GREEN STRIPE

VICTORIAN CURRICULUM
AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2023

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER									Letter

FURTHER MATHEMATICS

Written examination 2

Friday 26 May 2023

Reading time: 10.30 am to 10.45 am (15 minutes)
Writing time: 10.45 am to 12.15 pm (1 hour 30 minutes)

QUESTION AND ANSWER BOOK

Structure of book

Section A – Core	*Number of questions*	*Number of questions to be answered*	*Number of marks*
	8	8	36
Section B – Modules	*Number of modules*	*Number of modules to be answered*	*Number of marks*
	4	2	24
			Total 60

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 37 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination
- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION A – Core

Instructions for Section A

Answer **all** questions in the spaces provided.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.

In 'Recursion and financial modelling', all answers should be rounded to the nearest cent unless otherwise instructed.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Data analysis

Question 1 (6 marks)

In an investigation of foot sizes, data about 39 primary school students of the same age was recorded for the following six variables:

- *name*: name of student
- *gender*: boy, girl
- *foot length*: foot length, in centimetres
- *foot width*: foot width, in centimetres
- *longest foot*: left, right
- *dominant hand*: left, right

Table 1 shows data for 10 of these students.

Table 1

Name	***Gender***	***Foot length* (cm)**	***Foot width* (cm)**	***Longest foot***	***Dominant hand***
David	boy	24.4	8.4	left	right
Lars	boy	25.4	8.8	left	left
Zach	boy	24.5	9.7	right	left
Josh	boy	25.2	9.8	left	right
Lang	boy	25.1	8.9	left	right
Scotty	boy	25.7	9.7	right	right
Edward	boy	26.1	9.6	left	right
Caitlin	girl	23.0	8.8	left	right
Caroline	girl	24.0	8.7	right	left
Maggie	girl	24.7	8.8	right	right

Data: adapted from MC Meyer, 'Wider Shoes for Wider Feet?', *Journal of Statistics Education*, vol. 14(1), 2006, <www.amstat.org/publications/jse/v14n1/datasets.meyer.htm>

a. Write down the number of nominal variables in Table 1. 1 mark

b. Determine the mean and standard deviation of the *foot length* of the 10 students.
Round your answers to one decimal place. 1 mark

mean = ☐ standard deviation = ☐

c. Using the data in Table 1, determine the percentage of boys whose *dominant hand* is left.
Round your answer to the nearest percentage. 1 mark

d. Using the data in Table 1, complete the following two-way frequency table, Table 2. 1 mark

Table 2

	Dominant hand	
Longest foot	left	right
left		
right		

e. Determine the equation of the least squares regression line that could be used to predict the *foot width* from the *foot length* of the 10 students.
Write the equation in terms of the variables *foot width* and *foot length*.
Round the coefficients to three significant figures. 2 marks

Question 2 (5 marks)

The back-to-back ordered stem plot below shows the *foot width*, in cm, for each of the 19 girls and the 20 boys in this investigation. The *foot width* of three of the boys have been replaced by dashes.

key: 8|4 = 8.4

foot width (cm)

girls (n =19)		boys (n =20)
	7	
9 9	7	
3 1	8	4
8 8 8 7 6 6 5	8	0 6 – 8 9 9 9
3 3 3 0 0 0	9	0 1 2 3 4
5 5	9	5 6 7 7 – – 8
	10	

a. Write down the minimum *foot width*, in centimetres, for the girls. 1 mark

b. Use the information in the back-to-back stem plot above to complete the five-number summary for the boys. 2 marks

Minimum	**First quartile (Q_1)**	**Median**	**Third quartile (Q_3)**	**Maximum**
		9.15		

c. Does the information in the back-to-back stem plot support the contention that, at the same age, boys tend to have wider feet than girls?

Refer to the values of an appropriate statistic in your response. 2 marks

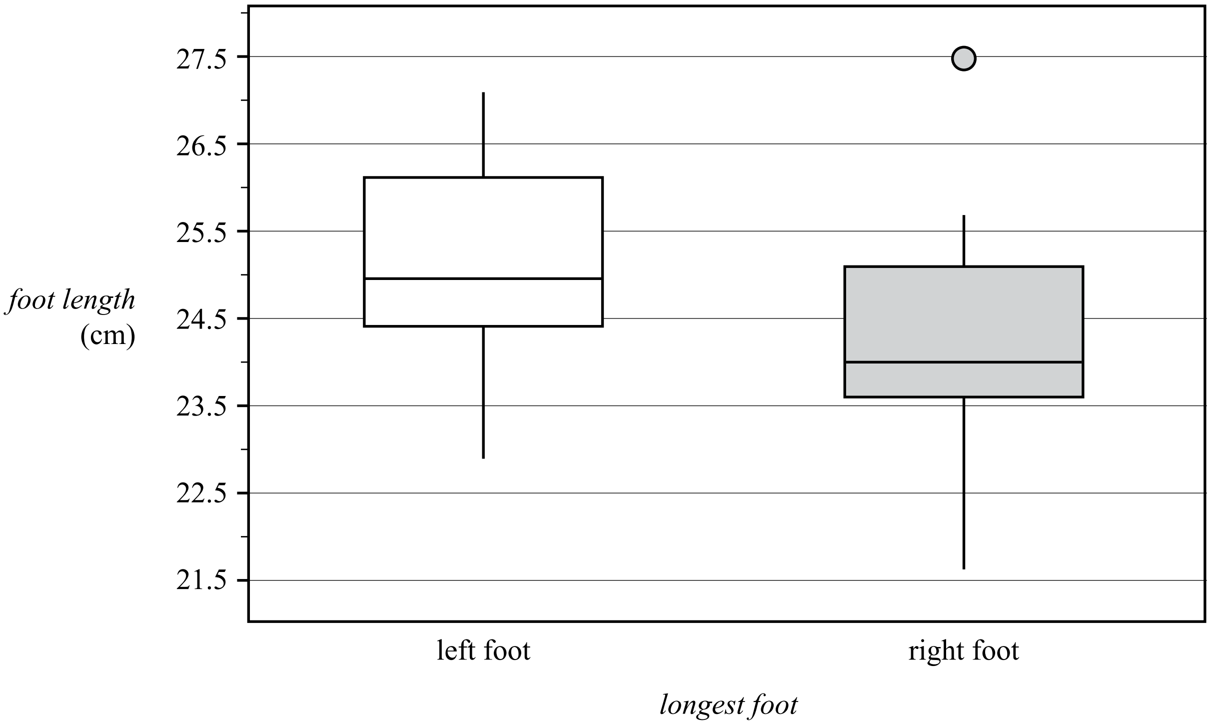
Question 3 (2 marks)

It is common for people to have one foot that is longer than the other.

The *foot length* recorded in this investigation was the longest foot of each student.

It was also recorded whether this was the student's left or right foot.

The resulting data was used to construct the parallel box plots below.



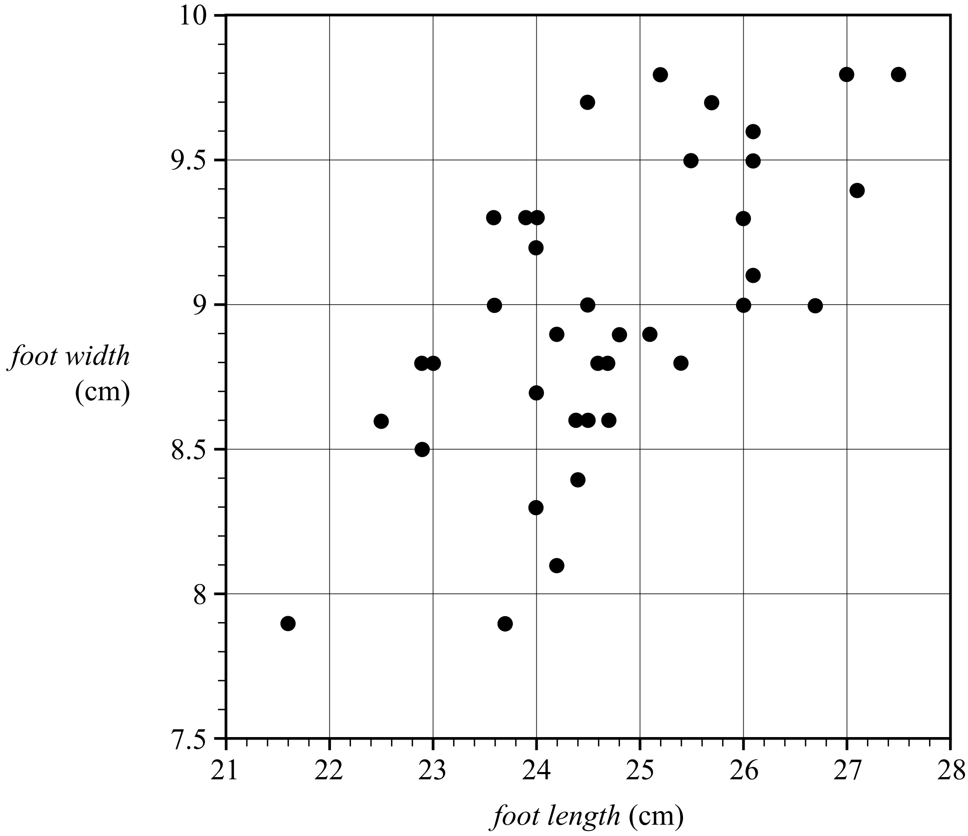
The five-number summary for the two box plots is shown below.

	Foot length				
Longest foot	**Minimum**	**First quartile (Q_1)**	**Median**	**Third quartile (Q_3)**	**Maximum**
left	22.9	24.4	24.95	26.1	27.1
right	21.6	23.6	24.0	25.1	27.5

Write down an appropriate calculation and use it to explain why the student with the longest right foot, who has a foot length of 27.5 cm, is an outlier for this group of students.

Question 4 (7 marks)

The scatterplot below shows *foot width*, in centimetres, plotted against *foot length*, in centimetres, for all 39 students in the study.



When a least squares line is fitted to the scatterplot, the equation of the least squares line is

$$\textit{foot width} = 2.862 + 0.2479 \times \textit{foot length}$$

a. Draw the graph of this least squares line on the **scatterplot above**. 1 mark

(*Answer on the scatterplot above.*)

b. Use the equation of the least squares line to show that, if a student has a *foot length* of 27.8 cm, their *foot width* is predicted to be 9.8 cm, rounded to one decimal place. 1 mark

c. Identify whether the prediction in **part b.** is an interpolation or an extrapolation. 1 mark

d. Interpret the slope of the least squares line in terms of *foot width* and *foot length*. 1 mark

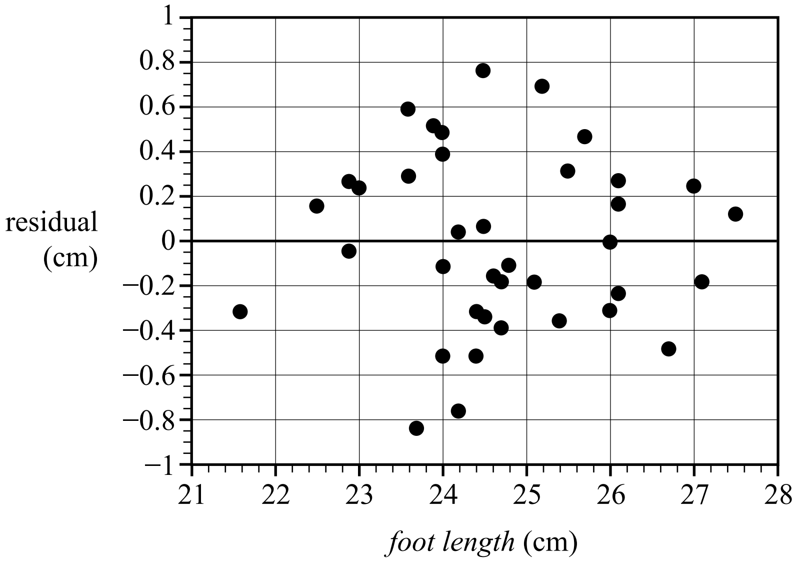
e. The summary statistics for the data in the scatterplot on page 6 are shown in the table below.

	***foot length* (cm)**	***foot width* (cm)**
mean	24.47	8.992
standard deviation	1.317	0.5096

Show that the correlation coefficient, r, is equal to 0.641, rounded to three significant figures. 1 mark

f. What percentage of the variation in *foot width* is **not** explained by the variation in *foot length*? Round your answer to one decimal place. 1 mark

g. The residual plot associated with fitting a least squares line to the data in the scatterplot on page 6 is shown below.



In **part a.**, a least squares line was fitted to the scatterplot. Does this residual plot justify this?

Briefly explain your answer. 1 mark

Question 5 (4 marks)

a. In a new study, the foot lengths of 20 000 women were found to be approximately normally distributed, with a mean of 24.5 cm and a standard deviation of 1.2 cm.

i. Use the 68–95–99.7% rule to determine the number of women in the study who are expected to have a foot length shorter than 20.9 cm. 1 mark

ii. A randomly selected woman from this large study had a foot length of 27.5 cm.

Determine the standard *z*-score for this woman's foot length. 1 mark

b. In another study, the foot lengths of 20 000 men were recorded.

Among these men:

- 500 were recorded as having a foot length longer than 30 cm
- 3200 were recorded as having a foot length shorter than 25.5 cm.

Assuming that the foot lengths for the men are normally distributed, use the 68–95–99.7% rule to determine the mean and standard deviation of this distribution. 2 marks

Recursion and financial modelling

Question 6 (5 marks)

Ramon took out a reducing balance loan of $650 000 with interest calculated monthly.

The balance of the loan, in dollars, after n months, V_n, can be modelled by the recurrence relation

$$V_0 = 650\,000, \qquad V_{n+1} = 1.00275V_n - 3184.75$$

a. What does the number 3184.75 represent? 1 mark

b. Showing recursive calculations, determine the balance of the loan after two months.
Round your answer to the nearest cent. 1 mark

c. Show that the annual compound interest rate for this loan is 3.3% per annum. 1 mark

d. Ramon is scheduled to repay the loan in full in 25 years.

i. Determine the balance halfway through the 25-year term of the loan.
Round your answer to the nearest cent. 1 mark

ii. Explain why the balance halfway through the 25-year term of the loan is not half of the original amount borrowed. 1 mark

Question 7 (4 marks)

Ramon owns a delivery van, which he purchased for $40 000 in 2018.

Ramon depreciates the value of the van using the reducing balance method.

A rule for the value of the van after *n* years is

$$V_n = 40\,000 \times 0.85^n$$

a. Determine the value of the van after two years. 1 mark

b. What is the annual rate of depreciation that Ramon uses? 1 mark

c. Ramon plans to sell the van at the end of the year in which the van's value first falls below $10 000.

At the end of which year will Ramon sell the van? 1 mark

d. Write a recurrence relation in terms of V_0, V_{n+1} and V_n that models the value of the van from year to year. 1 mark

Question 8 (3 marks)

Ramon is planning to invest in an annuity.

The interest rate for this annuity is 3.9% per annum, compounding weekly.

Under this plan, he will receive a regular weekly payment of $660 for 20 years.

a. Determine the amount of money Ramon is planning to invest.
Round your answer to the nearest cent. 1 mark

b. Ramon decides to invest $480 094.50 into the annuity instead of the amount originally planned. The term of the annuity remains 20 years and the interest rate remains 3.9% per annum, compounding weekly.

i. Determine the total amount of interest earned by this annuity over the 20 years.
Round your answer to the nearest cent. 1 mark

ii. Let R_n be the balance of Ramon's annuity after n weeks.

Write a recurrence relation in terms of R_0, R_{n+1} and R_n that models this balance from week to week. 1 mark

END OF SECTION A

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SECTION B – Modules

Instructions for Section B

Select **two** modules and answer **all** questions within the selected modules.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Contents **Page**

Module 1 – Matrices

Question 1 (3 marks)

A transport company has two types of buses for hire: coaches and minibuses.

The buses can be hired either per hour or per day.

The cost of hiring each type of bus per hour or per day is shown in the table below.

Bus type	Cost per hour	Cost per day
coach	$180	$920
minibus	$160	$850

The matrix product (P) that will give the total cost, in dollars, of hiring a coach for either eight hours or for one day is shown below.

$$P = \begin{bmatrix} 180 & 920 \end{bmatrix} \times \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

a. What are the values of a and b in the matrix equation above? 1 mark

$a =$ ☐ $b =$ ☐

b. Explain what the value of $p_{11} - p_{12}$ represents in matrix P. 1 mark

c. The transport company offers a discount of 20% on Mondays.

In the box below, complete the expression by writing down the scalar value that will calculate the discount of 20%. 1 mark

$$\square \times \begin{bmatrix} 180 & 920 \\ 160 & 850 \end{bmatrix}$$

Question 2 (4 marks)

Drivers at the transport company can hold either a light rigid vehicle licence (L), a medium rigid vehicle licence (M) or a heavy rigid vehicle licence (H).

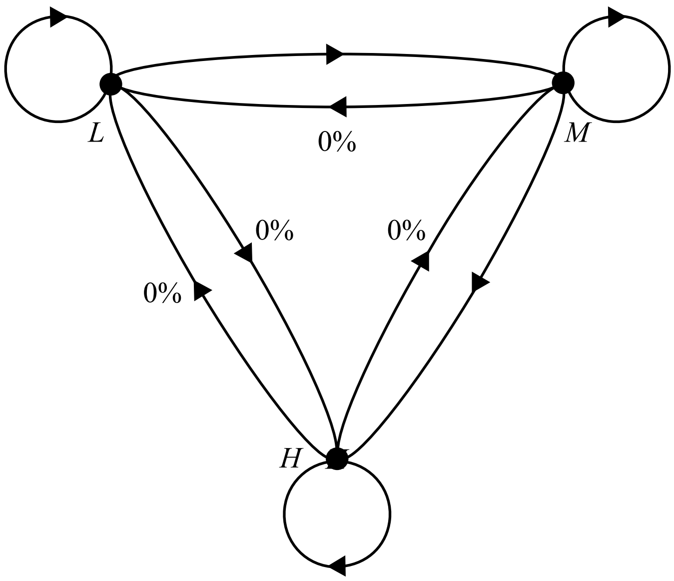
The proportion of drivers who hold each licence type changes from year to year, as shown in the transition matrix, T, below.

$$\begin{array}{c} \textit{this year} \\ \begin{array}{ccc} L & M & H \end{array} \end{array}$$

$$T = \begin{bmatrix} 0.4 & 0 & 0 \\ 0.6 & 0.7 & 0 \\ 0 & 0.3 & 1 \end{bmatrix} \begin{matrix} L \\ M \\ H \end{matrix} \quad \textit{next year}$$

a. Interpret what the 0 in row 3, column 1 means. 1 mark

b. An incomplete transition diagram for matrix T is shown below.



Complete the **transition diagram above** by adding all the missing values, as percentages, on the edges. 1 mark

(*Answer on the transition diagram above.*)

When the transport company first began, in January 2020, it employed 30 bus drivers.

Ten of the drivers held a light rigid vehicle licence and 20 held a medium rigid vehicle licence.

No drivers held a heavy rigid vehicle licence.

c. How many drivers held a heavy rigid vehicle licence by January 2021? 1 mark

d. From January 2023, the transport company will employ an additional three light rigid vehicle licensed drivers and five medium rigid vehicle licensed drivers each year.

They also anticipate that two heavy rigid vehicle licensed drivers will leave the company each year.

Let S_n be the state matrix that shows the number of drivers who held each licence type at the beginning of the n^{th} year after 2023.

The number of drivers who held each licence type for 2023 is shown in matrix S_0 below.

The recurrence relation that is used to predict how many drivers of each licence type is given by

$$S_{n+1} = T S_n + B$$

where

$$S_0 = \begin{bmatrix} 1 \\ 12 \\ 17 \end{bmatrix} \begin{matrix} L \\ M \\ H \end{matrix} \qquad T = \overset{\textit{this year}}{\overset{\begin{matrix} L & M & H \end{matrix}}{\begin{bmatrix} 0.4 & 0 & 0 \\ 0.6 & 0.7 & 0 \\ 0 & 0.3 & 1 \end{bmatrix}}} \begin{matrix} L \\ M \\ H \end{matrix} \; \textit{next year} \qquad B = \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \end{bmatrix} \begin{matrix} L \\ M \\ H \end{matrix}$$

Matrix B is incomplete.

How many light rigid vehicle licensed drivers is the company expected to have in January 2025?

Round your answer to the nearest whole number. 1 mark

Question 3 (5 marks)

Drivers at the company can work a morning shift, an afternoon shift or an evening shift.

The following table shows the shifts one driver, Bill, will work in Week 1. A tick indicates a shift that Bill will do and a cross indicates a shift that Bill will not do.

	Monday	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
Morning shift	✓	×	✓	×	✓
Afternoon shift	✓	✓	×	✓	×
Evening shift	×	✓	×	✓	✓

The following permutation matrix, P, is used to rotate the morning shifts, afternoon shifts and evening shifts.

$$P = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

To determine the shifts that Bill will work in subsequent weeks, the permutation matrix is multiplied by a 3×5 matrix created from the table above.

a. On which day(s) will Bill's shifts never change? 1 mark

b. On which day(s) will Bill be working a morning shift in Week 2? 1 mark

SECTION B – Module 1 – Question 3 – continued

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c. The manager of the transport company, Natalia, can work seven days a week. Each day she works a different number of hours per shift.

Natalia's shifts also change from week to week.

The following table shows the hours per shift that Natalia will work each day in Week 1 and Week 2.

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Week 1	5	7	6	8	0	3	4
Week 2	5	0	8	6	7	4	3

The hours that Natalia will work in Week 1 are also displayed in Matrix N below.

$$N = [5 \;\; 7 \;\; 6 \;\; 8 \;\; 0 \;\; 3 \;\; 4]$$

A different permutation matrix is multiplied by Matrix N, showing the change in hours for Natalia from Week 1 to Week 2.

Write the permutation matrix that is multiplied by Matrix N. 1 mark

$$\begin{bmatrix} \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ \end{bmatrix}$$

d. Drivers are paid at a different rate of pay for weekdays and the weekend.

On a weekday, drivers are paid x dollars per hour.

On the weekend, drivers are paid y dollars per hour.

To determine the weekday hourly rate and the weekend hourly rate, the following system of simultaneous equations is used.

$$\begin{bmatrix} 30 & 8 \\ 22 & f \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1084 \\ 1180 \end{bmatrix}$$

Given that

$$\det \begin{bmatrix} 30 & 8 \\ 22 & f \end{bmatrix} = 304$$

what is the weekday hourly rate of pay and the weekend hourly rate of pay? 2 marks

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Module 2 – Networks and decision mathematics

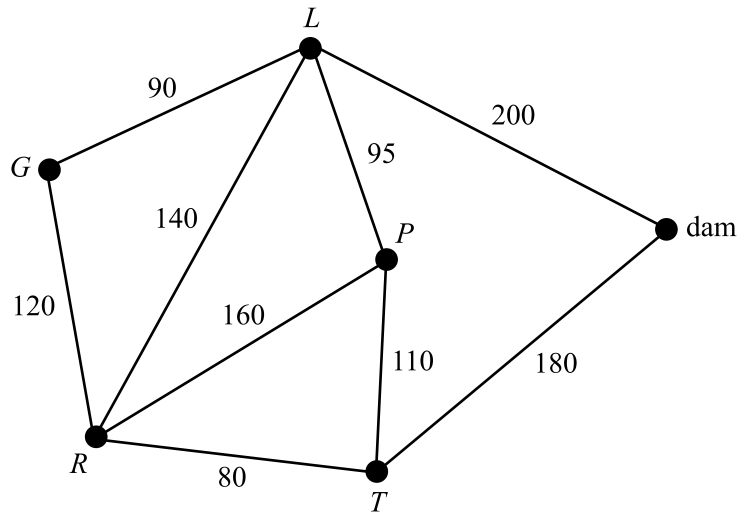
Question 1 (5 marks)

A flower farm has a dam and five different flower gardens: the gerbera (G), lavender (L), petunia (P), rose (R), and tulip (T) gardens.

The edges on the graph below represent the paths between the dam and the flower gardens.

The numbers on each edge represent the length, in metres, of each path.

Visitors to the flower farm can use only these paths to travel around the farm.



a. How many edges does this network have? 1 mark

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b. Identify the vertices that have an odd degree. 1 mark

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c. Determine the shortest distance of the journey that starts and finishes at the dam and visits all the flower gardens, in metres. 1 mark

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d. An adjacency matrix for this network is shown below. This matrix is incomplete.

	G	*L*	*P*	*R*	*T*	dam
G	___	___	___	___	___	___
L	1	___	___	___	___	___
P	0	1	___	___	___	___
R	1	1	1	___	___	___
T	0	0	1	1	___	___
dam	0	1	0	0	___	0

Complete the **adjacency matrix above**. 2 marks

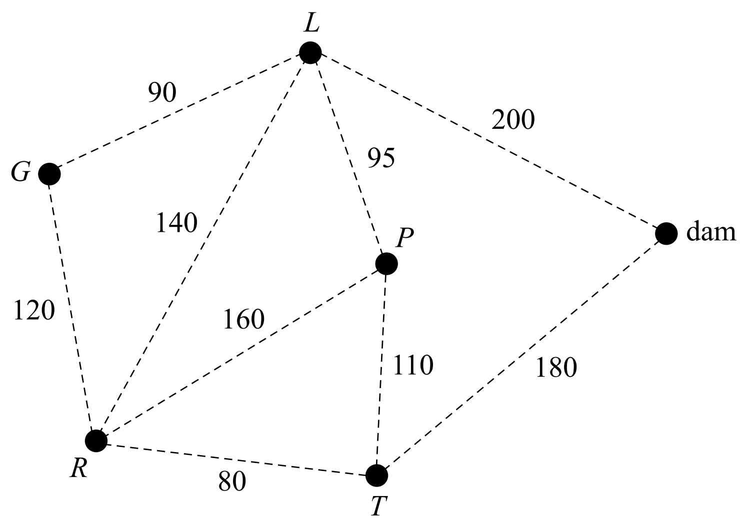
(*Answer on the adjacency matrix above.*)

Question 2 (2 marks)

To ensure that all the flower gardens are regularly watered, a watering system will be installed on the farm.

The cheapest installation only requires pipes to be installed along some of the paths in the network.

a. **i.** On the diagram below, draw the minimum length of pipe that is needed to supply water to all the flower gardens. 1 mark



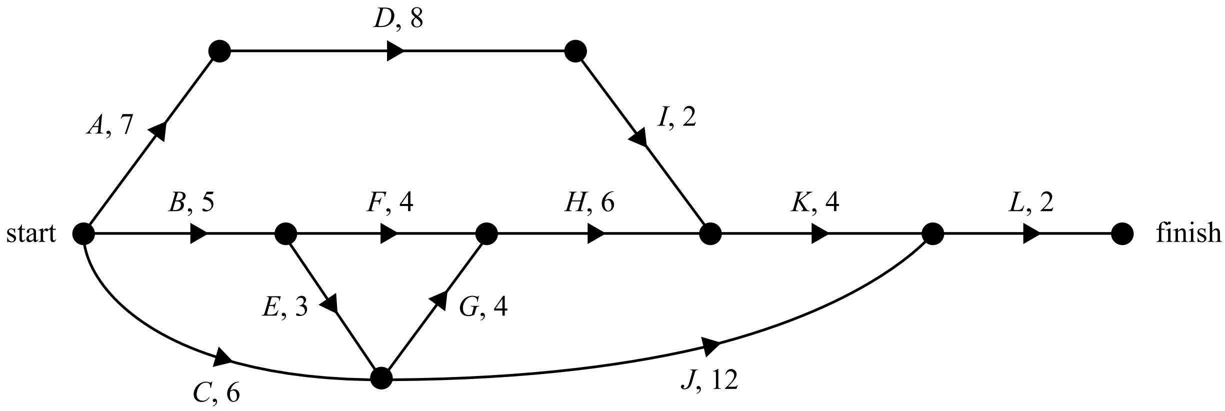
ii. What is the mathematical term that is used to describe this minimum length of pipe drawn in **part a.i.**? 1 mark

Question 3 (5 marks)

The installation of the watering system is a large project that involves 12 activities: *A* to *L*.

The watering system will comprise a series of pipes, pumps and tanks.

The directed network below shows these activities and their completion times, in days.



a. Determine the minimum time to complete this project, in days. 1 mark

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b. Determine the latest start time of activity *D*. 1 mark

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c. Which **two** activities have a float time of two days each? 1 mark

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The owner would like to reduce the completion time for the project.

Five activities, *A*, *B*, *F*, *H* and *K*, all involve the installation of pumps.

If the owner installs a new brand of pump, it is possible to reduce the completion time of each of these five activities by a maximum of two days each.

d. What is the maximum amount of time that can be saved by using the new pumps, in days? 1 mark

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e. The installation of this new brand of pump and the associated reduction in the completion time for the five activities, *A*, *B*, *F*, *H* and *K*, will incur an additional cost.

The table below shows the five activities that can have their completion times reduced by a maximum of two days and the associated daily cost, in dollars.

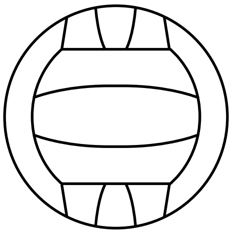
Activity	**Daily cost ($)**
A	400
B	450
F	200
H	350
K	300

What is the minimum amount of money that the owner will need to spend to save the maximum amount of time? 1 mark

Module 3 – Geometry and measurement

Question 1 (3 marks)

For players over the age of 12 years, netball is played with a spherical ball with a radius of 11.3 cm.



a. Calculate the volume, in cubic centimetres, of the ball.
Round your answer to the nearest whole number. 1 mark

b. For players under the age of 12 years, netball is played with a smaller ball.
The radius of the smaller ball and the radius of the larger ball are in the ratio **1 : 1.1**

i. Calculate the radius, in centimetres, of the smaller ball.
Round your answer to one decimal place. 1 mark

ii. By what value can the surface area of the smaller ball be multiplied to give the surface area of the larger ball? 1 mark

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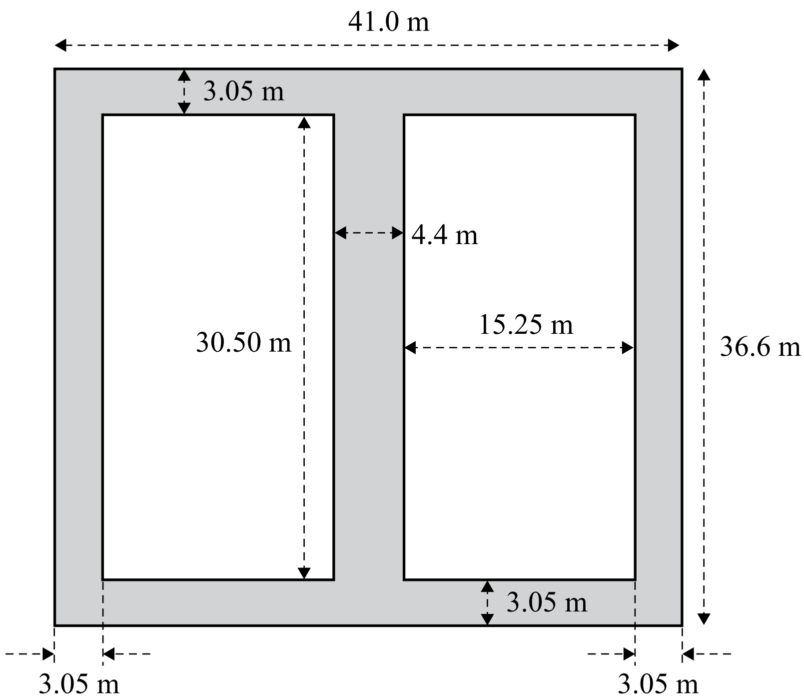
Question 2 (3 marks)

Springtown Council plans to build two all-weather netball courts.

Each of the two netball courts will be 30.50 m long and 15.25 m wide.

Each court is positioned on a rectangular concrete slab.

The layout, with measurements, is shown in the diagram below.



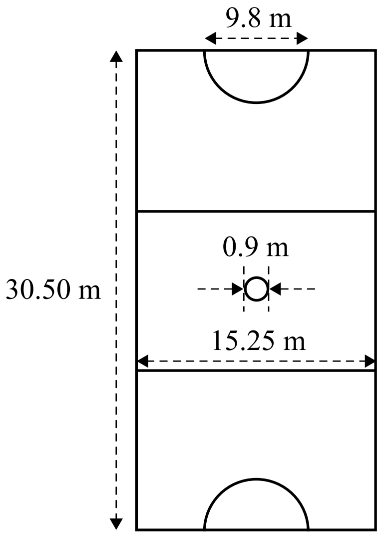
a. The concrete slab will be covered with a synthetic playing surface.
The area around the netball courts, shaded on the diagram above, will be grey in colour.

Determine the total area of the grey surface, in square metres. 1 mark

Lines are to be painted onto the synthetic playing surface to mark the netball courts.

The solid lines on the diagram below represent the lines that are to be painted on one court, based on the following information:

- the length of a court is 30.50 m and the width is 15.25 m
- the centre circle has a diameter of 0.9 m
- the semicircles at each end of the court are based on a diameter of 9.8 m.



b. Write a calculation that shows that the total length of the lines that are to be painted on one court, correct to one decimal place, is 155.6 m. 1 mark

c. All the lines that are to be painted must be 5 cm wide.

The line paint that is to be used covers 7 m^2 for every litre of paint.

The line paint is only available in 0.5 litre tins.

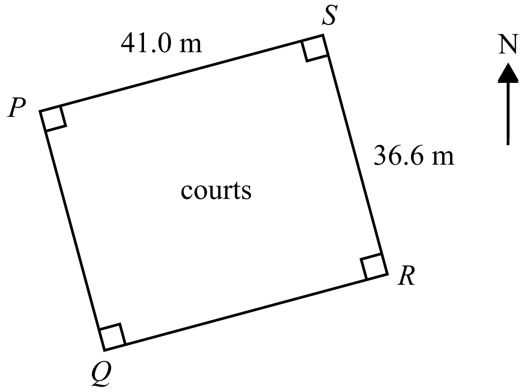
Calculate the minimum number of 0.5 litre tins that must be purchased to apply one coat of paint to each line. 1 mark

Question 3 (2 marks)

The diagram below shows the orientation of the concrete slab on which the netball courts are painted.

P, *Q*, *R* and *S* are the four corners of the slab.

Point *P* is on a bearing of 343° from point *Q*.



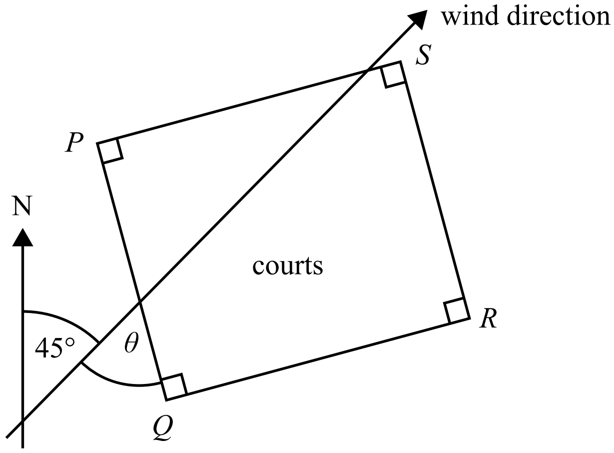
a. Determine the bearing of point *P* from point *S*. 1 mark

b. The prevailing wind during the winter netball season is from the south-west.

This means that the wind is heading across the concrete slab on a bearing of 045°.

The angle, θ, is between the angle of the wind direction and side *PQ*.

This information is shown on the diagram below.

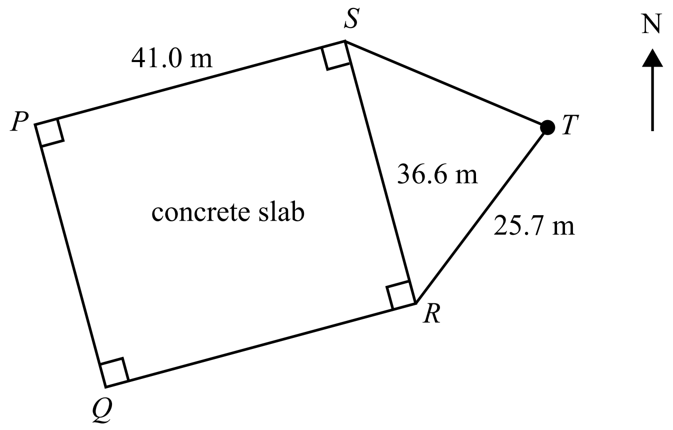


What is the value of θ, in degrees? 1 mark

Question 4 (4 marks)

There is an old tree on one side of the concrete slab.

Council engineers produce the following diagram to illustrate the situation.



T is a point at the base of the tree.

P, *Q*, *R* and *S* are the four corners of the concrete slab.

PS is 41.0 m, *SR* is 36.6 m and *RT* is 25.7 m.

P is on a bearing of 343° from *Q*.

T is on a bearing of 018° from *R*.

Point *R* and point *T* are at the same ground level.

The tree is 15.1 m tall.

a. What is the angle of elevation, in degrees, of the top of the tree from the ground at point *R*?
Round your answer to one decimal place. 1 mark

b. **i.** Show that the angle *SRT* is 35°. 1 mark

ii. Calculate the distance *ST*, in metres.
Round your answer to one decimal place. 1 mark

SECTION B – Module 3 – Question 4 – continued

c. Is it possible for the tree to hit the concrete slab if it falls?
Explain your answer showing appropriate calculations. 1 mark

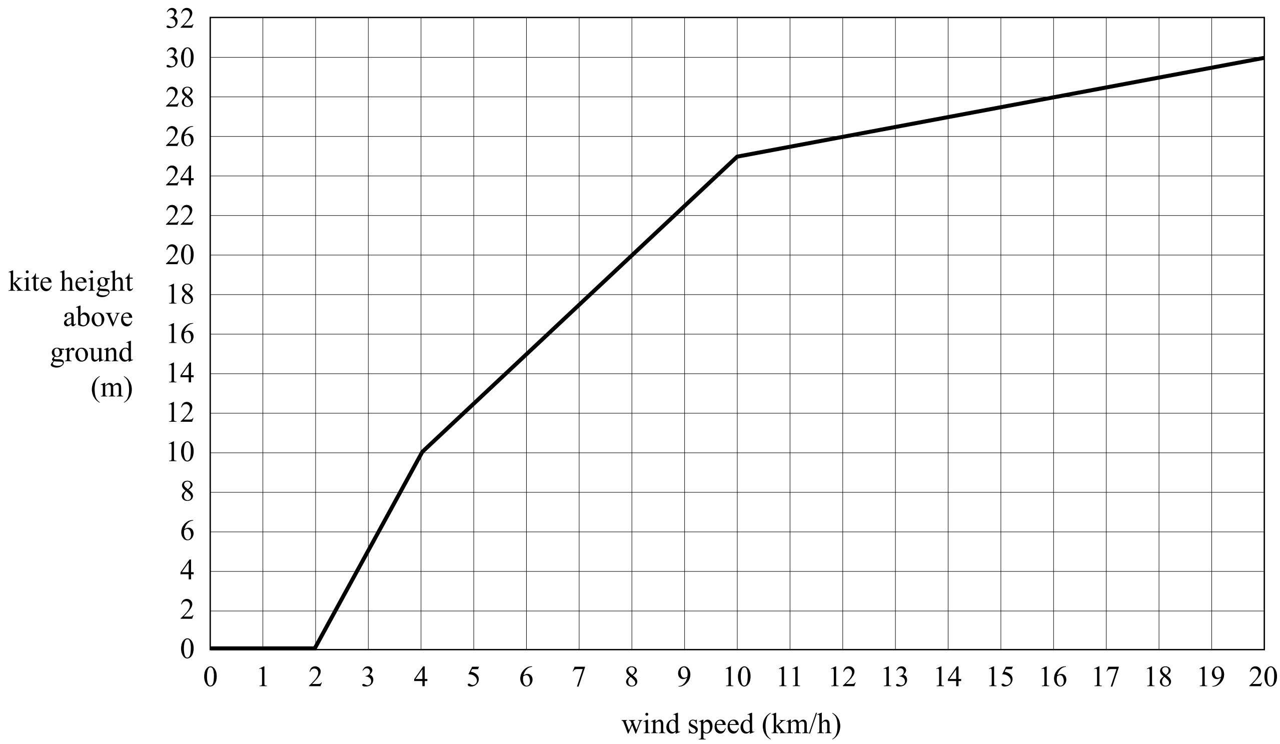
Module 4 – Graphs and relations

Question 1 (2 marks)

Vijay flies kites and is a member of a competitive kite flying club.

Competitive kite flying is performed over a large, flat horizontal space.

The graph below shows the height above ground of a kite, in metres, for the wind speed, in km/h.



a. At 9 am the wind speed was 4 km/h.

What was the height above ground of the kite, in metres? 1 mark

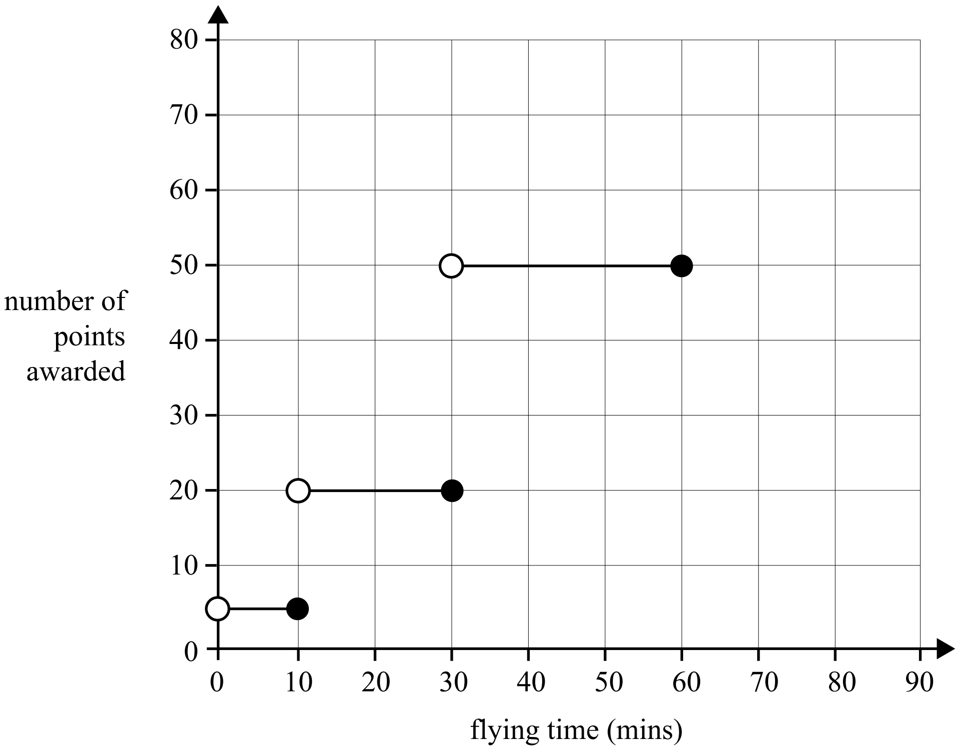
b. At 10 am the height above ground of the kite was 28 m and at 11 am the height above ground of the kite was 20 m.

How much faster was the wind speed at 10 am than it was at 11 am, in kilometres per hour? 1 mark

Question 2 (1 mark)

Vijay's kite club has regular competitions.

Points are awarded based on how long a kite flyer can keep their kite flying.



Kite flyers who can keep their kite flying for more than 60 minutes are awarded 80 points if they do not fly longer than 90 minutes.

Represent this information on the **graph above**.

(*Answer on the graph above.*)

Question 3 (4 marks)

A kite manufacturer produces two different types of kites: parafoil kites and rokkaku kites.

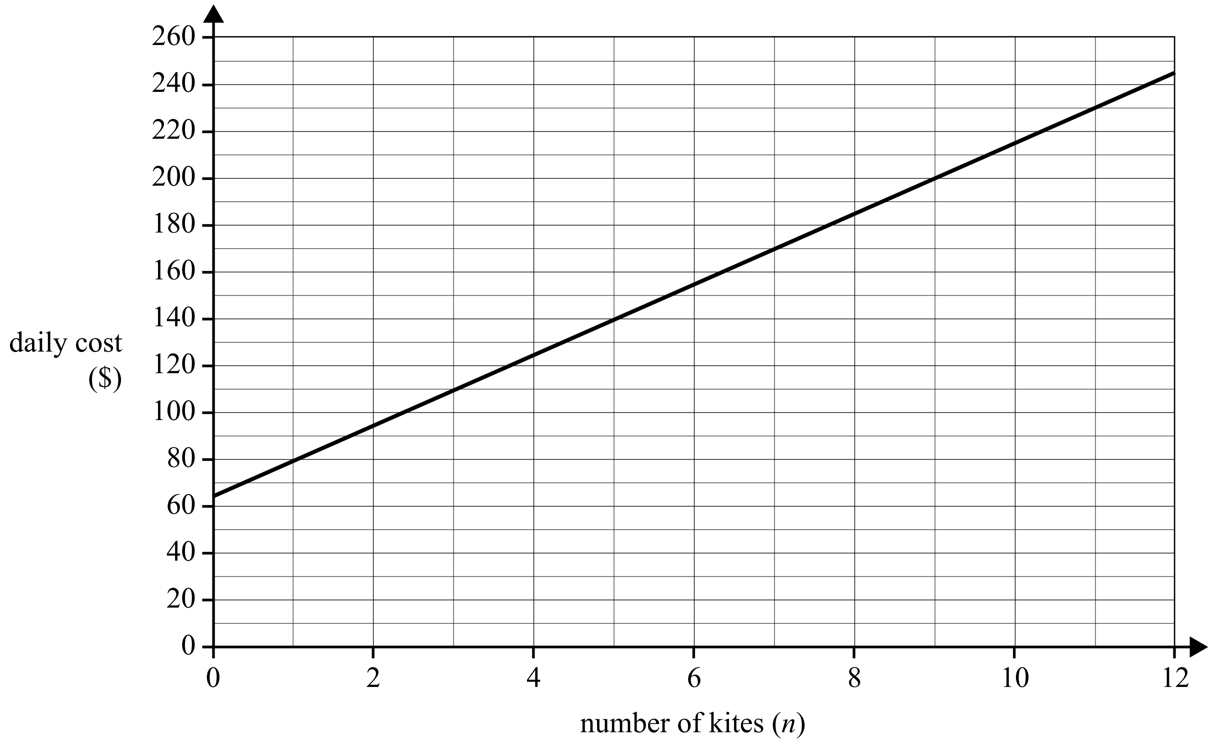
Each day, it costs the manufacturer a flat fee of \$65, plus \$15 per kite, to produce parafoil kites.

a. The daily cost, PC, to produce parafoil kites can be represented as a linear equation where n represents the number of kites produced.

Complete the equation for the daily cost to produce parafoil kites below. 1 mark

$PC = \square \times n + \square$

The following graph displays the daily cost, in dollars, to produce parafoil kites.



The daily cost, RC, for the manufacturer to produce rokkaku kites can be calculated using the following formula.

$$RC = 20n + 30$$

b. Sketch the line representing the daily cost of manufacturing rokkaku kites on the **graph above**. 1 mark

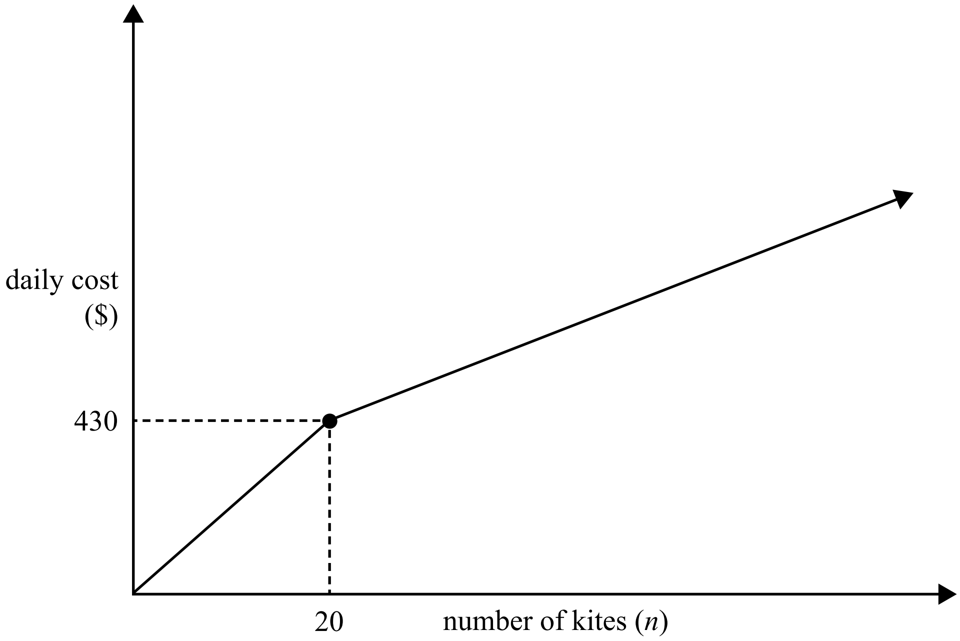
(*Answer on the graph above.*)

c. For what number of kites will the daily cost of production be the same for both types of kites? 1 mark

d. When producing a higher number of rokkaku kites per day, the daily cost of production varies.
The *daily cost* is now given by

$$daily\ cost = \begin{cases} 20n + 30, & 0 \le n \le 20 \\ mn + p, & n \ge 20 \end{cases}$$

This is represented on the graph below.



If 57 kites can be made for a daily cost of \$911, what is the value of *m*? 1 mark

Question 4 (5 marks)

Vijay's kite club is purchasing some new kites.

They are purchasing parafoil kites for beginners and rokkaku kites for experts.

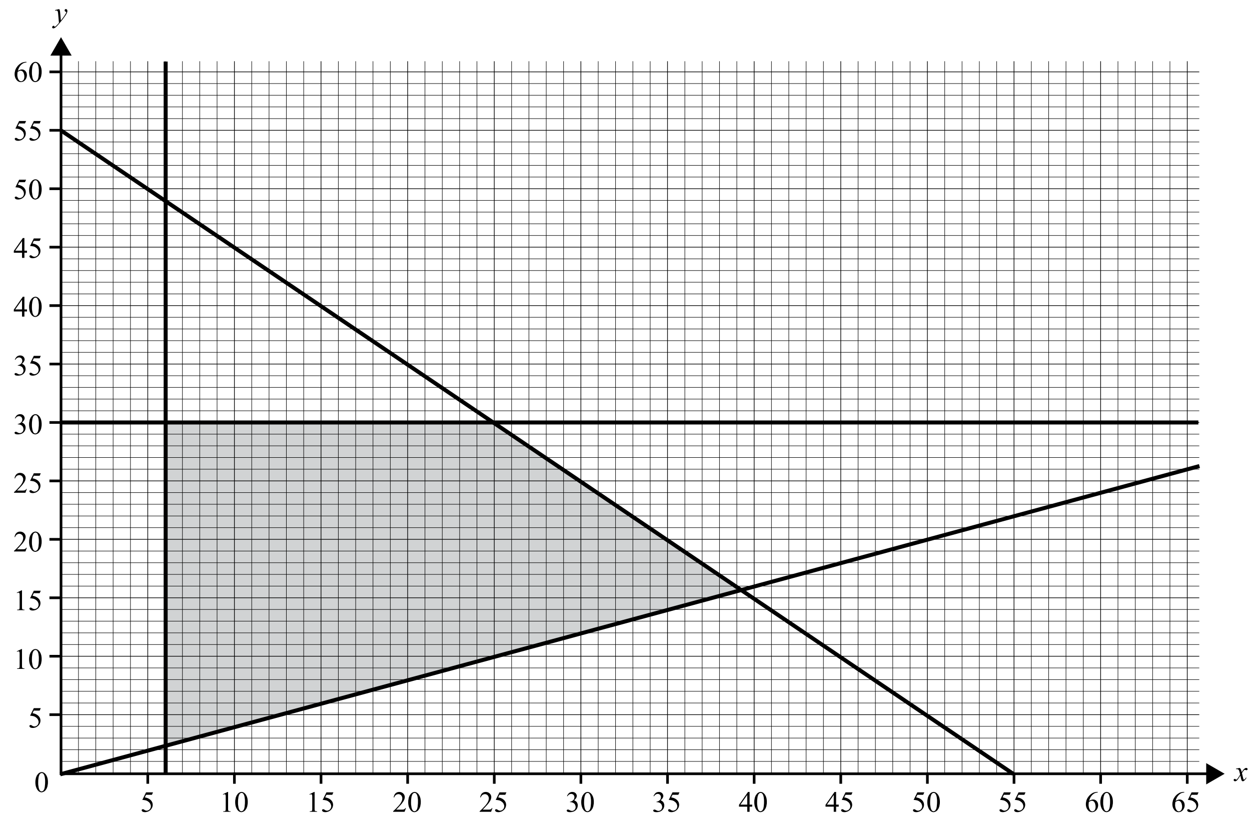
Let x be the number of parafoil kites purchased.

Let y be the number of rokkaku kites purchased.

There are constraints on the number of parafoil kites and rokkaku kites purchased based on the number of beginners and experts among the club members.

Inequality 1	$x \geq 6$
Inequality 2	$y \leq 30$
Inequality 3	$x + y \leq 55$
Inequality 4	$5y \geq 2x$

The graph below shows the lines that represent the boundaries of inequalities 1 to 4. The feasible region has been shaded.



a. Explain the meaning of Inequality 3 in terms of kite purchases. 1 mark

DO NOT WRITE IN THIS AREA

b. The team considers buying four rokkaku kites.

What is the maximum number of parafoil kites they could purchase? 1 mark

c. Parafoil kites cost $88 and rokkaku kites cost $119.

The kite club is planning to purchase both parafoil and rokkaku kites.

What is the difference, in dollars, between the maximum and minimum cost? 2 marks

d. In order to minimise the cost of purchasing new kites, the kite club negotiates new prices for the kites.

The club purchases 35 parafoil kites and 20 rokkaku kites for a total cost of $5225.

This is the maximum number of kites that the club could purchase, given the constraints above.

What is the new reduced price of the rokkaku kites? 1 mark

END OF QUESTION AND ANSWER BOOK

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2023

FURTHER MATHEMATICS

Written examination 2

FORMULA SHEET

Instructions

This formula sheet is provided for your reference.
A question and answer book is provided with this formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Further Mathematics formulas

Core – Data analysis

standardised score	$z = \dfrac{x - \bar{x}}{s_x}$
lower and upper fence in a boxplot	lower $Q_1 - 1.5 \times IQR$ upper $Q_3 + 1.5 \times IQR$
least squares line of best fit	$y = a + bx$, where $b = r\dfrac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$
residual value	residual value = actual value – predicted value
seasonal index	seasonal index = $\dfrac{\text{actual figure}}{\text{deseasonalised figure}}$

Core – Recursion and financial modelling

first-order linear recurrence relation	$u_0 = a, \quad u_{n+1} = bu_n + c$
effective rate of interest for a compound interest loan or investment	$r_{effective} = \left[\left(1 + \dfrac{r}{100n}\right)^n - 1\right] \times 100\%$

Module 1 – Matrices

determinant of a 2 × 2 matrix	$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$
inverse of a 2 × 2 matrix	$A^{-1} = \dfrac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, where $\det A \neq 0$
recurrence relation	S_0 = initial state, $S_{n+1} = T S_n + B$

Module 2 – Networks and decision mathematics

Euler's formula	$v + f = e + 2$

Module 3 – Geometry and measurement

area of a triangle	$A = \frac{1}{2} bc \sin(\theta^\circ)$
Heron's formula	$A = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{1}{2}(a+b+c)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule	$a^2 = b^2 + c^2 - 2bc \cos(A)$
circumference of a circle	$2\pi r$
length of an arc	$r \times \frac{\pi}{180} \times \theta^\circ$
area of a circle	πr^2
area of a sector	$\pi r^2 \times \frac{\theta^\circ}{360}$
volume of a sphere	$\frac{4}{3}\pi r^3$
surface area of a sphere	$4\pi r^2$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a prism	area of base × height
volume of a pyramid	$\frac{1}{3}$ × area of base × height

Module 4 – Graphs and relations

gradient (slope) of a straight line	$m = \frac{y_2 - y_1}{x_2 - x_1}$
equation of a straight line	$y = mx + c$

END OF FORMULA SHEET

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