VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2018

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

Letter

STUDENT NUMBER

MATHEMATICAL METHODS

Written examination 1

Friday 1 June 2018

Reading time: 2.00 pm to 2.15 pm (15 minutes)
Writing time: 2.15 pm to 3.15 pm (1 hour)

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 13 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination
- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (4 marks)

a. Let $f(x) = \dfrac{e^x}{\left(x^2 - 3\right)}$.

Find $f'(x)$. 2 marks

b. Let $y = (x + 5)\log_e(x)$.

Find $\dfrac{dy}{dx}$ when $x = 5$. 2 marks

Question 2 (4 marks)

Let $f(x) = -x^2 + x + 4$ and $g(x) = x^2 - 2$.

a. Find $g(f(3))$. 2 marks

b. Express the rule for $f(g(x))$ in the form $ax^4 + bx^2 + c$, where a, b and c are non-zero integers. 2 marks

Question 3 (2 marks)

Evaluate $\int_0^1 e^x - e^{-x} dx$.

Question 4 (3 marks)

Solve $\log_3(t) - \log_3(t^2 - 4) = -1$ for t.

DO NOT WRITE IN THIS AREA

Question 5 (3 marks)

Let $h: R^+ \cup \{0\} \rightarrow R,\ h(x) = \dfrac{7}{x+2} - 3$.

a. State the range of h. 1 mark

b. Find the rule for h^{-1}. 2 marks

DO NOT WRITE IN THIS AREA

Question 6 (4 marks)

The discrete random variable X has the probability mass function

$$\Pr(X = x) = \begin{cases} kx & x \in \{1, 4, 6\} \\ k & x = 3 \\ 0 & \text{otherwise} \end{cases}$$

a. Show that $k = \frac{1}{12}$. 2 marks

b. Find $\text{E}(X)$. 1 mark

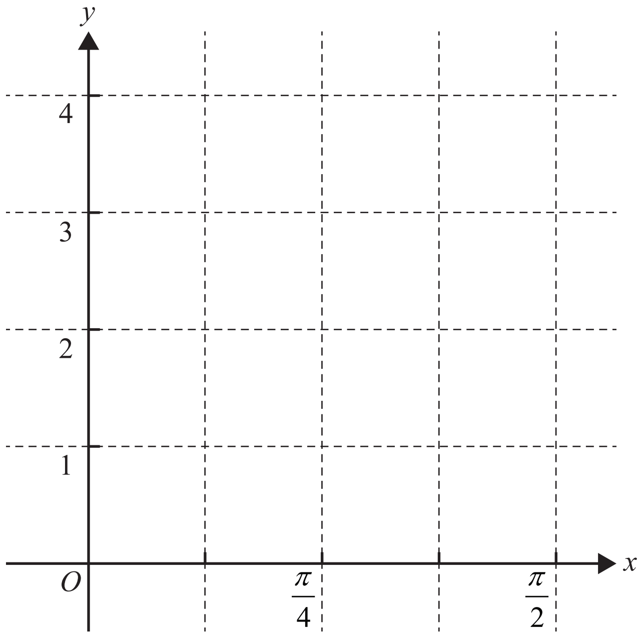
c. Evaluate $\Pr(X \geq 3 \mid X \geq 2)$. 1 mark

DO NOT WRITE IN THIS AREA

Question 7 (9 marks)

Let $f:\left[0, \frac{\pi}{2}\right] \to R,\ f(x) = 4\cos(x)$ and $g:\left[0, \frac{\pi}{2}\right] \to R,\ g(x) = 3\sin(x)$.

a. Sketch the graph of f and the graph of g on the axes provided below. 2 marks



b. Let c be such that $f(c) = g(c)$, where $c \in \left[0, \frac{\pi}{2}\right]$.

Find the value of $\sin(c)$ and the value of $\cos(c)$. 3 marks

c. Let A be the region enclosed by the horizontal axis, the graph of f and the graph of g.

i. Shade the region A on the axes provided in **part a.** and also label the position of c on the horizontal axis. 1 mark

ii. Calculate the area of the region A. 3 marks

Question 8 (3 marks)

Let $\hat{P}$ be the random variable that represents the sample proportions of customers who bring their own shopping bags to a large shopping centre.

From a sample consisting of all customers on a particular day, an approximate 95% confidence interval for the proportion p of customers who bring their own shopping bags to this large shopping centre was determined to be $\left(\frac{4853}{50000}, \frac{5147}{50000}\right)$.

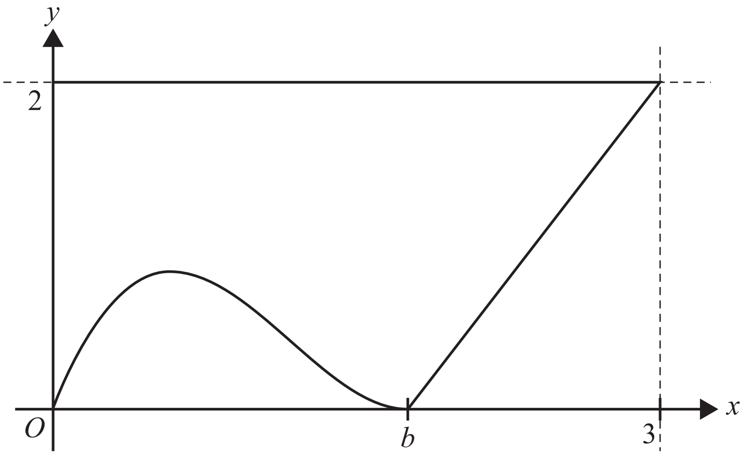
a. Find the value of $\hat{p}$ that was used to obtain this approximate 95% confidence interval. 1 mark

b. Use the fact that $1.96 = \frac{49}{25}$ to find the size of the sample from which this approximate 95% confidence interval was obtained. 2 marks

DO NOT WRITE IN THIS AREA

Question 9 (8 marks)

The diagram below shows a trapezium with vertices at (0, 0), (0, 2), (3, 2) and (b, 0), where b is a real number and $0 < b < 2$.

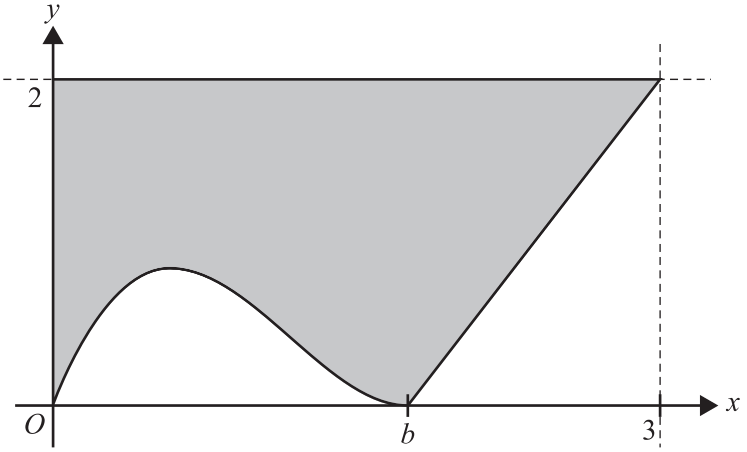


On the same axes as the trapezium, part of the graph of a cubic polynomial function is drawn. It has the rule $y = ax(x - b)^2$, where a is a non-zero real number and $0 \leq x \leq b$.

a. At the local maximum of the graph, $y = b$.

Find a in terms of b. 3 marks

The area between the graph of the function and the x-axis is removed from the trapezium, as shown in the diagram below.



b. Show that the expression for the area of the shaded region is $b+3-\frac{9b^2}{16}$ square units. 3 marks

Question 9 – continued

c. Find the value of b for which the area of the shaded region is a maximum and find this maximum area. 2 marks

END OF QUESTION AND ANSWER BOOK

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MATHEMATICAL METHODS

Written examination 1

FORMULA SHEET

Instructions

This formula sheet is provided for your reference. A question and answer book is provided with this formula sheet.

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Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$		$\int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$	
$\frac{d}{dx}\left((ax+b)^n\right) = an(ax+b)^{n-1}$		$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, n \neq -1$	
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$		$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$	
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$		$\int \frac{1}{x} dx = \log_e(x) + c,\ x > 0$	
$\frac{d}{dx}\left(\sin(ax)\right) = a\ \cos(ax)$		$\int \sin(ax) dx = -\frac{1}{a}\cos(ax) + c$	
$\frac{d}{dx}\left(\cos(ax)\right) = -a\ \sin(ax)$		$\int \cos(ax) dx = \frac{1}{a}\sin(ax) + c$	
$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\ \sec^2(ax)$			
product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$		

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A \mid B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = \mathrm{E}(X)$	variance	$\operatorname{var}(X) = \sigma^2 = \mathrm{E}((X - \mu)^2) = \mathrm{E}(X^2) - \mu^2$

Probability distribution		**Mean**	**Variance**
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x - \mu)^2\, p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x)dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$\mathrm{E}(\hat{P}) = p$
standard deviation	$\operatorname{sd}(\hat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$

END OF FORMULA SHEET