2009 Further Mathematics GA 3: Written examination 2

GENERAL COMMENTS

There were 26 206 students who sat the Further Mathematics examination 2 in 2009, compared with 25 746 students in 2008. The selection of modules by students in 2008 and 2009 is shown in the table below.

MODULE	% 2008	% 2009
1 – Number patterns	37	37
2 – Geometry and trigonometry	86	85
3 – Graphs and relations	49	49
4 – Business-related mathematics	48	46
5 – Networks and decision mathematics	43	46
6 – Matrices	36	48

The decreasing trend in the proportion of students failing to complete three modules continued in 2009. In general, students were able to answer the first questions in each module very well but found the subsequent more complex questions challenging.

Failure to completely read a question continues to be a concern, with a large number of students apparently not seeing questions such as Data analysis – Questions 1a. and 3a., Graphs – Question 1b. (two parts) and Matrices – Question 4d. Students need to practise reading and analysing the exact question that is being asked.

When asked to use three-median smoothing to smooth a time series, many students confused this with the three-median technique for fitting a three-median regression line or applied three-mean smoothing instead. Students who applied three-mean smoothing also unnecessarily and inappropriately listed the coordinates of all of the points on the scatterplot as a step in the smoothing process.

Questions that required written explanations seemed to be better handled this year than in previous years. However, it was evident that some students were simply transcribing statements from their bound reference notes, regardless of their relevance to the question. Further Mathematics students are expected to explain their understanding or to write an analysis of a calculation result within a real life context.

A significant number of students who gave incorrect answers forfeited the chance to gain any method or consequential marks by not showing working. All Further Mathematics students are strongly encouraged to communicate their mathematical thinking by showing clear and logical written mathematical steps, where appropriate, toward the required solutions. Consequential marks may only be available if the mathematical reasoning shows how a previous student-generated answer was used to generate the answer to the current question.

Rounding off continues to be an issue, especially for students who do not show working and only write their final answer. For example, if the correct answer to a question is 7.81 and a student writes an answer of 7.82, then the answer is wrong. If, however, the student shows working out to indicate its origins, $7.8149 \approx 7.82$, then the rounding error origin of 7.82 is clearly evident.

Students are expected to bring a ruler into the examination to rule straight lines when necessary. Quite a few students drew a line on a graph without using a ruler and the accuracy of the response was insufficient for the question.

In Business-related mathematics, many students continued to inappropriately round sums of money to the nearest five cents, despite the instruction to give an answer correct to the nearest one cent. In practice, Australians round sums of money to the nearest five cents only for cash sales. In answering questions in this module, rounding instructions must be followed.

Areas of strength

Data analysis

- finding the median of a set of one-variable data
- finding a percentage
- calculating a missing seasonal index in a table

Number patterns

- applying the formula for the *n*th term of an arithmetic sequence
- applying the formula for the *n*th term of a geometric sequence

Geometry and trigonometry

- marking in a bearing of 135°
- determining 315° as the correct back bearing of 135°
- determining the volume of a cylinder

Graphs and relations

- drawing in a missing section of a step graph
- substitution into a linear formula that does not require transposition
- finding the coordinates of the intersection of two lines

Business-related mathematics

- finding discount as a percentage of price
- using a TVM function on a calculator

Networks and decision mathematics

- explaining the significance of a zero in an adjacency matrix
- identifying reachability of one point from another
- degree of a vertex
- finding the earliest starting time of an activity in a sequence of activities

Matrices

- identifying the order of a matrix
- recognising the meaning of the elements of a product row matrix within the given context
- recognising the meaning of the elements of a transition matrix within the given context

Areas of weakness

Data analysis

- describing a general pattern in a time series involving more than a single trend
- plotting two distinct points to produce an accurate straight line from an equation
- understanding that a causal statement cannot be used when referring to the value of r^2
- interpreting seasonal indices

Number patterns

- working with difference equations in general
- confusing arithmetic and geometric sequences
- understanding that a negative value of r is evident from the negative trend shown on the graph

Geometry and trigonometry

- understanding the difference between 'angle of elevation' and 'angle of depression'
- understanding that 'the angle between LY and LF' means the angle YLF
- applying appropriate lengths to find the area of a triangle when using $A = \frac{1}{2}bh$
- determining the volume of a hemisphere

Graphs and relations

- using a straight edge to draw a straight line rather than presenting a freehand line
- setting up simultaneous equations

Business-related mathematics

- rounding a monetary sum to the nearest **one** cent when required. Many students rounded to the nearest five cents
- finding the annual flat rate of interest when given the principal and monthly interest amount
- showing a clear calculation that demonstrates the mathematical logic in more complex questions such as Questions 3e. and 4a.
- considering the reasonableness of a solution

Networks and decision mathematics

- determining the float times of all activities not on a critical path
- fitting a new directed edge to a network diagram from a description of its earliest and latest starting times

Matrices

- solving a 3×3 matrix equation by matrix methods on the calculator
- showing a clear calculation that demonstrates the mathematical logic in a more complex question
- finding the second and subsequent transitions using $L_{n+1} = \begin{bmatrix} 0.85 & 0.25 \\ 0.15 & 0.75 \end{bmatrix} L_n - \begin{bmatrix} 5 \\ 7 \end{bmatrix}$

As recommended in several past Assessment Reports, teachers could usefully address the following issues with students.

- Some students again relied heavily on using formulas, many of which they did not understand. This applied especially in geometry and business mathematics.
- Failure to read a question completely and/or carefully was again evident on many papers. Students should be instructed on the effective use of reading time.
- If an answer to a previous question had to be rounded off and then used in a subsequent question, the rounded answer could be used. However, the correct use of an unrounded answer does not preclude full marks if full working out is shown.
- A number (for example, length) found during working out **within** a question should **not** be rounded off when used to determine the subsequent answer for that question.
- If working out is absent or difficult to follow, then rounding error allowances, method and consequential marks cannot be applied if the answer is incorrect.
- When using a TVM function, a table to show students' inputs should be written as 'working out', including relevant negative signs and decimal places for inputs as applicable.

SPECIFIC INFORMATION

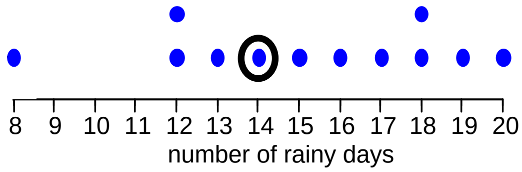
Section A

Data analysis

Questions 1 and 2

Marks	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**Average**
%	0	1	8	23	40	7	15	5	**4.1**

Question 1
1a.



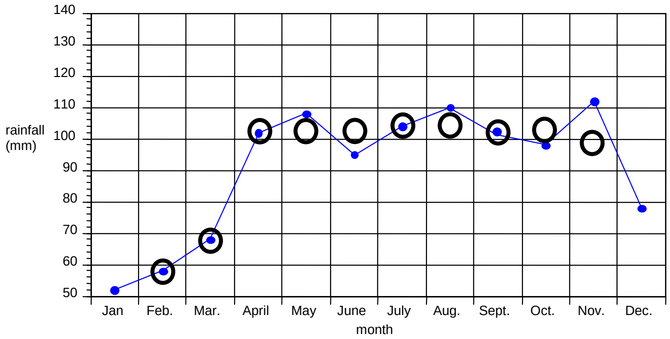
A number of students apparently did not see this question and left it blank, even though the verb 'circle' started the sentence and was in bold type.

1bi.
15.5

1bii.
92%

Question 2
2a.
November

2b.



As stated in the study design, median smoothing is to be treated as a graphical technique. That is, when students are given a time series graph to smooth using median smoothing, they are expected to locate medians directly from the graph by inspection, not by listing out the coordinates of each point and then performing the task numerically. Tackling such problems by first listing out the coordinates is unnecessary, time-consuming and usually inaccurate. This approach is not consistent with the intention that a time series plot should be able to be smoothed quickly without doing any calculations. In the 2009 examination, the process of listing out the data points first seemed to lead students to do three-mean smoothing rather than three-median smoothing as required.

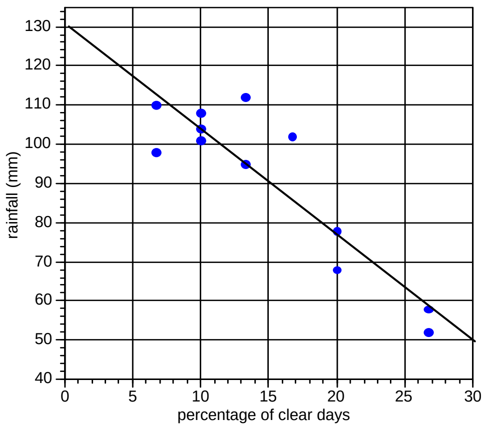
2c.
In the smoothed time series, there are two key trends. Until April, there is an increase in monthly rainfall. It then remains relatively constant for the remainder of the year.

In answering such questions, students must always refer to the given data rather than their own personal experiences. Some students referred to the seasons of the year but there was no indication in the data to suggest that these figures belonged to the southern hemisphere. Some students referred to rainfall in January and/or December but these months did not belong to the smoothed data plot.

Questions 3 and 4

Marks	0	1	2	3	4	5	6	7	8	Average
%	11	11	11	11	12	13	13	12	5	3.8

Question 3
3a.



By choosing two points that were close to each other, many students were unable to plot this line within an acceptable accuracy. Some students apparently failed to see this question and left it unanswered.

Students are expected to draw straight lines with the aid of a ruler or other straight edge. Freehand drawings are inaccurate and will generally lead to errors that are penalised.

3b.
37.2 mm, correct to one decimal place

3ci.
80.81% of the variation in the **rainfall** can be explained by the variation in the **percentage of clear days**.

The coefficient of determination refers to the relationship in which the variation in the dependent variable can be explained by the variation in the independent variable. It does not imply any causation between the two variables. Unacceptable causation terms for this relationship include 'is determined by', 'is due to', 'is caused by' and 'is because of'.

Other common errors included students reversing the variables, incorrectly suggesting that 80.81% of the variation in percentage of clear days (independent value) was explained by the variation in the rainfall (dependent value). Others incorrectly referred to the absolute values of one or both of the variables rather than to the variation in them.

It was apparent that some students transcribed material from their bound reference notes and gave inappropriate answers that made reference to variables such as age, cost or profit.

3cii.
-0.899, correct to three decimal places

Many students failed to include the negative sign as indicated by a generally negative slope of the data in the graph.

Question 4
4a.
1.10

$0.78+1.05+1.07+s = 4$
$\Rightarrow s = 1.1$

A common incorrect answer was 1.09.

4b.
241 mm

4c.
The autumn rainfall is 5% above the average for the four seasons of the year.

A common incorrect answer suggested that the autumn rainfall was 5% above the **monthly** average.

Students are expected to be clear in their explanations. It was again evident that some students simply copied material from their bound reference notes and gave answers that referred to autumn 'sales'.

Module 1 – Number patterns

Question 1
1a–dii.

Marks	0	1	2	3	4	5	Average
%	2	6	11	18	21	43	3.8

1a.
37

1b.
43

$70 = 28 + (n - 1)$
$\therefore n = 43$

A common incorrect answer was 70 – 28 = 42.

1c.
750

$S_{20} = \frac{20}{2}(2 \times 28 + (20 - 1)) = 750$

1di.
$2828

$2800 \times 1.01 = 2828$

1dii.
$3032

$2800 \times 1.01^{9-1} = 3031.9988$

Question 1e–2b.

Marks	0	1	2	3	4	5	Average
%	29	14	13	14	15	16	2.2

1e.
$97 398

1f.
$67.75

Revenue = $2800 \times 1.01^{25} = 3590.81$
Seats in row 26 = 28 + (26-1) = 53

Cost/seat = $\frac{3590.81}{53}$ = \$67.7511

Many students were able to find the correct number of seats in row 26 but could not correctly complete the answer to this question.

2a.
2%

A common incorrect answer was 1.02%.

2b.
$2500

$$R_4 = \frac{2601}{1.02^2} = 2500$$

Question 3a–d.

Marks	0	1	2	3	4	5	Average
%	38	9	16	15	9	12	**1.8**

3a.
9480

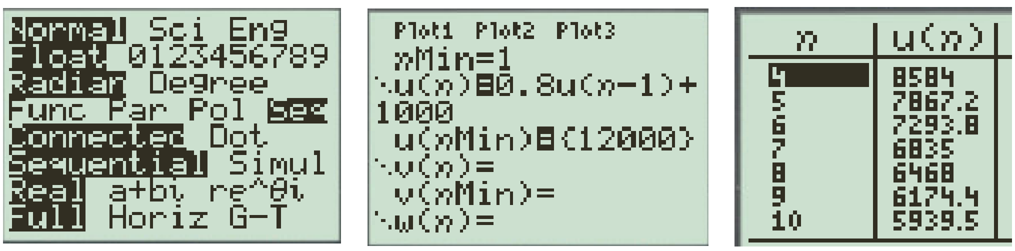
$T_2 = 0.8(12\,000) + 1000 = 10\,600$

$T_3 = 0.8(10\,600) + 1000 = 9480$

3b.
$T_3 - T_2 = 9480 - 10\,600 = -1120$ and $T_2 - T_1 = 10\,600 - 12\,000 = -1400$
Since $-1120 \neq -1400$, there is no common difference, hence the sequence is not arithmetic.

Students had to show two appropriate calculations and a conclusion that referred to the results of the calculations. Some students calculated $T_3 - T_2$ and then $T_1 - T_2$. As one of these subtractions must be positive and the other negative they could not be equal, even if an arithmetic progression existed, and so are not applicable in determining the existence of a common difference. A number of calculations showed that the sequence was not geometric but concluded that it was not arithmetic.

3c.
Week 10



The sequence function on the calculator is very useful here, as illustrated above, or similar.

3d.
When $T_n = 5000$, $T_{n+1} = 0.8 \times 5000 + 1000 = 5000$
That is, $T_n = T_{n+1} = 5000$ and so the performance season will continue indefinitely as the audience will never fall below 5000.

Students were required to do an appropriate calculation and provide a correct conclusion that was supported by a clear calculation.

Module 2 – Geometry and trigonometry

Students who choose this module must ensure their calculator is operating in degree mode. Where radians are used, students may miss out on some or all marks at each instance. The production of an inappropriate answer, such as an angle of elevation of 0.12°, should be seen as a possible indicator that radians have been used inadvertently.

Question 1a–c.

Marks	0	1	2	3	Average
%	7	12	22	59	2.3

1a.

$\frac{50}{400} = 0.125$

A surprising number of students did not attempt this question.

1b.

7.1°, correct to one decimal place

The answer was expected in degrees correct to one decimal place rather than in minutes and seconds of angle.

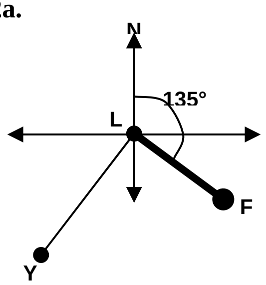
1c.

403.1 m, correct to one decimal place

Question 2a–d.

Marks	0	1	2	3	4	Average
%	13	10	19	27	32	2.6

2a.



2b.

75°

A significant number of students showed a poor understanding of terminology in this question by offering two answers of $LY = 30°$ and $LF = 45°$.

2c.

6.87 km, correct to two decimal places

Many students forgot to take the square root as the final step of the cosine rule and gave 47.13 m as their answer. Others showed poor arithmetic skills in presenting $9 + 49 - 42 \cos 75° = 16 \cos 75°$.

2d.

315°

Question 3a–c.

Marks	0	1	2	3	4	Average
%	26	13	12	17	31	**2.1**

3a.

$\frac{360}{5} = 72°$

A valid calculation that resulted in an answer of 72° was expected. The use of a formula that calculated the interior angle of a polygon required some explanation of the mathematical reasoning in this 'show that' question.

Circular arguments of this type were not acceptable:

$x + x + 72° = 180°$
$\Rightarrow \quad 2x = 108°$
$\Rightarrow \quad x = 54°$
$\Rightarrow \quad 54° + 54° + POQ = 180°$
$\Rightarrow \quad POQ = 72°$

3b.

$OP = \frac{15}{\sin 36^\circ} = 25.52$, correct to two decimal places

The sine rule could also have been used. However, the transposition to make *x* the subject is required in a 'show that' question:

$$\frac{30}{\sin 72^\circ} = \frac{x}{\sin 54^\circ}$$

$\therefore x = \frac{30 \sin 54^\circ}{\sin 72^\circ}$ (Some students did not show this step.)

$\therefore x = 25.52$, correct to two decimal places

3c.

1548 cm^2

The area of one triangle could be found with the sine rule or $\frac{1}{2}bh$. Some students who used $\frac{1}{2}bh$ incorrectly stated that the height of the isosceles triangle was 25.25 cm from Question 3b.

Question 4a–b.

Marks	0	1	2	3	4	Average
%	24	25	38	4	8	**1.5**

4a.

17 191 cm^3

Many students were only able to find the volume of the cylinder. A surprising number of students used $\frac{1}{2} \times \frac{4}{3}\pi r^2$ rather than $\frac{1}{2} \times \frac{4}{3}\pi r^3$ for the hemisphere, despite the formula for the volume of a sphere appearing on the formula sheet. Students should use the π constant from their calculator for calculations rather than $\pi \approx 3.14$, which is a simple approximation useful for hand calculation.

4b.
93.6%

Proportion of volume remaining = $\left(\frac{2}{5}\right)^3 = \frac{8}{125} = 0.064$

Therefore, proportion of volume removed = $\frac{125}{125} - \frac{8}{125} = \frac{117}{125} = 0.936$

A common incorrect answer was 60%, which represents the percentage of the height removed rather than the percentage of volume of oil removed. While the actual volume of a cone was not needed to answer this question, some students compared the volumes of two cones of *height* = 50 cm and *height* = 20 cm. Although the radius of the cone was not given, these students assumed it to be 12 cm and determined the volume of a cone of *height* = 50 cm. However, in finding a volume of a smaller cone of *height* = 20 cm, they again used a radius of 12 cm, which meant their two cones were not of similar proportions.

Module 3 – Graphs and relations

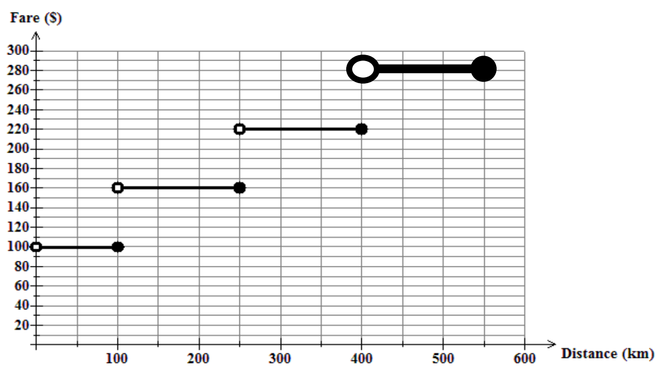
Question 1a–b.

Marks	0	1	2	Average
%	2	15	83	**1.8**

1a.
250 km

The most common incorrect answer was 249 km, suggesting confusion with a range for the *distance* expressed as $100 \le distance \le 250$.

1b.



Open and closed terminals were required as shown. The most common error showed the line ending at a distance of 525 km.

Question 1c–e.

Marks	0	1	2	3	4	5	Average
%	8	14	6	41	6	25	**3**

1c.
$30

1d.
360 km

$220 = 40 + 0.5d$
$\therefore\ d = 360$

Students must clearly identify their solution to a question. For this question, several answers consisted merely of the line $220 = 40 + 0.5\times360$.

1e.
$a = 60\ ,\ \ b = \frac{2}{5} = 0.4$

This involved solving two simultaneous equations selected from:

- $100 = a + 100b$
- $160 = a + 250b$
- $220 = a + 400b$.

Some students found only the equation $160 = a + 250b$ and then made up a value for a (usually $a = 0$) to find a value of b.

Question 2a.

Marks	0	1	Average
%	4	96	**1**

Correct point marked at (10, 45)

Question 2b–d.

Marks	0	1	2	3	Average
%	11	20	19	50	**2.1**

2b.
64 (on the table) and the point (64, 28.8) correctly located on the graph

Some students did not complete both parts of this question.

2c.
$k = 0.45$

2d.
$64.80

Question 3

Marks	0	1	2	Average
%	43	9	48	**1.1**

$m = 0.3$

Flight = $20 + 0.47 \times 450$ = $231.50
Left for luggage = $299 – 231.50$ = $67.50

$m = \frac{67.50}{15^2} = 0.3$

A common error assumed $299 was the charge for luggage.

Question 4

Marks	0	1	2	Average
%	87	9	5	**0.2**

$17 300

The objective function was P = 1300*x* + 2100*y*.
Substituting (2, 7) gave the maximum profit of $17 300.

Once the feasible region was identified, most students found the point of intersection at (4.5, 5.5) and substituted these values into the objective function without considering the context of the question. The two variables, number of crew and number of flights cannot have fractional values and so (4.5, 5.5) did not answer this question. Students had to find integer values **within** the correct feasible region. A method mark was available for applying any two applicable integer pairs other than (0, 0). Some students applied the point (0, 10), but this was not within the feasible region.

Module 4 – Business-related mathematics

A major issue with 'rounding to the nearest cent' resurfaced in 2009. Many students rounded all their answers to the nearest five cents, likely recalling their daily spending experiences where the one-cent coin no longer exists. Many students tended to show little or no working out in this module and merely wrote their final answers, very often to their detriment. Where students show neither working nor the correct answer before rounding to five cents, a rounding error cannot be applied and the answer mark for the question will not be awarded. No method mark is available when only a single and incorrect answer is presented as a solution.

In this module, students must be prepared to give answers correct to the stated accuracy. This may be to the nearest thousand dollars, nearest dollar, nearest cent or the nearest fraction of a cent. Sums of money correct to fractions of a cent are common in financial transactions. For instance, the price of petrol is given correct to the nearest tenth of a cent. For a cash payment, the cost of a petrol purchase is rounded to the nearest five cents, but it is rounded to the nearest cent for a credit/debit card payment.

Question 1–3b.

Marks	0	1	2	3	4	5	6	Average
%	1	6	13	19	23	21	17	**3.9**

1a.
$380

1b.
24%

A common incorrect answer came from $\frac{380}{500} = 76\%$.

2a.
$3.75

A common incorrect answer came from $\frac{250 \times 1.5 \times \frac{1}{12}}{100} = \$0.31$.

2b.
28.8%, correct to one decimal place

A common incorrect answer came from $\frac{6}{250} = 2.4\%$.

3a.

$\frac{4.4}{4} = 1.1$

3b.

\$3876.97

$3400 \times 1.011^{12} = 3876.973\,068\ldots$

Many students incorrectly rounded this result to the nearest five cents and gave \$3876.95.

Question 3c–4c.

Marks	0	1	2	3	4	5	6	Average
%	22	11	17	15	12	11	10	**2.6**

3c.

\$1020.86

FV after 24 periods = \$ 4420.858 874…
Interest = 4420.86 – 3400 = 1020.86

Many students incorrectly rounded this result to the nearest five cents and gave \$1020.85.

4a.

\$11 440

Amount of depreciation = 22 000 × 0.12 × 4 = 10 560
Depreciated value = 22 000 – 10 560 = 11 440

Many students did not fully answer this question and found only the amount of depreciation.

4b.

\$10 953.17

$22\,000 \times 0.84^{4} = 10\,953.169\,92\ldots$

Many students incorrectly rounded the result to the nearest five cents and gave \$10 953.15.

4c.

Reducing balance method by \$11 046.83

Question 5a–c.

Marks	0	1	2	3	Average
%	57	15	15	13	**0.9**

5a.

\$151 133.38

$N = 60$
$I\% = 4.65$
$PV = 200000$
$PMT = -1500$
$\boldsymbol{FV = 151133.38..}$
$P/Y = 12$
$C/Y = 12$

Many students incorrectly rounded this result to the nearest five cents and gave \$151 133.40.

The application of the annuities formula was again evident in some cases, despite this not being in the study design. The result was usually inaccurate due to rounding issues through the calculation. Students should use the TVM function of their calculator instead of the annuities formula.

5b.
$41 133.38

60 × 1500 – (200 000 – 151 133.38) = 41 133.38

Many students incorrectly rounded the result to the nearest five cents and gave $41 133.40.

5c.
$1825.03

N = 60
I% = 5.65
PV = 95200
***PMT* = -1825.0291..**
FV = 0
P/Y = 12
C/Y = 12

Many students incorrectly rounded the result to the nearest dollar rather than the nearest cent as required and gave $1825.

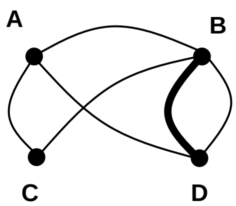
Module 5 – Networks

Question 1a–bi.

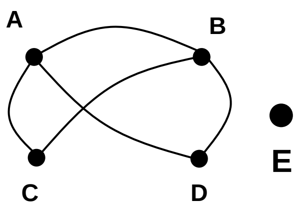
Marks	0	1	2	3	Average
%	5	22	13	60	2.3

1a.
There is no land border between *E* and any other suburb.

1bi.



1bii.



The vertex *E* had to be clearly isolated from all edges and other vertices.

Question 2a–b.

Marks	0	1	2	Average
%	2	13	85	1.8

2a.
K and *F*

2b.
(*K*)*JH*, (*K*)*FJH* and (*K*)*MJH*

Question 3a–cii.

Marks	0	1	2	3	4	5	Average
%	2	4	9	23	30	32	3.7

3a.
4

3bi.
P

3bii.
5

The landmarks are *N*, *T*, *R*, *P*, *U*.

3ci.
(*SR*) *QPONU* (*T*)

3cii.
Any three of:

- (*SR*)*QPUTNO*
- (*SR*)*QPONTU*
- (*SR*)*TUNOPQ*
- (*SR*)*UTNOPQ*.

Question 4a–e.

Marks	0	1	2	3	4	5	Average
%	14	22	26	21	9	9	2.2

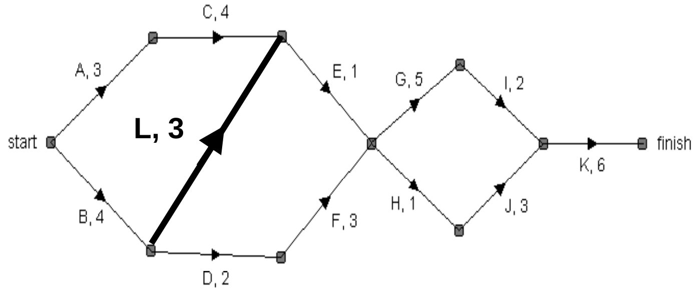
4a.
7

4b.
BDFGIK

4c.
3

H or *J* can be delayed for a maximum of three weeks.

4d.



The answer required the correct edge with an arrow marked in the correct direction.

Some students apparently failed to see this question and left it unanswered.

4e.
25 weeks

A new critical path is created through *BLEGIK* with duration of 4+7+1+5+2+6 = 25 weeks.

Module 6 – Matrices

Question 1a–bii.

Marks	0	1	2	3	Average
%	4	6	11	79	2.7

1a.
2×3

1bi.
$$\begin{bmatrix} 131.30 \\ 130.75 \end{bmatrix}$$

1bii.
Safeworth

Question 2a–b.

Marks	0	1	2	3	Average
%	7	33	7	53	2.1

2a.
35 and 2

2b.
$32

$$\begin{bmatrix} 283 & 28 & 5 \\ 35 & 4 & 2 \\ 84 & 3 & 7 \end{bmatrix}^{-1} \begin{bmatrix} 8712 \\ 1143 \\ 2609 \end{bmatrix} = \begin{bmatrix} 27 \\ 32 \\ 35 \end{bmatrix}$$

It was not sufficient for students to simply provide the column matrix as the solution without extracting and writing the cost of the teacher's ticket as required by the question. A surprising number of students were unable to use the matrix equation to solve it using technology, despite having the two correct numbers in Question 2a.

Question 3a–biii.

Marks	0	1	2	3	4	5	Average
%	2	4	13	41	6	35	3.5

3a.
500

3bi.
25%

A common incorrect answer was 0.25%.

3bii.
5%

3biii.
206

85% × 160 + 40% × 120 + 10% × 220 = 206

Question 3c–4b.

Marks	0	1	2	3	4	Average
%	24	17	33	13	13	**1.8**

3c.

$$S_1 = \begin{bmatrix} 310 \\ 130 \\ 60 \end{bmatrix}$$

3d.
361

$$T^3 S_0 = \begin{bmatrix} 361 \\ 91.1 \\ 47.9 \end{bmatrix}$$

Question 4
4a.
68

$$L_2 = T \times L_1 - \begin{bmatrix} 5 \\ 7 \end{bmatrix} \Rightarrow L_2 = \begin{bmatrix} 0.85 & 0.25 \\ 0.15 & 0.75 \end{bmatrix} \begin{bmatrix} 95 \\ 97 \end{bmatrix} - \begin{bmatrix} 5 \\ 7 \end{bmatrix} = \begin{bmatrix} 100 \\ 80 \end{bmatrix}$$

$$\therefore L_3 = T \times L_2 - \begin{bmatrix} 5 \\ 7 \end{bmatrix} \Rightarrow L_3 = \begin{bmatrix} 0.85 & 0.25 \\ 0.15 & 0.75 \end{bmatrix} \begin{bmatrix} 100 \\ 80 \end{bmatrix} - \begin{bmatrix} 5 \\ 7 \end{bmatrix} = \begin{bmatrix} 100 \\ 68 \end{bmatrix}$$

A common error was for students to square the transition matrix first and then use the initial state matrix.
Students should note that:

$$L_3 \neq T^2 \times L_1 - \begin{bmatrix} 5 \\ 7 \end{bmatrix} = \begin{bmatrix} 0.85 & 0.25 \\ 0.15 & 0.75 \end{bmatrix}^2 \times \begin{bmatrix} 95 \\ 97 \end{bmatrix} - \begin{bmatrix} 5 \\ 7 \end{bmatrix}.$$

4b.
12

The subtracted column matrix $\begin{bmatrix} 5 \\ 7 \end{bmatrix}$ indicates that, each week, another 5 + 7 = 12 students will no longer turn up to any rehearsal.