VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

2018 VCE Specialist Mathematics 1 examination report

General comments

In Specialist Mathematics examination 1 students were required to answer 10 questions worth a total of 40 marks. Students were not permitted to bring technology or notes into the examination.

Students are reminded to ensure that they present their responses in a clear and logical manner as assessors cannot award marks when they are unable to follow a student's working. In many instances, students made transcription errors that changed a problem into one that was either more difficult or easier. In either case, full marks could not be awarded. Students are reminded that if an answer is to be given in a specific form, then not doing so will result in full marks not being awarded.

Students should be able to perform basic arithmetic and algebraic calculations efficiently and accurately. Many students had difficulty applying index laws or solving simple equations. Some students did not use brackets correctly, leading to sign and other errors in calculations.

Examples of idiosyncratic notation were observed. This was most apparent in the labelling of the force diagram in Question 1. In some cases the student's intent was not clear and full marks were not awarded.

Students should ensure that they read questions carefully and consider the reasonableness of their answers. It may be beneficial for students to underline or highlight key information in questions. Many simple errors can be avoided in this way.

Areas of strength included:

- recognising that expressing complex numbers in polar form was required (Question 2b.)
- recognising the need to use the product and chain rule when differentiating implicitly (Question 3)
- knowing that the position vector needed to be differentiated in order to find the velocity vector and hence the momentum of a particle (Question 6)
- separating variables to solve a differential equation (Question 8b.)
- using trigonometric identities (Questions 7 and 9a.)

Areas of weakness included:

- not reading questions carefully. In Question 1b. specific information was required, which many students did not provide. Some students did not give answers in the required form (Questions 2b., 3 and 8b.)
- algebraic and arithmetic errors. In Questions 4 and 9b. students were required to solve quadratic equations. In many cases, students immediately applied the quadratic formula without noticing that the problem could be solved more quickly and efficiently by factorising the quadratic expression
- graph sketching (Question 5)
- not checking that the answer was reasonable (Question 10)

In questions that required a specific result to be shown (Questions 2, 8a. and 9a.), students are reminded that they must present sufficient evidence that the result has been shown. Most students included the dx in their integrals in Question 9c.; however, the dt was frequently missing from integrals in Question 10, and students should be mindful of this.

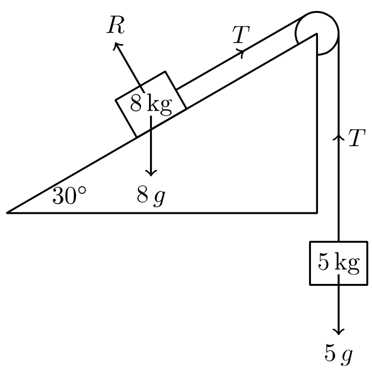
Specific information

This report provides sample answers or an indication of what answers may have included. Unless otherwise stated, these are not intended to be exemplary or complete responses.

The statistics in this report may be subject to rounding resulting in a total more or less than 100 per cent.

Question 1a.

Marks	0	1	Average
%	40	60	0.6



Students were expected to label the two weight forces, the normal reaction force on the 8 kg mass and the tension force in the string.

This question was answered reasonably well. Common errors included:

- omitting the normal reaction force on the 8 kg mass
- introducing extra forces, for example, a friction force
- failing to indicate that the tension was constant throughout the string
- failing to label the forces. In particular, some students used labels that did not distinguish between the two masses.

Question 1b.

Marks	0	1	2	3	Average
%	18	24	17	41	1.8

The 8 kg mass moves up the plane with acceleration $\frac{g}{13}$.

Students could resolve forces in order to find an equation of motion. Some students correctly determined the force $5g - 4g = g$ but did not use the combined mass of 13 kg in order to find the acceleration. In particular, $\frac{g}{8}$ was a common incorrect response. Many students neglected to give the magnitude and/or the direction of motion of the 8 kg mass.

Question 2a.

Marks	0	1	Average
%	17	83	0.9

Students were required to show that $1+i=\sqrt{2}\text{cis}\left(\frac{\pi}{4}\right)$.

This question was answered well by most students. A common incorrect response was to write $\tan\left(\frac{1}{1}\right)=\frac{\pi}{4}$ rather than $\arctan\left(\frac{1}{1}\right)=\frac{\pi}{4}$.

Question 2b.

Marks	0	1	2	3	Average
%	12	3	48	37	2.1

$-8-8\sqrt{3}i$

Most students realised that they needed to use polar form and de Moirvre's theorem. Quite a few students were not able to write $\sqrt{3}-i$ in polar form correctly with arguments of $\frac{\pi}{6}$, $\frac{5\pi}{6}$ and $\frac{\pi}{3}$ being given frequently. Students are reminded that a diagram placing the complex number in the correct quadrant can be helpful in avoiding errors. Of those students who obtained the result $16\text{cis}\left(-\frac{2\pi}{3}\right)$, some neglected to write the final answer in the required form or made errors in their attempt.

A small number of students attempted to expand brackets. This approach was rarely successful.

Question 3

Marks	0	1	2	3	4	Average
%	8	4	9	33	46	3.1

$\frac{dy}{dx}=\frac{-18}{\pi\sqrt{3}+6}$

Most students knew to use implicit differentiation in this problem and were successful in their application of the chain and product rules.

Many students attempted to find an expression for $\frac{dy}{dx}$ in terms of x and y. This was not necessary, with a more effective approach being to substitute $x=\frac{\pi}{6}$ and $y=\frac{\pi}{6}$ immediately following the implicit differentiation. Some students had difficulty with arithmetic.

Question 4

Marks	0	1	2	3	4	Average
%	5	8	8	36	43	3

$a = 2,\ b = 3$

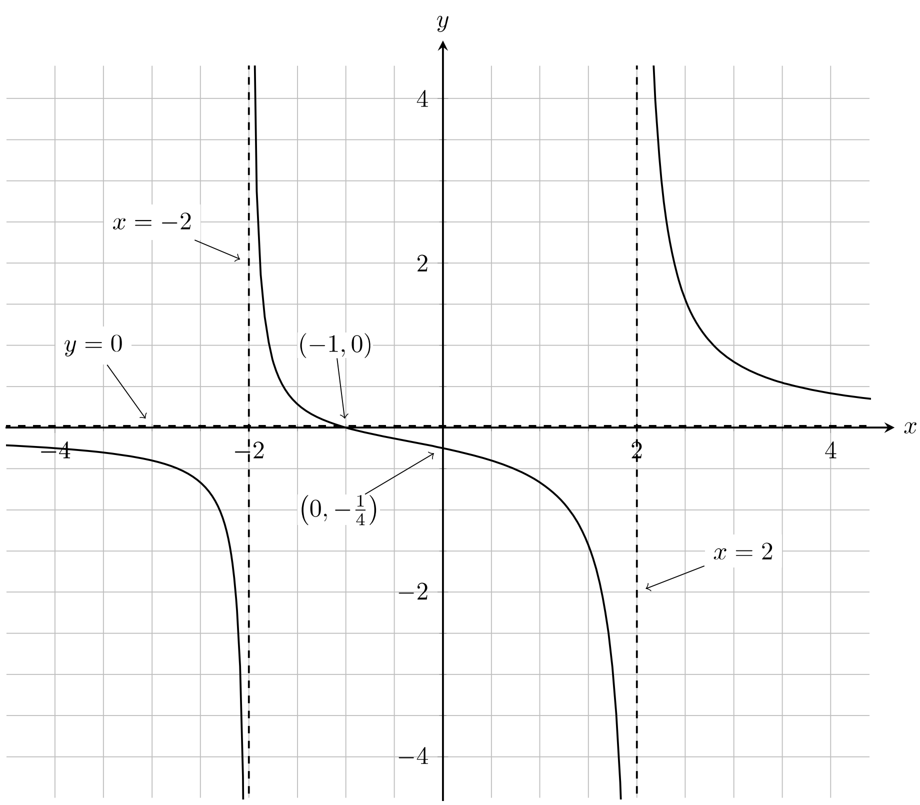
From the information given, students needed to write down a pair of simultaneous equations

$2a + 2b = 10,\ 2a^2 + 4b^2 = 44$ and then solve for a and b. Common problems included failing to reject the non-integer solution and only stating the solution with minimal or no working. Students are reminded that in a question worth more than one mark, appropriate working must be shown.

Algebraic errors were common, with some students having difficulty solving a quadratic equation. Quite a few students 'squared' both sides of the first equation to obtain $4a^2 + 4b^2 = 100$.

Question 5

Marks	0	1	2	3	4	Average
%	9	21	35	20	15	2.1



Most students realised that $x = -2$ and $x = 2$ were vertical asymptotes, although the horizontal asymptote $y = 0$ was often not stated. Students who found the axis intercepts were not always able to position them correctly on the axes. Some students showed a stationary point of inflection on their graph or were missing the outer branches.

Question 6

Marks	0	1	2	3	Average
%	12	14	42	31	1.9

$2\left(-\underset{\sim}{i}+\underset{\sim}{j}+\pi\underset{\sim}{k}\right)$

The majority of students correctly differentiated $\underset{\sim}{r}(t)$ to find $\dot{\underset{\sim}{r}}(t)=\cos t\underset{\sim}{i}-\sin t\underset{\sim}{j}+2t\underset{\sim}{k}$ and substituted $t=\frac{\pi}{2}$ and $t=\pi$. Some students made errors in their arithmetic when attempting to evaluate $2\left(\dot{\underset{\sim}{r}}(\pi)-\dot{\underset{\sim}{r}}\left(\frac{\pi}{2}\right)\right)$. A number of students were unable to evaluate the trigonometric expressions correctly. Some students thought that a scalar result was required.

Students who interpreted this question as asking for the average rate of change of momentum to be dimensionally consistent with the units and did this correctly were awarded marks accordingly.

Question 7

Marks	0	1	2	3	Average
%	10	6	19	66	2.4

$a=\frac{1}{2}$

This question was answered well, with most students converting $\cot(2x)$ into $\frac{1}{\tan(2x)}$ and using a double angle formula. A number of students used $\sin$ and $\cos$ but were less successful than those who used the more direct approach. A number of students thought that $\cot(2x)$ was equal to $\frac{1}{\cos(2x)}$. Some students gave their final answer as $a=2$.

Question 8a.

Marks	0	1	Average
%	56	44	0.5

$\frac{dQ}{dt}=0-\frac{3Q}{16+2t}=-\frac{3Q}{16+2t}$

This problem required students to recognise a difference of rates. The most common error was a failure to explicitly note that the rate in was zero.

Question 8b.

Marks	0	1	2	3	Average
%	27	12	38	23	1.6

$Q=\frac{32}{(16+2t)^{\frac{3}{2}}}$

The majority of students realised that this was a separable differential equation, but many made errors in the subsequent integration with the arbitrary constant of integration frequently missing. Some students made transcription errors that fundamentally changed the problem. Others encountered arithmetic or algebraic issues. Many students took the common factor of 2 from the $16+2t$ expression and evaluated $\frac{1}{2}\int \frac{1}{8+t}dt$. This unnecessary manipulation made subsequent calculations more difficult for these students.

Question 9a.

Marks	0	1	2	Average
%	13	14	73	1.6

This question was answered well, with most students realising that a substitution into a trigonometric identity was required.

Question 9b.

Marks	0	1	Average
%	30	70	0.7

$x=1,\ x=3$

Students needed to substitute $y=x-1$ into the equation $x^2-2y^2=1$ and solve the resulting quadratic equation for x. A number of students gave the coordinates of the points of intersection and in some cases did not do this correctly.

Question 9c.

Marks	0	1	2	Average
%	60	21	19	0.6

$\frac{2\pi}{3}$

Students found this question challenging. Some students did not apply the formula for the volume of a solid of revolution correctly. Many students made algebraic or arithmetic errors. A small number of students realised that the volume required could be found by finding the volume obtained by rotating the region bounded by the hyperbola, the x-axis and the lines $x=1$ and $x=3$ about the x-axis and then subtracting the volume of an appropriate cone.

Question 10

Marks	0	1	2	3	4	5	Average
%	35	22	24	17	0	2	1.3

$\int_0^{\frac{3}{4}}\left(2-t^2\right)dt$

Only a few students obtained full marks for this question. Most students recognised that the arc length formula needed to be applied, but some had difficultly differentiating $\arcsin(t)+t\sqrt{1-t^2}$. A number of students applied the product and chain rule correctly to the $t\sqrt{1-t^2}$ term and ignored the $\arcsin(t)$ term. Many students had difficulty simplifying $\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2$ and were unable to proceed further. Of those students who were able to find that $d=\int_{0}^{\frac{3}{4}}\sqrt{\left(t^{2}-2\right)^{2}}\,dt$, only a small number recognised the significance of the domain $0\le t\le 1$.