2008 Mathematical Methods (CAS) GA 2: Examination 1

GENERAL COMMENTS

The number of students who sat for the 2008 examination was 4108.

The overall quality of responses was similar to that of recent years but slightly improved on those in 2007. There were many very good responses and it was rewarding to see a substantial number of students who worked through to obtain full marks. There still appears to be a wide gap between students who understand course material and those who do not. Many students found difficulty in correctly carrying out simple arithmetic calculations especially involving decimals, or in correctly manipulating simple algebraic expressions. In Questions 7 and 8 answers were often left unsimplified. It was pleasing to see that few students dropped '*dx*' from integral expressions. This was a marked improvement from recent years.

Students need to be aware that the instruction to show working is applied rigorously when marking the papers. When more than one mark is available, and there is no working, students cannot gain full marks. Similarly, where there is incorrect or careless notation, students may not be able to gain full marks, for example, the lack or incorrect use of brackets in expressions and incorrect use of set notation for stating domains and ranges. Graphs should be drawn correctly, showing features such as smoothness and endpoints.

Students are strongly advised to reread a question upon completion to avoid missing some of the required answer; for example, in Question 10a., many students made an attempt at $f^{-1}(x)$ but did not specify the domain. Similarly, students should be discouraged from continuing to further engage with questions, especially in performing unnecessary algebraic manipulation, as errors can be made. As noted in previous Assessment Reports, there continues to be difficulty with algebraic skills, setting out, graphing skills and the proper use of mathematical notation.

SPECIFIC INFORMATION

Question 1

This was a straightforward differentiation question involving the use of the chain rule and the product rule, both of which were given on the formula sheet. Students need to be careful not to confuse the use of these rules.

Question 1a.

Marks	0	1	2	Average
%	17	27	57	1.5

Let $f(x) = (3x^2 - 5x)^5 = u^5$, $u = 3x^2 - 5x$

then $f^{1}(x) = 5(6x - 5)(3x^2 - 5x)^4$

While generally well done, some students had difficulty with brackets in this question, either using them poorly or omitting them. Some students carried out unnecessary incorrect simplification of their answer. A small number of students attempted to expand the expression and then differentiate, but this was often not successful.

The following expressions were commonly seen errors.

$$f^{1}(x) = 5\times 6x - 5\times\left(3x^2 - 5x\right)^4 = 30x - 5\left(3x^2 - 5x\right)^4$$

or $f^{1}(x) = 30x - 25\left(3x^2 - 5x\right)^4$

or $f^{1}(x) = 6x - 25\left(3x^2 - 5x\right)^4$

Question 1b.

Marks	0	1	2	3	Average
%	29	5	9	57	2.1

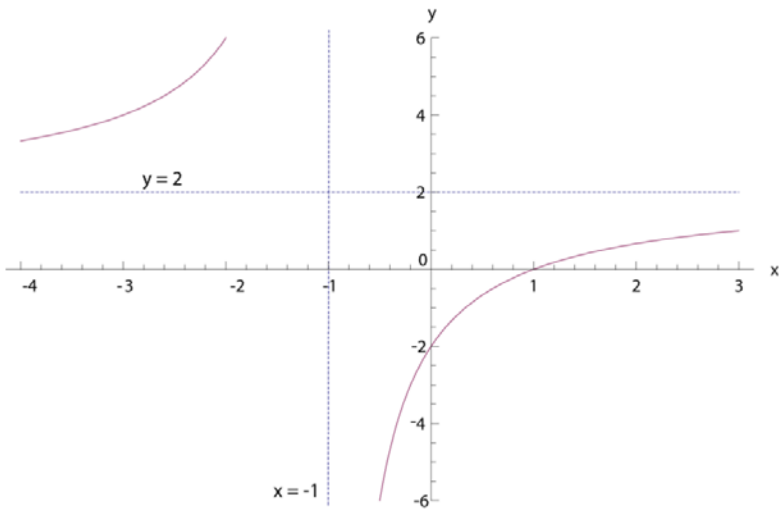
$f^{1}(x) = x.3e^{3x} + e^{3x}$

$f^{1}(0) = 1$

The product rule was needed to obtain an answer in this question, although a number of students did not recognise this. A common incorrect response for the derivative was $x3e^{3x}$. Some students left the answer as $f^{1}(x)$, and did not evaluate $f^{1}(0)$. Rereading the question may have avoided this.

Question 2

Marks	0	1	2	3	4	Average
%	13	8	7	15	57	3.1



This question was generally well done with most students recognising a hyperbola. Students should label asymptotes and/or axial intercepts. In this question students sometimes gave the wrong intercept ($x = 3$ was common), extra intercepts or no horizontal asymptote.

Question 3

Marks	0	1	2	Average
%	31	28	41	1.2

$$\cos\left(\frac{3x}{2}\right) = \frac{1}{2}, x \ \varepsilon \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$$

$$\frac{3x}{2} = \frac{-\pi}{3}, \frac{\pi}{3}, \frac{5\pi}{3} \ \ldots$$

$$x = \frac{-2\pi}{9}, \frac{2\pi}{9}, \frac{10\pi}{9} \ldots$$

$$x = \frac{-2\pi}{9} \text{ or } \frac{2\pi}{9}$$

Students were usually able to get one correct response or at least write down $\frac{3x}{2} = \frac{\pi}{3}$. Solving this expression caused some students problems when the given domain was ignored. It resulted in either only one solution or too many solutions. Occasionally students used incorrect values for $\cos^{-1}(0.5)$.

Question 4a.

Marks	0	1	2	Average
%	31	20	49	1.3

$\int_0^1 k\sin(\pi x)\, dx = 1$

$\left[-\frac{k}{\pi}\cos(\pi x)\right]_0^1 = 1$

$-\frac{k}{\pi}\cos(\pi) + \frac{k}{\pi}\cos(0) = 1$

$2\frac{k}{\pi} = 1$

$k = \frac{\pi}{2}$

Although many students answered this question satisfactorily, there was a significant number of vanishing negative signs, incorrect indefinite integrals, incorrect substitutions or incorrect algebraic manipulation. It was pleasing to see that only a few students left off the '*dx*'.

Question 4b.

Marks	0	1	2	3	Average
%	35	18	21	27	1.5

$\Pr(X \le \frac{1}{4} \mid X \le \frac{1}{2})$

$= \dfrac{Pr(x \le \frac{1}{4})}{\Pr(x \le \frac{1}{2})}$

$\dfrac{\int_0^{\frac{1}{4}} k\sin(\pi x)dx}{\int_0^{\frac{1}{2}} k\sin(\pi x)dx} = \dfrac{\int_0^{\frac{1}{4}} k\sin(\pi x)dx}{0.5}$

$= \dfrac{2-\sqrt{2}}{2}$

Few students used symmetry of the probability density function to obtain the denominator. However, it appeared that many students found this question both difficult and complicated. Many recognised the need for conditional probability but thought the required intersection was between $\frac{1}{4}$ and $\frac{1}{2}$ rather than between 0 and $\frac{1}{4}$. Some students correctly navigated complicated expressions and their evaluation without recognising how much simply cancelled (for example, *k*), while others, whose method was correct, were let down by poor manipulation skills and were left with π in the answer.

Question 5

Marks	0	1	2	3	Average
%	28	15	10	47	1.9

$$\int_0^c e^{2x}\,dx = \frac{5}{2}$$

$$\left[\frac{1}{2}e^{2x}\right]_0^c = \frac{5}{2}$$

$$e^{2c} - 1 = 5$$

$$c = \frac{1}{2}\ell o\varsigma_e(6) \text{ or } \ell o\varsigma_e(\sqrt{6})$$

This question was generally quite well done. Some students interchanged the $\frac{5}{2}$ and c. A common error was to see the derivative in the place of the antiderivative.

Question 6a.

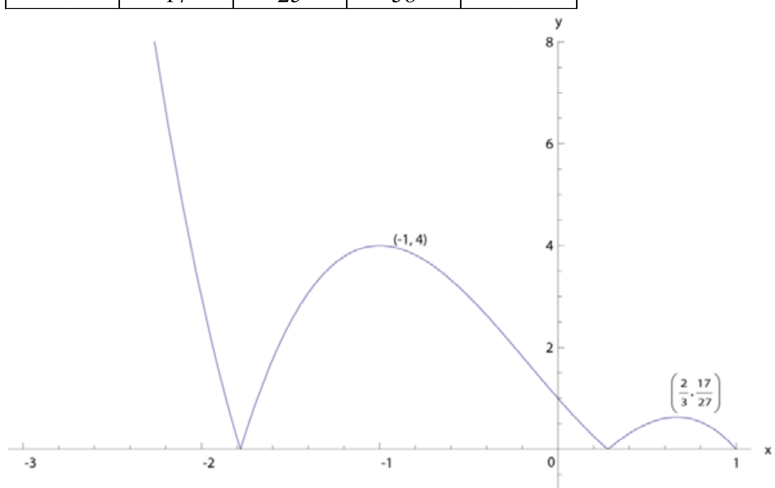
Marks	**0**	**1**	**Average**
%	60	40	**0.4**

$d_{f'} = R\backslash\{1, 2\}$

Incorrect notation in various possible ways – incorrect use of round, curly and square brackets, $\cap$ and $\cup$ being confused, expressions such as ‘ $\infty]$ ’, and so on – meant that many students did not gain marks for this question. The most common incorrect response was R\ {1}, with the 2 being overlooked as a point at which the function was not differentiable. It was also common to see $\frac{2}{3}$ being incorrectly excluded from the domain.

Question 6b.

Marks	**0**	**1**	**2**	**Average**
%	17	25	58	**1.5**



Most students realised the significance of the modulus sign and drew a graph on or above the x axis. Common mistakes included not labelling stationary points or incorrectly labelling the right stationary point as $\left(\frac{2}{3}, \frac{-17}{27}\right)$ despite it being obviously positive, or missing the open circle at (1, 0).

Question 7

Students who obtained zero for Question 7a. were often still able to correctly answer 7b.

Question 7a.

Marks	0	1	Average
%	32	68	0.7

mode = 3

Some students found the instruction regarding the mode difficult. Some attempted to find the mean or median while others stated probability values or wrote Pr ($X = 3$) = 0.4 without specifying which part was the mode.

Question 7b.

Marks	0	1	2	Average
%	41	10	49	1.1

Pr(0,0) + Pr(1,1) + Pr(2,2) + Pr(3,3)

$= 0.1^2 + 0.2^2 + 0.3^2 + 0.4^2$

= 0.3

Most students realised what was required to answer the question but the difficulty for some was in correctly performing arithmetic calculation involving decimals. For example, 0.1^2 was variously calculated as 0.1 (as expected) but also 0.2, 0.02 and even 0, and 0.4^2 came out as 0.016 at times. At times the four values were individually written down but no attempt was made to add them together.

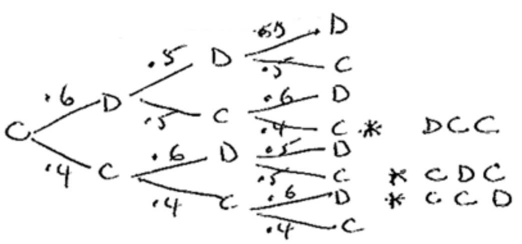
Question 8

Marks	0	1	2	3	Average
%	24	21	16	39	1.8

Pr (2 of next 3 Fridays) = CCD + CDC + DCC

= 0.4 x 0.4 x 0.6 + 0.4 x 0.6 x 0.5 + 0.6 x 0.5 x 0.4

= 0.336



Nearly all students drew a tree diagram. However, a simpler approach would have been to only write down the three appropriate choices or list all possibilities and then select the three required. Some students were unable to assign correct probabilities and a number had difficulty with completing the arithmetic calculation correctly. A small number of students attempted to use binomial probability.

Question 9a.

Marks	0	1	2	Average
%	24	42	35	1.2

$$V = 1000 = \frac{x}{2} \cdot \frac{\sqrt{3}x}{2} \cdot y$$

so $y = \frac{4000\sqrt{3}}{3x^2}$

Quite a few students had difficulty finding the volume of the prism; some used the volume of the triangular pyramid given on the formula sheet, others attempted to use surface area or perimeter. Finding the area of the triangle proved particularly challenging, with many using the formula $A = \frac{1}{2} ab\sin C$, often with x as both the angle and the side. Most students, however, did equate their volume expression with 1000. Students are expected to have adequate algebraic facility with these sorts of expressions.

Question 9b.

Marks	0	1	2	Average
%	41	28	31	**0.9**

$$Area = 2\left(\frac{x^2}{2} \cdot \frac{\sqrt{3}}{2}\right) + 3xy$$

$$= \frac{4000\sqrt{3}}{x} + \frac{x^2\sqrt{3}}{2}$$

Many students realised that the area required the sum of the area of three rectangles and two triangles.

Question 9c.

Marks	0	1	2	3	Average
%	45	14	11	30	**1.3**

Solve $\frac{dA}{dx} = \frac{-4000\sqrt{3}}{x^2} + x\sqrt{3} = 0$

gives $x = 10\sqrt[3]{4}$ or $\sqrt[3]{4000}$ or $4000^{\frac{1}{3}}$

Most students realised that to solve the question they needed to differentiate A and then solve $\frac{dA}{dx}$ equal to zero.

Students who were able to do this correctly almost always obtained the correct answer. A common incorrect response was to attempt to solve for $A = 0$, with some students believing A and its derivative were the same.

Question 10a.

Marks	0	1	2	Average
%	26	29	45	**1.3**

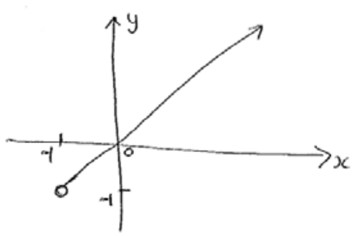
$d_{f^{-1}} = r_f = (-1, \infty)$

$f^{-1}(x) = \frac{1}{2}\log_e(x+1)$ or $\log_e\left(\sqrt{x+1}\right)$

Many students did correctly find the rule for the inverse function, but neglected to give the domain. Students must be careful to explicitly use brackets for compound arguments of functions. Common incorrect responses were $\frac{1}{2}\log_e(x-1)$ and $\frac{1}{2}\log_e(x)+1$.

Question 10b.

Marks	0	1	Average
%	81	19	0.2



Students should be aware that the composition of a function and its inverse results in the function $y = x$ over the appropriate domain. Many graphs were either exponential or logarithmic. Students who drew the line $y = x$ often drew it over R even if they had stated the correct domain in part *a*.

Question 10c.

Marks	0	1	2	Average
%	52	28	20	0.7

$$f(-f^{-1}(2x)) = f\left(\frac{-\log_e(2x+1)}{2}\right)$$

$$= \frac{-2x}{2x+1}$$

A number of students were able to find $-f^{-1}(2x)$ but then did not know how to proceed or were unable to simplify their expressions correctly, with -1 often disappearing during the process. Again many students found the algebra for this question difficult.