2022 VCE Mathematical Methods 1 (NHT) external assessment report

Specific information

This report provides sample answers or an indication of what answers may have included. Unless otherwise stated, these are not intended to be exemplary or complete responses.

Question 1a.

This question involved the chain rule for differentiation. A common error included writing a 2 rather than $2x$.

$2x\cos(x^2+1)$

Question 1b.

This question involved the product rule for differentiation.

Students are reminded to use the notation given to name the derivative.

$f'(x) = 2x\log_e(x) + x$

$f'(e) = 2e\log_e e + e$

$f'(e) = 3e$

Question 2

It is important to include the constant of integration (+*c*) and remember that the base *e* needs to be written as a subscript. A bracket around the (*x* + 1) term ensures the answer is interpreted correctly.

$f(x) = 2\log_e(x+1) + 2\sin(x) + c$

At $f(0) = 3$

$\therefore 3 = 0 + 0 + c$

$\therefore c = 3$

$\therefore\ f(x) = 2\log_e(x+1) + 2\sin(x) + 3$

Question 3

Students are reminded that the formula for calculating the confidence interval is found on the formula sheet. It is important that students familiarise themselves with this material.

$n = 100, \hat{p} = 0.1$

$$CI = \left(\frac{1}{10} - 2\sqrt{\frac{0.1 \times 0.9}{100}}, \frac{1}{10} + 2\sqrt{\frac{0.1 \times 0.9}{100}}\right)$$

$CI = \left(\frac{1}{10} - \frac{6}{100}, \frac{1}{10} + \frac{6}{100}\right)$

$CI = (0.04, 0.16) = \left(\frac{1}{25}, \frac{4}{25}\right)$ or equivalent

Question 4a.

This was a 'show that' question, so the working needed to be clear and explicit, leading to the correct answers.

$a\sin\left(\frac{\pi}{2}\right) + b = 2$

$\therefore a + b = 2$

$b = 2 - a$ (1)

$a\sin\left(\frac{3\pi}{2}\right) + b = -8$

$\therefore -a + b = -8$ (2)

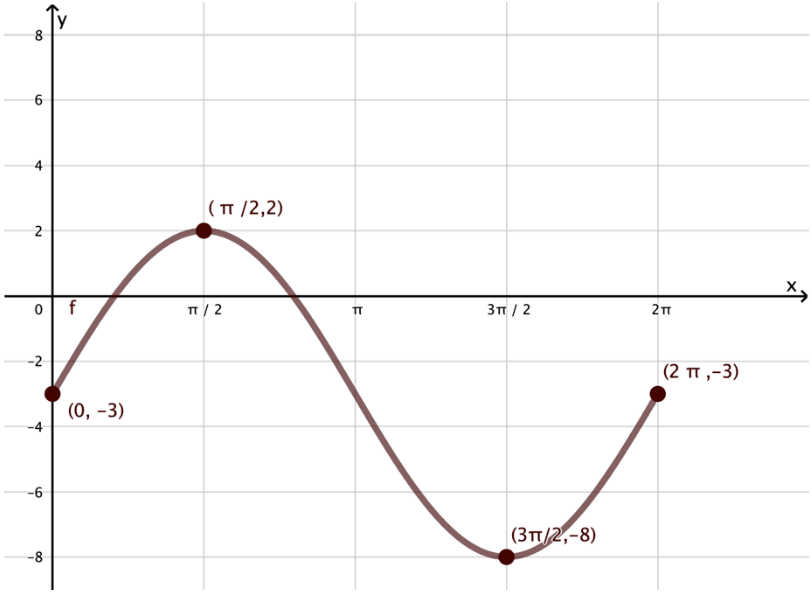
Sub (1) into (2)

$2 - 2a = -8$

$2a = 10$

$\therefore a = 5,\ b = -3$ from (1)

Question 4b.



Question 4c.

Students are encouraged to tackle this type of question visually, rather than try to use an algebraic approach. There were two intervals for values of *k* that needed to be found.

$(-\infty, -2) \cup (8, \infty)$ or $k < -2$ *and* $k > 8$

Question 4d.

Most students included the dx in the setting up of the integral.

$A = \int_0^{\pi} \big(5\sin(x) + (m-3)\big)dx$

$= [-5\cos(x) + (m-3)x]_0^{\pi}$

$= \big(-5(-1) + (m-3)\pi - (-5(1) - 0)\big)$

$= 10 + (m-3)\pi = 0$

$\therefore m = 3 - \dfrac{10}{\pi} = \dfrac{3\pi - 10}{\pi}$

Question 5a.

This question required a particular format for the answer and most students answered as requested.

$e^{1-x} \div e^{1+x}$

$= e^{1-x-(1+x)}$

$= e^{-2x}$

Question 5b.

This question is a ‘show that’ question and students needed to indicate clearly, through logical and explicit steps, that the left-hand side is equal to the right-hand side.

$LHS = e^{1-a} \times e^{1-b}$

$= e^{1-a+1-b}$

$= e^{2-a-b}$

$RHS = e^{1-(a+b-1)}$

$= e^{2-a-b} = LHS$

Question 5c.

This question required the answer in a particular form.

$e^{1-x} = 2$

$\Rightarrow 1 - x = \log_e(2)$

$\therefore x = 1 - \log_e(2)$

$\therefore x = \log_e(e) - \log_e(2)$

$\therefore x = \log_e\left(\frac{e}{2}\right)$

Question 6ai.

Students are encouraged to use a diagram to solve probability questions; in this case a tree diagram was helpful.

$\frac{x^3}{4}$

Question 6aii.

Students are reminded to check the reasonableness of their answers; probability values need to lie within the interval [0,1].

$\frac{x^3}{4} = \frac{1}{4}$

$\Rightarrow x^3 = 1$

$\therefore x = 1$

Question 6bi.

$\frac{x^3}{4} + (1 - x^2)x$

$= -\frac{3x^3}{4} + x$

Question 6bii.

This question required two answers: the value of *x* and the probability for this value of *x*. Many students did not give an answer for the probability of the value of *x*. Students are reminded to check they have answered all parts of a question.

Let $y = -\frac{3x^3}{4} + x$

$\therefore \frac{dy}{dx} = -\frac{9x^2}{4} + 1$

$0 = -\frac{9x^2}{4} + 1$

$\therefore x^2 = \frac{4}{9}$

$\therefore x = \frac{2}{3},\ as\ x \in [0,1]$

At $x = \frac{2}{3}$

$-\frac{3\left(\frac{2}{3}\right)^3}{4} + \frac{2}{3}$

$= -\frac{3 \times 2^3}{2^2 \times 3^3} + \frac{2}{3}$

$= -\frac{2}{3^2} + \frac{2}{3} = -\frac{2}{9} + \frac{6}{9} = \frac{4}{9}$

Question 7ai.

This question involved recognition of the inverse case.

If $p^{-1}(a) = 2 \Rightarrow p(2) = a$

$p(2) = 2^4 - 2^3 - 2^2 + 2 + 1 = a$,

$\therefore a = 7$

Question 7aii.

$b^4 - b^3 - b^2 + b + 1 = 1$

$b^4 - b^3 - b^2 + b = 0$

$b(b^3 - b^2 - b + 1) = 0$

$b(b-1)(b^2 - 1) = 0$

$\therefore b = -1, 0, 1$ and $b > 0$ thus

$\therefore b = 1$

Question 7b.

Students are reminded to take care with their use of notation. Setting $y = f(x)$ and then later writing $y =$ the inverse function is a contradiction of ideas and should be avoided. Students need to be using $f^{-1}(x)$ as the preferred naming notation for the inverse function.

Let $y = f(x)$,

Swap *x* and *y* for inverse

$x = \frac{y+3}{y-2}$

$xy - 2x = y + 3$

$-2x = (1 - x)y + 3$

$(1 - x)y = -(2x + 3)$

$\therefore y = -\frac{2x+3}{1-x} = \frac{2x+3}{x-1}$

$f^{-1}(x) = \frac{2x+3}{x-1}$

Domain $R\backslash\{1\}$

Alternative:

Let $y = f(x) = 1 + \frac{5}{x-2}$

Swap *x* and *y* for inverse

$$x = 1 + \frac{5}{y-2}$$

$$f^{-1}(x) = \frac{5}{x-1} + 2$$

Question 8a.

This question involved using the null factor law and solving $\sin(x) = 0$. The domain of the function meant that only solutions in the interval $(-\infty, 0]$ were valid.

$0 = e^x \sin(x)$

$\therefore \sin(x) = 0$ as $e^x \neq 0$

$\therefore x = n\pi, n \in \boldsymbol{Z}\ and\ n \leq 0\ (0\text{r}\ n \in Z^- \cup \{0\})$

Question 8b.

Other equivalent solutions are possible.

$f'(x) = e^x \cdot \sin(x) + e^x \cdot \cos(x)$

$0 = e^x \cdot \sin(x) + e^x \cdot \cos(x)$

$\sin(x) = -\cos(x)$

$\tan(x) = -1$

$x = -\frac{\pi}{4} + n\pi,\ n \in \boldsymbol{Z}\ and\ n \leq 0$ or

$$x = \frac{3\pi}{4} + n\pi, n \in \boldsymbol{Z}\ and\ n < 0$$

Question 8ci.

This is a ‘show that’ question. Lack of brackets for some students meant that the $-\sin(x)$ term caused confusion; $e^x - \sin(x) \neq -e^x \sin(x)$. It is best to put the $-\sin(x)$ term in a bracket as $e^x(-\sin(x))$ for clarity.

$\frac{d}{dx}(e^x \cdot \sin(x) - e^x \cdot \cos(x))$

$= e^x \cdot \sin(x) + e^x \cdot \cos(x) - e^x \cdot \cos(x) + e^x \cdot \sin(x)$

$= e^x \cdot \sin(x) + e^x \cdot \sin(x)$

$= 2e^x \cdot \sin(x)$

Question 8cii.

Students are reminded to take care with setting up their integrals to calculate the area. The region being considered is below the *x*-axis so any ‘Area’ statement needs to be positive.

$\left|\int_{-\pi}^{0} e^x \cdot \sin(x)\, dx\right|$ or

$-\int_{-\pi}^{0} e^x \cdot \sin(x)\, dx$ or

$\int_{0}^{-\pi} e^x \cdot \sin(x)\, dx$

Area $= \frac{1}{2}\left|\int_{-\pi}^{0} 2e^x \cdot \sin(x)\, dx\right|$

$= \frac{1}{2}|[e^x \cdot \sin(x) - e^x \cdot \cos(x)]_{-\pi}^{0}|$

$= \frac{1}{2}|(0 - 1 - (0 - e^{-\pi}(-1))|$

$= \frac{1}{2} + \frac{1}{2e^{\pi}} = \frac{1 + e^{\pi}}{2e^{\pi}} = \frac{e^{-\pi} + 1}{2}$

Question 8d.

It was helpful for students to see a connection between answers they obtained in Question 8b. and x_m discussed here. Often, earlier parts of a question provide clues or information to assist with solutions to subsequent parts.

$Pr\left(x < max\left(g(x)\right)\right) = \Pr\left(-\pi \leq x \leq -\frac{\pi}{4}\right) = \int_{-\pi}^{-\frac{\pi}{4}} g(x)dx$

$= -\frac{2e^{\pi}}{1+e^{\pi}}\int_{-\pi}^{-\frac{\pi}{4}} e^x \sin(x)dx$

$= -\frac{2e^{\pi}}{1+e^{\pi}} \cdot \frac{1}{2}[e^x \cdot \sin(x) - e^x \cdot \cos(x)]_{-\pi}^{-\frac{\pi}{4}}$

$= -\frac{e^{\pi}}{1+e^{\pi}}\left(e^{-\frac{\pi}{4}}\left(-\frac{1}{\sqrt{2}}\right) - e^{-\frac{\pi}{4}}\left(\frac{1}{\sqrt{2}}\right) - \left(0 - e^{-\pi}(-1)\right)\right)$

$= \frac{2e^{\pi}}{1+e^{\pi}}\left(\frac{1}{\sqrt{2}}e^{-\frac{\pi}{4}} + \frac{1}{2}e^{-\pi}\right) = \frac{e^{\pi}}{1+e^{\pi}}\left(\sqrt{2}e^{-\frac{\pi}{4}} + e^{-\pi}\right) = \frac{\sqrt{2}e^{\frac{3\pi}{4}}+1}{1+e^{\pi}}$