6}$ Actual area = $\int_0^{\frac{\pi}{2}} f(x)dx=\frac{3\pi}{2}$ $\frac{\frac{7\pi}{6}}{\frac{3\pi}{2}}=\frac{7}{9}$
17	17	16	45	14	8	1	
18	17	22	24	21	14	2	$f'(d)=g'(d)$ for some $d\in(1,\infty)$ is false. Options A to D could be seen to be true by substituting in values.
19	3	4	41	9	42	1	Area = $\int_0^{\frac{1}{3}}(f(x)-g(x)dx)-2\int_{\frac{1}{3}}^{1}(f(x)-g(x)dx-\int_{\frac{5}{3}}^{3}(f(x)-g(x))dx$
20	20	26	21	21	11	1	Gradient $=m=4$ reflect in the *y*-axis: $m=-4$. Dilate by a factor of 2 from the *y*-axis: $m=-\frac{4}{2}=-2.$ Dilate by a factor of $\frac{1}{2}$ from the *x*-axis: $m=-1$. The matrix $\begin{bmatrix}-2 & 0\\ 0 & \frac{1}{2}\end{bmatrix}$ represents this transformation.

Section B

Question 1a.

Marks	0	1	Average
%	5	95	1

$f:R\to R, f(x)=3x^4+4x^3-12x^2$, $(-2,\ -32)$

This question was answered well. Some students only gave the *x* value when coordinates were required.

Question 1b.

Marks	0	1	Average
%	35	65	0.7

$b > 32$

This question was answered well. Common incorrect answers were $(-\infty, 32)$, $b = 32$, $b \geq 32$, $[33, \infty)$ and $b = 33$. Others used the *x*-coordinate and gave *x* > 2 as their answer.

Question 1c.

Marks	0	1	Average
%	27	73	0.8

$l(x) = \frac{80}{9}x + \frac{41}{27}$

An equation and exact values were required.

Question 1d.

Marks	0	1	2	Average
%	20	13	67	1.5

Solve $l(x) = f(x)$ for x, $x = \frac{-1 \pm \sqrt{42}}{3}$

Exact values were required. There were many sign errors, for example $x = \frac{1 \pm \sqrt{42}}{3}$. Some students found the values of *x* where the gradient of *l* was equal to the gradient of *f*.

Question 1e.

Marks	0	1	2	Average
%	38	13	49	1.1

Area $= \int_{\frac{-1-\sqrt{42}}{3}}^{\frac{-1+\sqrt{42}}{3}} \left(l(x) - f(x)\right) dx = \frac{784\sqrt{42}}{135}$

Students who answered Question 1d. correctly were generally able to answer this question correctly. Some students split the integral, which was unnecessary. Others put a negative sign in front of the integral for the bounded area below the *x*-axis. Some had their terminals or expressions the reverse of what was required.

Question 1f.

Marks	0	1	Average
%	51	49	0.5

$a = 0$

Some students gave an additional expression $a = -6x(x-2)$, which was obtained if technology was used rather than equating coefficients.

Question 1g.

Marks	0	1	Average
%	43	57	0.6

$x=1,\ x=-1\pm\sqrt{1-a}$

A common error was $x=1\pm\sqrt{1-a}$. $x=\dfrac{-1\pm\sqrt{9-4a}}{2}$, $x=0$ was often given. This comes from forgetting to differentiate $-12ax$ when differentiating *p*(*x*).

Question 1hi.

Marks	0	1	Average
%	82	18	0.2

$1-a<0,\ a>1$

This question was not answered well. Common incorrect answers were $a=1$, $a=0$ or $a>0$.

Question 1hii.

Marks	0	1	Average
%	47	53	0.6

$a=2$, $p(x)=3x^4+4x^3-24x+4$, $p(1)=-13$, the minimum value is -13.

This question was answered well. The minimum value needed to be stated, not just the coordinates of the turning point.

Question 1hiii.

Marks	0	1	2	Average
%	92	4	4	0.2

Solve $p(x)>0$ for a when $x = 1$, $a>\sqrt{14}+3$ as $a>1$

This question was not answered well. Many students did not attempt this question. Some students solved $p(x)=0$ or $p'(x)=0$ for *x*. Others tried to apply the discriminant to a cubic equation. Others, who used a correct method, sometimes gave an incorrect inequality, for example $a<\sqrt{14}+3$.

Question 2a.

Marks	0	1	2	Average
%	19	8	73	1.6

$b'(t)=0$, $t=\dfrac{10\log_e(4.5)}{7}$ hours

This question was answered well. An exact answer was required. Some students converted *t* = 2.148… to 2 hours and 15 minutes.

Question 2b.

Marks	0	1	2	Average
%	16	15	70	1.6

Average rate of change = $\frac{b(6)-b(2)}{6-2} = -33.5$ mg/h, correct to one decimal place

Some students used the average value of the function. Others made substitution errors. A common incorrect answer was 33.5. Several students used the graph to approximate values rather than find *b*(6) and *b*(2). Some students found the average of the gradient at *b* = 2 and *b* = 6.

Question 2c.

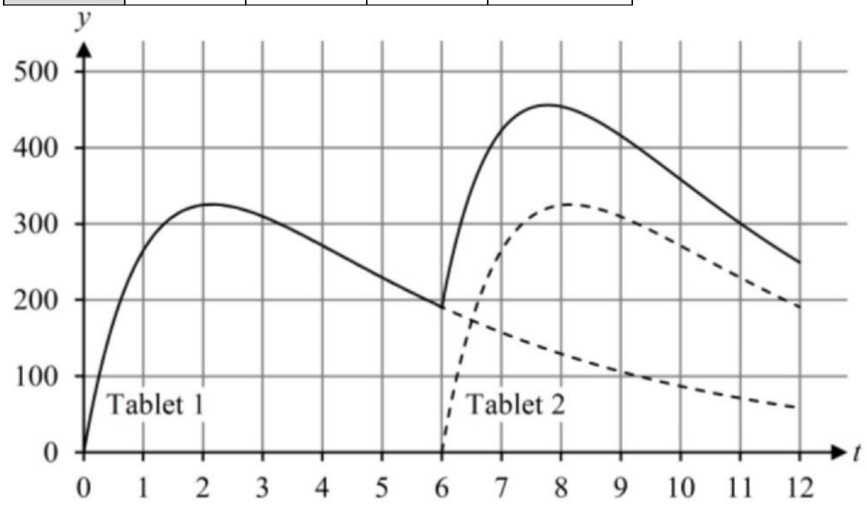
Marks	0	1	2	Average
%	39	6	56	1.2

Average amount of drug = $\frac{1}{6}\int_0^6 \left(b(t)\right)dt = 256$ mg to the nearest integer

Some students used the interval [2, 6] from Question 2c., instead of [0, 6]. Others did not divide by 6, which gave 1535.1… mg. $\frac{b(0)+b(1)+b(2)+b(3)+b(4)+b(5)+b(6)}{6}$ was often given. Some thought that the first six hours meant *t* = 1 to *t* = 6 instead of *t* = 0 to *t* = 6. Others evaluated $\int_0^6 t \times b(t)dt$ or found the average rate of change.

Question 2di.

Marks	0	1	2	Average
%	35	29	35	1



Many students were able to trace over the first part of the graph from *t* = 0 to *t* = 6. Some did not join the two sections at *x* = 6, with some starting at the intersection of the two graphs. Students could use addition of ordinates or define the function $b_2(t) = b(t) + b(t-6)$ and use technology to sketch the graph and find the position of turning point. Some students shaded the area under the graph.

Question 2dii.

Marks	0	1	2	Average
%	74	6	19	0.5

Total amount of drug = $b(t)+b(t-6)$, maximum amount of drug is 455.82 mg, correct to two decimal places, t = 7.78 h correct to two decimal places

This question was not answered well. Many students were unable to find the new rule and solved $b'(t)=0$ for t, getting 324.34 mg for the maximum amount of drug. Some then added six to this answer, 324.34 + 6 = 330.34 mg. Some assumed that t = 8. Answers were required to two decimal places. Other students gave only one answer.

Question 3a.

Marks	0	1	Average
%	5	95	1

$a=75$

This question was answered well.

Question 3b.

Marks	0	1	Average
%	22	78	0.8

Translation of 35 m to the right

This question was answered well. Some students wrote 35 m to the right, omitting translation. A few wrote to the left or 5 m to the left or right. Some students had a correct statement followed by an incorrect expression $x\to x-35$.

Question 3c.

Marks	0	1	2	3	Average
%	8	8	18	66	2.4

$5\times110-3\int_5^{35}\left(h_1(x)\right)dx = 264\text{ m}^2$, correct to the nearest square metre

This question was answered reasonably well. There were some rounding errors. Some students had the correct expression but the incorrect answer. Others had incorrect terminals for the area of the arch, for example $\int_0^{30}\left(h_1(x)\right)dx$. Some students left their answer as $550-\frac{900}{\pi}$.

Question 3d.

Marks	0	1	Average
%	39	61	0.6

$\tan\left(\frac{\pi}{90}\right)=0.035$, correct to three decimal places

This question was generally well answered. Some students evaluated $\frac{\pi}{90}$ or $\sin\left(\frac{\pi}{90}\right)$. There were some rounding errors and 0.036 was sometimes given.

Question 3e.

Marks	0	1	2	Average
%	46	13	41	1

$h_2'(x) = \tan\left(\frac{\pi}{90}\right)$, $x = 54.36\ \ldots$, $h_2(54.3626...) = 4.99\ \ldots$, $P\ (54.36,\ 4.99)$, correct to two decimal places

Many students were able to equate their answer to Question 3d. to $h_2'(x)$. Some students were possibly estimating values for the coordinates from the graph as *P* (54, 5) was sometimes given without working.

Question 3f.

Marks	0	1	2	3	Average
%	56	17	8	19	0.9

$y_1 = \tan\left(\frac{\pi}{90}\right)x + 5$, $y_2 - 4.988... = -\frac{1}{\tan\left(\frac{\pi}{90}\right)}\left(x - 54.363...\right)$, $Q\ (54.29\ldots,\ 6.896\ldots)$,

Distance $PQ = \sqrt{(54.36... - 54.29..)^2 + (4.98... - 6.896...)^2} = 1.91\ \text{m}$, correct to two decimal places

There were a number of other approaches to this question. Many students were able to get the negative reciprocal of their answer to Question 3d. Other students incorrectly thought *P* was on the line *y* = 5 and used a trigonometric ratio to find the distance.

Question 4a.

Marks	0	1	Average
%	13	87	0.9

$M \sim N(68, 64)$, $\Pr(60 < M < 90) = 0.838$, correct to three decimal places

This question was answered well.

Question 4bi.

Marks	0	1	Average
%	43	57	0.6

$\Pr(H \mid S) = \frac{\Pr(H \cap S)}{\Pr(S)} = \frac{0.09}{0.29} = 0.310$, correct to three decimal places

This question was answered reasonably well. Some students gave their answer as 0.31. A common mistake was $\frac{\Pr(H)}{\Pr(S)} = \frac{0.1587}{0.29} = 0.547$ or $\frac{\Pr(H \cap S)}{\Pr(S)} = \frac{0.9}{0.1857}$, giving an answer greater than 1.

Question 4bii.

Marks	0	1	Average
%	56	44	0.5

No, the events are not independent, $\Pr(H \mid S) = \dfrac{\Pr(H \cap S)}{\Pr(S)} \neq \dfrac{\Pr(H) \times \Pr(S)}{\Pr(S)} = \Pr(H)$,

$0.310... \neq 0.1587$, $\Pr(H \cap S) \neq \Pr(H) \times \Pr(S)$, $0.09 \neq 0.1587 \times 0.29 = 0.046...$

A mathematical explanation was required. Some students confused mutually exclusive events with independent events. A common mistake was $\Pr(H \mid S) = \Pr(S)$.

Question 4ci.

Marks	0	1	2	Average
%	25	9	66	1.4

$X \sim \text{Bi}(16, 0.1587)$, $\Pr(X = 1) = 0.190$, correct to three decimal places

This question was reasonably well done. A method was required to get full marks. Stating the correct n and p value was sufficient. Some students gave their answer as 0.19.

Question 4cii.

Marks	0	1	2	Average
%	54	9	36	0.8

$\Pr(\hat{P} > 0.1) = \Pr(X > 1.6) = \Pr(X \geq 2) = 0.747$, correct to three decimal places

Some students used the normal approximation to the binomial distribution. There was poor use of variables, for example, $\Pr(\hat{P} > 0.1) = \Pr(\hat{P} > 1.6) = \Pr(\hat{P} \geq 2)$.

Question 4ciii.

Marks	0	1	2	Average
%	86	6	8	0.2

$\Pr\left(\hat{P}_n > \dfrac{1}{n}\right) > 0.99$, $\Pr(X > 1) = \Pr(X \geq 2) > 0.99$, $1 - \left(\Pr(X = 0) + \Pr(X = 1)\right) > 0.99$

$\Pr(X = 0) + \Pr(X = 1) < 0.01$, $(1 - 0.1587)^n + \dbinom{n}{1} 0.1587(1 - 0.1587)^{n-1} < 0.01$, $n = 38.925\ldots$, $n = 39$

This question was not answered well. Many students appeared to be confused by the terminology $\Pr\left(\hat{P}_n > \dfrac{1}{n}\right)$. $1 - \Pr(X = 0) < 0.01$ was often evaluated, giving n = 27. Trial and error was an acceptable method.

Question 4di.

Marks	0	1	Average
%	55	45	0.5

$\hat{p} = \dfrac{0.102 + 0.145}{2} = 0.1235$

Many students tried to find the sample size rather than the proportion. n = 900 was often given.

Question 4dii.

Marks	0	1	Average
%	89	11	0.1

The 95% confidence interval for Statsville, (0.102, 0.145), does not contain the Mathsland proportion, which is 0.1587.

The confidence interval needed to be referred to in the answer.

Question 4e.

Marks	0	1	2	Average
%	38	9	53	1.2

$\int_0^{\infty}\left(t \times M(t)\right)dt = 44.6$, correct to one decimal place

Some students found the median or the mode. Others found the area under the curve. Some had one of the terminals incorrect, for example, $\int_0^{437}\left(t \times M(t)\right)dt$. There were rounding errors; 44.7 was occasionally given.

Question 4f.

Marks	0	1	Average
%	44	56	0.6

$\int_0^{15}\left(M(t)\right)dt = 0.0266$**,** correct to four decimal places

This question was answered reasonably well. $\int_0^{15}\left(t \times M(t)\right)dt = 0.2991$ was a common incorrect answer.

Question 4g.

Marks	0	1	2	Average
%	89	6	5	0.2

$0.05 \times \frac{1}{7} + x \times \frac{6}{7} = 0.0266...$, $x = 0.0227$, correct to four decimal places

This question was not answered well. There were a number of other approaches to this question, for example, Karnaugh maps, tree diagrams or a conditional probability statement. A common incorrect answer was $\frac{6}{7} \times 0.0266 = 0.0228$.

Question 5a.

Marks	0	1	2	Average
%	32	19	49	1.2

$\left(\frac{2a}{3}, \frac{3}{a}\right)$

This question was answered reasonably well. Some students also gave the coordinates of the local minimum, (0, 0). Others just gave the x value of the local maximum. $\left(\frac{2a}{3}, 3a\right)$ was a common incorrect answer. This occurs if $4a^2$ is used for the denominator of $f(x)$ instead of $4a^4$. Some found a in terms of x and gave an answer of $a = \frac{3x}{2}$.

Question 5b.

Marks	0	1	Average
%	38	62	0.6

$f(x) = h(x)$, $x = \frac{2a}{3}$ or $x = \frac{a}{3}$ or $x = 0$

This question was generally well answered. Some students wrote $x = \frac{2}{3}$, $x = \frac{1}{3}$ or $x = 0$. A common incorrect answer was $x = \frac{\pm\sqrt{9a^2 - 8} + 3}{3}$ or $x = 0$. As in Question 5b., this occurs if $4a^2$ is used for the denominator of $f(x)$ instead of $4a^4$.

Question 5c.

Marks	0	1	2	Average
%	60	6	34	0.8

$$\int_0^{\frac{a}{3}} \left(h(x) - f(x)\right) dx + \int_{\frac{a}{3}}^{\frac{2a}{3}} \left(f(x) - h(x)\right) dx = \frac{1}{8}$$

Some students found only half of the area. Others had the correct expression for the area but did not have a correct answer. A common mistake was that students had the functions in the incorrect order, either both intervals were $h(x) - f(x)$ or the other way around. Other students substituted a value for a and found the area.

Question 5d.

Marks	0	1	Average
%	36	64	0.7

$$\frac{2a}{3} \times g\left(\frac{2a}{3}\right) = 2$$

This question was answered well.

Question 5e.

Marks	0	1	2	Average
%	87	6	7	0.2

$$\frac{2a}{3} \times \frac{3}{a} - \int_0^{\frac{2a}{3}} \left(g(x)\right) dx = 1$$

There were a number of other approaches to this question, using symmetry. This question was not answered well.

Question 5f.

Marks	0	1	Average
%	92	8	0.1

$a = \frac{3\sqrt{2}}{2}$

This question was not answered well. Many students did not attempt this question.

Question 5g.

Marks	0	1	Average
%	97	3	0.1

$\frac{1}{4}$

This question was not answered well. Many students did not attempt this question.