VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

**Victorian Certificate of Education Year**

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

Letter

**STUDENT NUMBER**

# GENERAL MATHEMATICS

## Written examination 2

**Day Date**

**Reading time: *.** to *.** (15 minutes)**
**Writing time: *.** to *.** (1 hour 30 minutes)**

## QUESTION AND ANSWER BOOK



**Structure of book**

| *Number of questions* | *Number of questions to be answered* | *Number of marks* |
|---|---|---|
| 18 | 18 | 60 |
| | | Total 60 |

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

**Materials supplied**
- Question and answer book of 23 pages
- Formula sheet
- Working space is provided throughout the book.

**Instructions**
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

**At the end of the examination**
- You may keep the formula sheet.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**



March 2023

## Instructions

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required, you should only round your answer when instructed to do so.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

## Data analysis

**Question 1** (4 marks)

The table below displays the *average sleep time*, in hours, for a sample of 19 types of mammals.

| *Type of mammal* | *Average sleep time* (hours) |
|---|---|
| cat | 14.5 |
| squirrel | 13.8 |
| mouse | 13.2 |
| rat | 13.2 |
| grey wolf | 13.0 |
| arctic fox | 12.5 |
| raccoon | 12.5 |
| gorilla | 12.0 |
| jaguar | 10.8 |
| baboon | 9.8 |
| red fox | 9.8 |
| rabbit | 8.4 |
| guinea pig | 8.2 |
| grey seal | 6.2 |
| cow | 3.9 |
| sheep | 3.8 |
| donkey | 3.1 |
| horse | 2.9 |
| roedeer | 2.6 |

Data: T Allison and DV Cicchetti,
'Sleep in Mammals: Ecological and Constitutional Correlates',
in *Science*, American Association for the Advancement of
Science, vol. 194, no. 4266, pp. 732–734, 12 November 1976;
accessed from OzDASL, StatSci.org,
<www.statsci.org/data/general/sleep.html>



**Question 1** – continued

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**a.** Which of the two variables, *type of mammal* or *average sleep time*, is a nominal variable? 1 mark

**b.** Determine the mean and standard deviation of the variable *average sleep time* for this sample of mammals.

Write your answers in the boxes provided below.

Round your answers to one decimal place. 1 mark

mean = ☐ hours standard deviation = ☐ hours

**c.** The average sleep time for a human is eight hours.

What percentage of this sample of mammals has an *average sleep time* that is less than the average sleep time for a human?

Round your answer to one decimal place. 1 mark

**d.** The sample is increased in size by adding in the average sleep time of the little brown bat. Its average sleep time is 19.9 hours.

By how many hours will the range for *average sleep time* increase when the average sleep time for the little brown bat is added to the sample? 1 mark

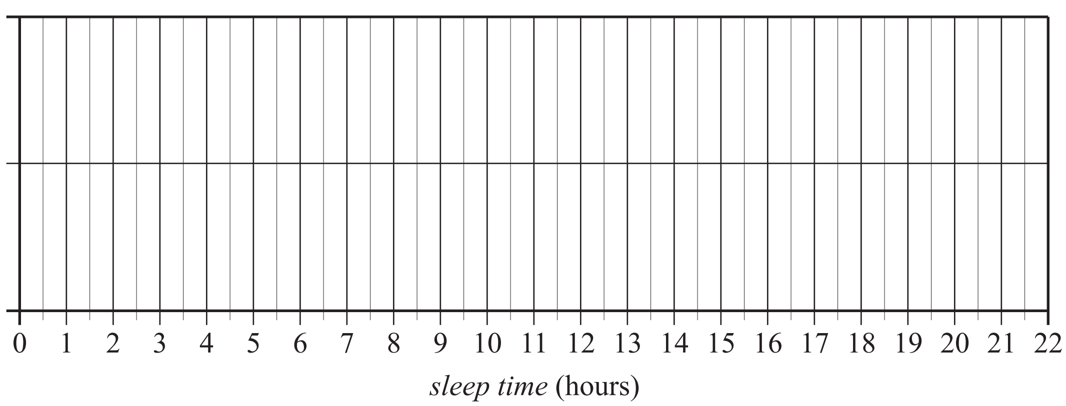
**Question 2** (3 marks)

The five-number summary below was determined from the *sleep time*, in hours, of a sample of 59 types of mammals.

| Statistic | *Sleep time* (hours) |
|---|---|
| minimum | 2.5 |
| first quartile | 8.0 |
| median | 10.5 |
| third quartile | 13.5 |
| maximum | 20.0 |

**a.** Show, with calculations, that a boxplot constructed from this five-number summary will not include outliers. 2 marks

**b.** Construct the boxplot below. 1 mark



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**Question 3** (4 marks)

The *life span*, in years, and *gestation period*, in days, for 19 types of mammals are displayed in the table below.

| ***Life span* (years)** | ***Gestation period* (days)** |
|---|---|
| 3.20 | 19 |
| 4.70 | 21 |
| 7.60 | 68 |
| 9.00 | 28 |
| 9.80 | 52 |
| 13.7 | 63 |
| 14.0 | 60 |
| 16.2 | 63 |
| 17.0 | 150 |
| 18.0 | 31 |
| 20.0 | 151 |
| 22.4 | 100 |
| 27.0 | 180 |
| 28.0 | 63 |
| 30.0 | 281 |
| 39.3 | 252 |
| 40.0 | 365 |
| 41.0 | 310 |
| 46.0 | 336 |

**a.** A least squares line that enables *life span* to be predicted from *gestation period* is fitted to this data.

Name the explanatory variable in the equation of this least squares line. 1 mark

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**b.** Determine the equation of the least squares line in terms of the variables *life span* and *gestation period*.

Write your answers in the appropriate boxes provided below.

Round the numbers representing the intercept and slope to three significant figures. 2 marks

[ ] = [ ] + [ ] × [ ]

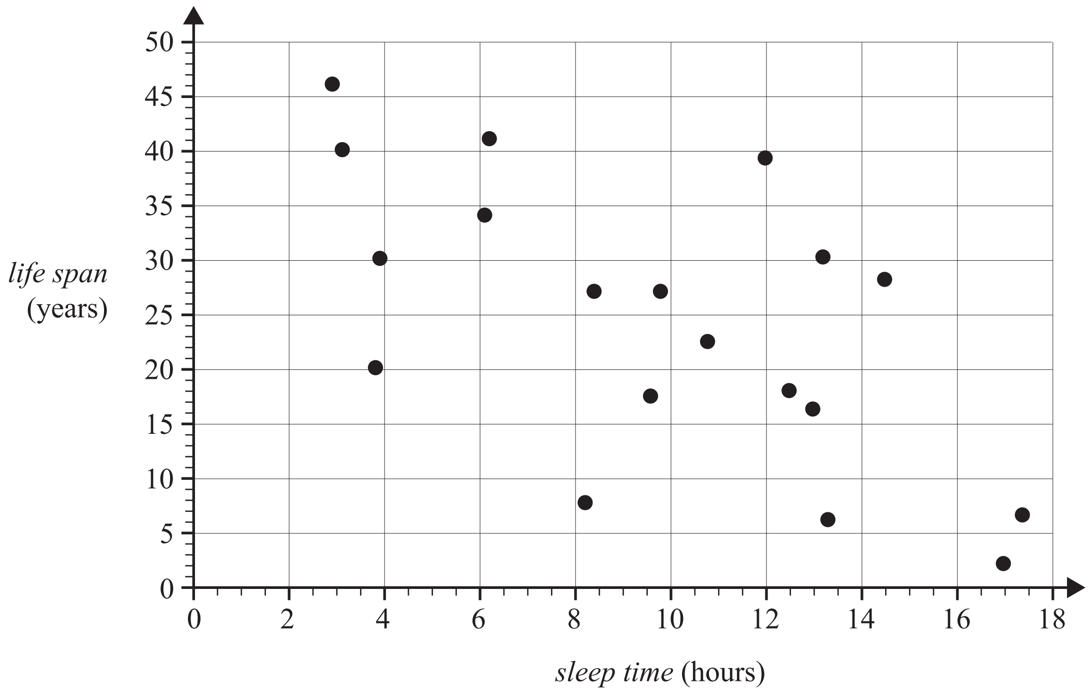
**c.** Write the value of the correlation coefficient rounded to three decimal places. 1 mark

$r =$ [ ]

**TURN OVER**

**Question 4** (8 marks)

The scatterplot below plots the variable *life span*, in years, against the variable *sleep time*, in hours, for a sample of 19 types of mammals.




On the assumption that the association between *sleep time* and *life span* is linear, a least squares line is fitted to this data with *sleep time* as the explanatory variable.

The equation of this least squares line is

$$\textit{life span} = 42.1 - 1.90 \times \textit{sleep time}$$

The coefficient of determination is 0.416

**a.** Draw the graph of the least squares line on the scatterplot above. 1 mark

(*Answer on the scatterplot above.*)

**b.** Describe the linear association between *life span* and *sleep time* in terms of strength and direction. 2 marks

**c.** Interpret the slope of the least squares line in terms of *life span* and *sleep time*. 2 marks



**Question 4** – continued

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**d.** Interpret the coefficient of determination in terms of *life span* and *sleep time*. 1 mark

**e.** The life span of the mammal with a sleep time of 12 hours is 39.2 years.

Show that, when the least squares line is used to predict the life span of this mammal, the residual is 19.9 years. 2 marks

**Question 5** (5 marks)

A random sample of 12 mammals drawn from a population of 62 types of mammals was categorised according to two variables:

- *likelihood of attack* (low = 1, medium = 2, high = 3)
- *exposure to attack during sleep* (low = 1, medium = 2, high = 3)

The data is shown in the following table.

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ***Likelihood of attack*** | 2 | 2 | 1 | 3 | 2 | 3 | 1 | 3 | 1 | 1 | 3 | 3 |
| ***Exposure to attack during sleep*** | 3 | 1 | 1 | 1 | 3 | 3 | 1 | 3 | 1 | 1 | 3 | 3 |

**a.** Use this data to complete the two-way frequency table below. 1 mark

| ***Likelihood of attack*** | ***Exposure to attack during sleep*** | | |
|---|---|---|---|
| | low (=1) | medium (=2) | high (=3) |
| low (=1) | | 0 | 0 |
| medium (=2) | | 0 | |
| high (=3) | | 0 | |

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**Question 5** – continued

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**b.** The following two-way frequency table was formed from the data generated when the entire population of 62 types of mammals was similarly categorised.

| ***Likelihood of attack*** | ***Exposure to attack during sleep*** | | |
|---|---|---|---|
| | low ( =1) | medium (=2) | high (=3) |
| low (=1) | 31 | 8 | 2 |
| medium (=2) | 2 | 0 | 2 |
| high (=3) | 1 | 1 | 15 |

**i.** How many of these 62 mammals had both a high *likelihood of attack* and a high *exposure to attack during sleep*? 1 mark

**ii.** Of those mammals that had a medium *likelihood of attack*, what percentage also had a low *exposure to attack during sleep*? 1 mark

**iii.** Does the information in the table above support the contention that *likelihood of attack* is associated with *exposure to attack during sleep*? Justify your answer by quoting appropriate percentages. It is sufficient to consider only one category of *likelihood of attack* when justifying your answer. 2 marks

**TURN OVER**

## Recursion and financial modelling

**Question 6** (4 marks)

Samuel owns a printing machine.

The printing machine is depreciated in value by Samuel using flat rate depreciation.

The value of the machine, in dollars, after $n$ years, $V_n$, can be modelled by the recurrence relation

$$V_0 = 120\,000, \qquad V_{n+1} = V_n - 15\,000$$

**a.** By what amount, in dollars, does the value of the machine decrease each year? 1 mark

**b.** Showing recursive calculations, determine the value of the machine, in dollars, after two years. 1 mark

**c.** What annual flat rate percentage of depreciation is used by Samuel? 1 mark

**d.** The value of the machine, in dollars, after $n$ years, $V_n$, could also be determined using a rule of the form $V_n = a + bn$.

Write down this rule for $V_n$. 1 mark

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**Question 7** (3 marks)

Samuel has a reducing balance loan.

The first five lines of the amortisation table for Samuel's loan are shown below.

| Payment number | Payment ($) | Interest ($) | Principal reduction ($) | Balance ($) |
|---|---|---|---|---|
| 0 | 0.00 | 0.00 | 0.00 | 320 000.00 |
| 1 | 1600.00 | 960.00 | 640.00 | 319 360.00 |
| 2 | 1600.00 | 958.08 | 641.92 | 318 718.08 |
| 3 | 1600.00 | 956.15 | | 318 074.23 |
| 4 | 1600.00 | | | |

Interest is calculated monthly and Samuel makes monthly payments of $1600.

Interest is charged on this loan at the rate of 3.6% per annum.

**a.** **i.** Using the values in the amortisation table, calculate the principal reduction associated with payment number 3 — 1 mark

**ii.** Using the values in the amortisation table, calculate the balance of the loan after payment number 4 is made.

Round your answer to the nearest cent. — 1 mark

**b.** Let $S_n$ be the balance of Samuel's loan after $n$ months.

Write down a recurrence relation, in terms of $S_0$, $S_{n+1}$ and $S_n$, that could be used to model the month-to-month balance of the loan. — 1 mark

**Question 8** (3 marks)

Samuel now invests \$500 000 in an annuity from which he receives a regular monthly payment.

The balance of the annuity, in dollars, after $n$ months, $A_n$, can be modelled by a recurrence relation of the form

$$A_0 = 500\,000, \qquad A_{n+1} = kA_n - 2000$$

**a.** Calculate the balance of this annuity after two months if $k = 1.0024$ 1 mark

**b.** Calculate the annual compound interest rate percentage for this annuity if $k = 1.0024$ 1 mark

**c.** For what value of $k$ would this investment act as a simple perpetuity? 1 mark

**Question 9** (2 marks)

Some time later, Samuel takes out a new reducing balance loan.

The interest rate for this loan was 4.1% per annum, compounding monthly.

The balance of the loan after four years of monthly repayments is \$329 587.25

The balance of the loan after seven years of monthly repayments is \$280 875.15

Samuel will continue to make the same monthly repayment.

To ensure the loan is fully repaid, to the nearest cent, the required final repayment will be lower.

In the first seven years, Samuel makes 84 monthly repayments.

From this point on, how many more monthly repayments will Samuel make to fully repay the loan?

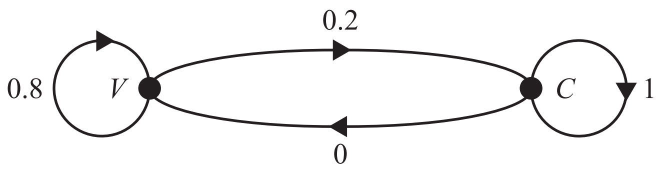
## Matrices

**Question 10** (3 marks)

An animal sanctuary is run by both volunteers and employed conservationists.

People volunteer ($V$) at the sanctuary before they become conservationists ($C$).

The transition diagram below shows the way in which employees are expected to transition between groups from month to month.




**a.** Interpret the value on the loop at $V$. 1 mark

**b.** Write the transition matrix, $T$, that represents the transition diagram above. 1 mark

$T =$

**c.** According to the transition diagram, what is expected to happen to each group of workers in the long term? 1 mark

**Question 11** (4 marks)

The sanctuary is trying to increase the population of an endangered fish.

During this process:

- eggs (*E*) may die (*D*) or they may live and become baby fish (*B*)
- baby fish (*B*) may die (*D*) or they may live and become adult fish (*A*)
- adult fish (*A*) continue to live for a period of time but will eventually die (*D*).

From year to year, changes to this fish population can be modelled by the recurrence relation

$$S_{n+1} = TS_n,$$

$$\text{where } T = \begin{array}{c} \textit{this year} \\ \begin{array}{cccc} E & B & A & D \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.4 & 0 & 0 & 0 \\ 0 & 0.25 & 0.5 & 0 \\ 0.6 & 0.75 & 0.5 & 1 \end{bmatrix} \end{array} \begin{array}{c} E \\ B \\ A \\ D \end{array} \textit{ next year}$$

The initial state matrix for this fish population, $S_0$, is

$$S_0 = \begin{bmatrix} 10\,000 \\ 1000 \\ 800 \\ 0 \end{bmatrix} \begin{array}{c} E \\ B \\ A \\ D \end{array}$$

**a.** Complete the following multiplication for the row that determines the number of adult fish predicted to be in the population after one year. 1 mark

$\square \times 10\,000 + \square \times 1000 + \square \times 800 + \square \times 0$

**Question 11** – continued

**b.** To take into account the new eggs added to the population when the adult fish begin to breed, the following matrix recurrence relation is used.

$$R_{n+1} = T \times R_n + B,$$

$$\text{where } T = \begin{array}{c} \textit{this year} \\ \begin{array}{cccc} E & B & A & D \end{array} \\ \left[\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0.4 & 0 & 0 & 0 \\ 0 & 0.25 & 0.5 & 0 \\ 0.6 & 0.75 & 0.5 & 1 \end{array}\right] \end{array} \begin{array}{c} E \\ B \\ A \\ D \end{array} \textit{ next year}, \quad B = \begin{bmatrix} k \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad R_0 = \begin{bmatrix} 10000 \\ 1000 \\ 800 \\ 0 \end{bmatrix}$$

The extra number of eggs expected to be laid by the adult fish each year is represented by $k$.

**i.** If $R_1 = \begin{bmatrix} 500 \\ 4000 \\ 650 \\ 7150 \end{bmatrix}$, show that $k = 500$. 1 mark

**ii.** Determine how many adult fish there are expected to be after two years. 1 mark

**iii.** Which populations, $E$, $B$, $A$ and $D$, remain the same in the long term? 1 mark

**Question 12** (3 marks)

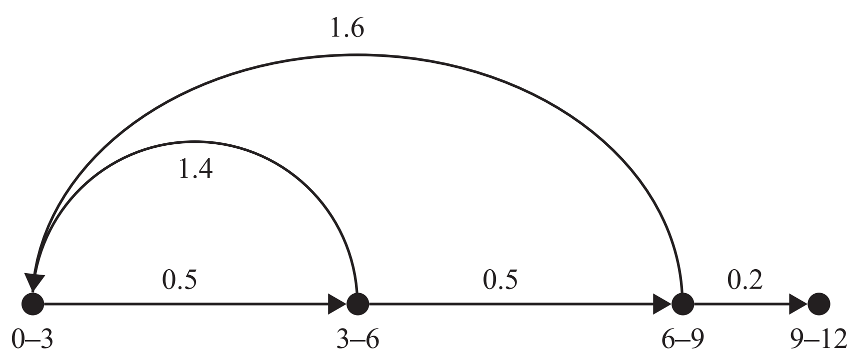
The sanctuary monitors the breeding patterns of the female population of a species of marsupial on a nearby island.

These marsupials have a life span of 12 years and have been categorised into four age groups: 0−3 years, 3−6 years, 6−9 years and 9−12 years.

The Leslie matrix, $L$, that models the breeding patterns for this female marsupial population is as follows.

$$L = \begin{bmatrix} 0 & 1.4 & 1.6 & 0 \\ 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.2 & 0 \end{bmatrix}$$

The same information is presented in the transition diagram below.




**a.** The following table shows the *birth rate*, for each of the four age groups, the average proportion of offspring born to each age group, and the *survival rate*, the average proportion of offspring to survive to the next age group.

Some values in the table are missing.

Use the Leslie matrix to complete the table below. 1 mark

| | Age group (years) | | | |
|---|---|---|---|---|
| | **0−3** | **3−6** | **6−9** | **9−12** |
| ***Birth rate*** | | 1.4 | | 0 |
| ***Survival rate*** | 0.5 | | | 0 |

**Question 12** – continued

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The initial state matrix for this female marsupial population, $S_0$, can be written as follows.

$$S_0 = \begin{bmatrix} 20 \\ 40 \\ 50 \\ 15 \end{bmatrix}$$

The recurrence relation to model the breeding patterns of this marsupial population is as follows.

$$S_{n+1} = LS_n$$

**b.** By calculating $S_1$, determine how many marsupials there are aged 0–3 years after one three-year time period. 1 mark

**c.** What percentage of this female marsupial population are aged 9–12 years after the second three-year time period? Round your answer to one decimal place. 1 mark

**Question 13** (2 marks)

The four cleaners at the sanctuary, Trinh, Koby, Sam and Mon, take turns cleaning four different areas, A, B, C and D.

The cleaning roster for four weeks in February is shown in the table below.

| | **Trinh** | **Koby** | **Sam** | **Mon** |
|---|---|---|---|---|
| **Week 1** | A | B | C | D |
| **Week 2** | B | C | D | A |
| **Week 3** | C | D | A | B |
| **Week 4** | D | A | B | C |

The information from this table for Week 1 is presented in matrix $N$.

$$N = \begin{bmatrix} A & B & C & D \end{bmatrix}$$

**a.** When matrix $N$ is multiplied by the permutation matrix, $P$, the cleaning roster will change from Week 1 to Week 2.

Write down permutation matrix $P$ in the space provided. 1 mark

$P =$

**b.** The transpose of the permutation matrix, $P^T$, was used to create a new roster for March.

The table below shows the cleaning roster for March. Some values in the table are missing.

Complete the cleaning roster for weeks 2, 3 and 4 in March. 1 mark

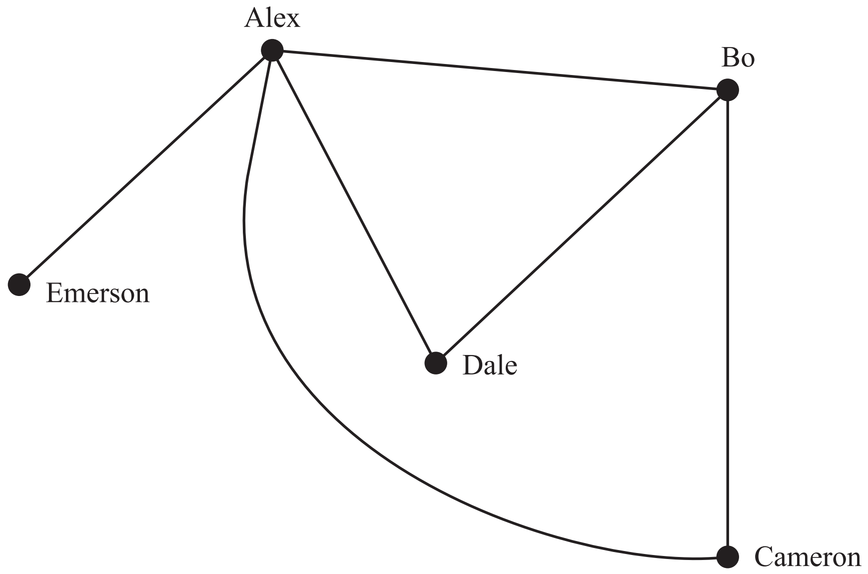
| | **Trinh** | **Koby** | **Sam** | **Mon** |
|---|---|---|---|---|
| **Week 1** | A | B | C | D |
| **Week 2** | | | | |
| **Week 3** | | | | |
| **Week 4** | | | | |

## Networks and decision mathematics

**Question 14** (2 marks)

The Sunny Coast Cricket Club has five new players join its team: Alex, Bo, Cameron, Dale and Emerson.

The graph below shows the players who have played cricket together before joining the team. For example, the edge between Alex and Bo shows that they have previously played cricket together.




**a.** How many of these players had Emerson played cricket with before joining the team? 1 mark

**b.** Who had played cricket with both Alex and Bo before joining the club? 1 mark

**Question 15** (1 mark)

A cricket team has 11 players, who are each assigned to a batting position.

Three of the new players, Alex, Bo and Cameron, can bat in position 1, 2 or 3.

The table below shows the average scores, in runs, for each player for the batting positions 1, 2 and 3.

| Player | Batting position 1 | Batting position 2 | Batting position 3 |
|---|---|---|---|
| Alex | 22 | 24 | 24 |
| Bo | 25 | 25 | 21 |
| Cameron | 24 | 25 | 19 |

Each player will be assigned to one batting position.

To which position should each player be assigned to maximise the team's score? Write your answer in the table below.

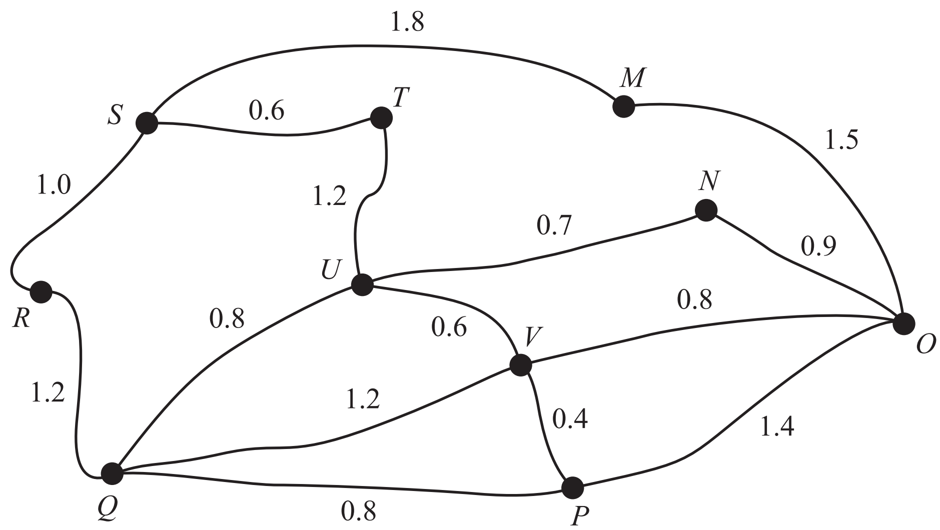
| Player | Batting position |
|---|---|
| Alex | |
| Bo | |
| Cameron | |

**Question 16** (3 marks)

A local fitness park has 10 exercise stations: *M* to *V*.

The edges on the graph below represent the tracks between the exercise stations.

The number on each edge represents the length, in kilometres, of each track.




The Sunny Coast cricket coach designs three different training programs, all starting at exercise station *S*.

| Training program number | Training details |
|---|---|
| 1 | The team must run to exercise station *O*. |
| 2 | The team must run along all tracks just once. |
| 3 | The team must visit each exercise station and return to exercise station *S*. |

**a.** What is the shortest distance, in kilometres, covered in training program 1? 1 mark

**b.** At which exercise station would training program 2 finish? 1 mark

**c.** To complete training program 3 in the minimum distance, one track will need to be repeated.
Complete the following sentence by filling in the boxes provided. 1 mark

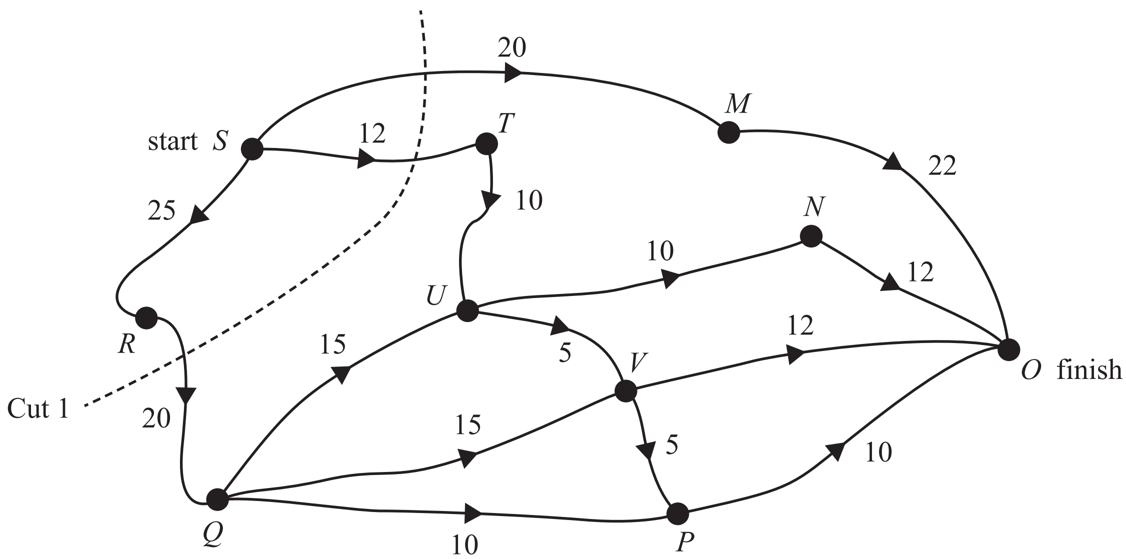
This track is between exercise station ☐ and exercise station ☐.

**Question 17** (2 marks)

Training program 1 has the cricket team starting from exercise station *S* and running to exercise station *O*.

For safety reasons, the cricket coach has placed a restriction on the maximum number of people who can use the tracks in the fitness park.

The directed graph below shows the capacity of the tracks, in number of people per minute.




When considering the possible flow of people through this network, many different cuts can be made.

**a.** Determine the capacity of Cut 1, shown above. 1 mark

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**b.** What is the maximum flow from *S* to *O*, in number of people per minute? 1 mark

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**Question 18** (4 marks)

The Sunny Coast cricket clubrooms and oval are undergoing a major works project.

This project involves nine activities: *A* to *I*.

The table below shows the earliest start time (EST) and duration, in months, for each activity.

The immediate predecessor(s) is also shown.

The duration for activity *C* is missing.

| **Activity** | **EST** | **Duration** | **Immediate predecessor(s)** |
|---|---|---|---|
| *A* | 0 | 2 | – |
| *B* | 0 | 5 | – |
| *C* | 5 | | *A*, *B* |
| *D* | 7 | 7 | *C* |
| *E* | 7 | 9 | *C* |
| *F* | 5 | 3 | *B* |
| *G* | 14 | 4 | *D* |
| *H* | 8 | 9 | *F* |
| *I* | 18 | 2 | *E*, *G*, *H* |

The information in the table above can be used to complete a directed network.

This network will require a dummy activity.

**a.** Complete the following sentence by filling in the boxes provided. 1 mark

This dummy activity could be drawn as a directed edge from the end of activity [ ] to the start of activity [ ].

**b.** What is the duration, in months, of activity *C*? 1 mark

**c.** Name the four activities that have a float time of at least one month. 1 mark

**d.** The project is to be crashed by reducing the completion time of one activity only.

What is the minimum time, in months, in which the project can be completed? 1 mark



**END OF QUESTION AND ANSWER BOOK**

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