2009 Mathematical Methods (CAS) GA 3: Examination 2

GENERAL COMMENTS

There were 7059 students who sat the Mathematical Methods (CAS) examination in 2009. Marks ranged from 2 to 80 out of a possible score of 80. Student responses showed that the paper was accessible and that it provided an opportunity for students to demonstrate their knowledge.

The mean for 2009 was 44. The median score for the paper was 44 marks. Of the whole cohort, around 10% of students scored 87% or more of the available marks, and 26% scored 77% or more of the available marks.

The mean score for the Multiple-choice section was 13.6 out of 22. Only five of the multiple-choice questions were distinctive from the Mathematical Methods paper – Questions 1, 4, 5, 12 and 18. Question 1 was answered correctly by only 49% of students.

As stated in the instructions on the examination paper, students must show appropriate working for questions worth more than one mark. This was done better than in 2008, but there was still a number of students who gave answers only or did not give sufficient working for Questions 3a. and 3b. In Questions 1dii., 2d., 2e. and 4eii., writing out the expression to be evaluated or the equation to be solved was considered sufficient working. Students should be encouraged to attempt to write out an expression or equation, even if they think their answer to a previous question is incorrect, because marks can be awarded, for example, in Questions 1dii., 2d., 2e. and 4eii. Students must ensure that they provide sufficient working in 'show that' questions such as Question 2aii.

Students must ensure they read questions carefully so that they give the answers over the required interval. This was done better than in 2008, particularly in Questions 1a., 2biii. and 4cii. Errors relating to this were made in Question 2e.

Correct mathematical notation is expected and should always be used. Technology syntax should not be used, as sometimes occurred in Questions 2aii., 3a., 3b., 3d. and 3g.

Students must use the units that are given in the question, especially in questions like Question 2bii. and Question 4. They do not need to convert units unless the question asks for the answer to be in a specific unit. It was pleasing to see that many students used the correct variables this year.

From 2010, it will be assumed that students will provide exact answers to questions unless specified otherwise. It was still apparent that a number of students do not know the meaning of the word 'exact' when it appears in a question. Many students gave the exact answer and then wrote down an approximate answer as their final answer. This occurred in Questions 1c., 1di., 1dii. and 4cii.

Students should retain sufficient decimal place accuracy in computation to ensure that they can provide numerical answers to a specified accuracy; for example, in Question 3ci., the conditional probability uses results of previous calculations.

Students should take care when sketching graphs. If they are subsequently required to sketch over part of a graph, they need to make the relevant parts visible, as in Question 1b. More care needs to be taken with closed and open circles for end points.

2009 Assessment Report

SPECIFIC INFORMATION

Section 1

The table below indicates the percentage of students who chose each option. The correct answer is indicated by shading.

Question	% A	% B	% C	% D	% E	% No Answer	Comments
1	18	49	11	14	8	1	Using multiples of coefficients and combining equations, a unique solution exists when $k(k+2)-15$ is non zero, that is, $k \in R \setminus \{-5, 3\}$. Alternatively, solve $\begin{vmatrix} k & -3 \\ 5 & -k-2 \end{vmatrix} = 0$ $k=-5$ or $k=3$ (infinite number of solutions). A unique solution will occur if $k \in R \setminus \{-5, 3\}$.
2	1	2	95	2	0	0	This was the best answered question on the paper.
3	6	86	2	4	1	0	
4	55	13	8	14	9	1	The solution to $\sin(2x)=-1$ could be found either directly using technology or by hand. $2x = -\frac{\pi}{2} + 2\pi n$, $x = -\frac{\pi}{4} + n \times \text{period} = -\frac{\pi}{4} + n\pi$, $n \in Z$
5	8	14	13	53	13	0	$f(x-y) = (x-y)^2 = x^2 - 2xy + y^2$ $f(x) - f(y) = x^2 - y^2$ $f(x-y) \neq f(x) - f(y)$
6	4	3	23	64	6	0	
7	6	84	3	1	6	0	
8	4	9	79	5	3	0	
9	11	8	8	24	49	1	The curve of $y = f(x)$ has been translated 2 units to the right and 3 units up. The image of the point (1, 5) is (3, 8). Hence the gradient of $y = f(x-2)+3$ at the point (3, 8) will be 2. The equation of the tangent at (3, 8) is $y = 2x+2$.
10	5	72	13	6	3	0	
11	10	6	15	61	7	1	
12	17	10	6	61	6	1	B was also accepted as it leads to an equivalent expression.
13	3	17	71	4	4	0	
14	4	17	11	59	9	0	
15	4	15	4	2	74	0	
16	4	8	74	6	8	0	
17	31	43	10	10	6	1	For independent events $\Pr(A \cap B) = \Pr(A) \times \Pr(B)$ Let $A = \{1, 3, 5, 7, 9, 11\}$ and $B = \{1, 4, 7, 10\}$ $\Pr(A) = \frac{1}{2}$ and $\Pr(B) = \frac{1}{3}$ $\Pr(A \cap B) = \frac{1}{6}$

							$\Pr(A)\times\Pr(B)=\frac{1}{2}\times\frac{1}{3}=\frac{1}{6}$ Hence $\Pr(A\cap B)=\Pr(A)\times\Pr(B)$. Many students chose option B, which contained mutually exclusive events, for which $\Pr(A\cap B)=0$.
18	18	52	8	19	3	1	Solve $\frac{1}{k-0}\int_0^k\left(\frac{1}{2x+1}\right)dx=\frac{1}{6}\log_e(7)$ for *k*. $k=3$
19	13	5	12	3	66	0	
20	11	28	16	11	34	0	The number of solutions to $\lvert a\cos(x)\rvert=\lvert a\rvert$ where $x\in[-2\pi,2\pi]$ is 9. The horizontal line $y=\lvert a\rvert$ touches the graph of $y=\lvert a\cos(x)\rvert$ at the two end points and the seven turning points.
21	8	43	24	11	14	1	$f'(x)>0$ for $x\in(-3,2)$, $f'(x)=0$ at $x=2$ and $f'(x)<0$ for $x>2$. Hence the graph of *f* has a local maximum at $x=2$.
22	10	15	21	45	7	1	$\int_0^3\left(e^x+1\right)dx=e^3+2$

Section 2

Question 1

1a.

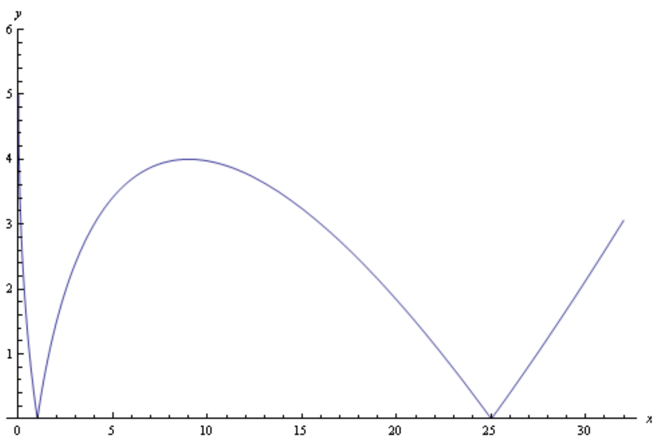
Marks	0	1	2	Average
%	19	8	72	**1.6**

$f'(x)=0$, $x=9$, $[9,\infty)$ The interval for which the graph is strictly decreasing is when $f(b)<f(a)$ for $b>a$ and $a,\ b\in R$.

This question was well done. Some students gave the domain and range, while others gave $(\infty,9)$, $(9,-\infty)$ or $(9,4)$. Some students appeared to have guessed the answer, giving $(10,\infty)$ or $(8.99,\infty)$.

1b.

Marks	0	1	2	Average
%	8	38	55	**1.5**



This question was generally answered well. Students were required to draw over the middle section of the original graph (this should have been clearly indicated) and have the closed circle at the end point (0, 5). A number of students did not draw a smooth curve. Some had the end point (0, 5) below the turning point (9, 4). The cusps were generally drawn well. Students were not expected to label the graph.

1c.

Marks	0	1	2	Average
%	16	24	60	**1.5**

$\int_1^{25} \left(f(x)\right) dx = 64$, $AD = \frac{64}{24} = \frac{8}{3}$ or average value $= \frac{1}{25-1}\int_1^{25} \left(f(x)\right) dx = \frac{8}{3}$

Many students gave $\frac{64}{25}$ as the answer or used $\int_0^{25} \left(f(x)\right) dx$. It was pleasing to see some students recognise that the question was asking for the average value of the function. Some students answered the question without using CAS and this would likely have been time-consuming. An exact answer was required.

1di.

Marks	0	1	Average
%	18	82	**0.8**

$m = \frac{3-0}{16-25} = -\frac{1}{3}$

Some students gave two answers to this question. Others gave -3 as the answer, using $m = \frac{x_2 - x_1}{y_2 - y_1}$ or simplified $-\frac{3}{9}$ to -3. An exact answer was required.

1dii.

Marks	0	1	2	Average
%	22	18	60	**1.4**

$f'(a) = -\frac{1}{3}$, $a = \frac{81}{4}$

Many students were able to equate $f'(a)$ to their answer to Question 1di. but some failed to see the connection. Some students gave lengthy solutions, which would have been time-consuming and often led to incorrect answers; for example, $\sqrt{a} = \frac{9}{2}$ therefore $a = \sqrt{\frac{9}{2}}$. Others integrated or attempted to find the equation of the line passing through the points *P* and *B*.

1ei.

Marks	0	1	2	Average
%	8	7	85	**1.8**

$f(g(x)) = 6\sqrt{x^2} - x^2 - 5$ or $f(g(x)) = 6|x| - x^2 - 5$ or $f(g(x)) = \begin{cases} 6x - x^2 - 5, & \text{for } x \geq 0 \\ -6x - x^2 - 5, & \text{for } x < 0 \end{cases}$ were acceptable equivalent forms.

This question was generally well done. Some students gave the expression $6\sqrt{x^2} - x^2 - 5$ when the rule was required.

1eii.

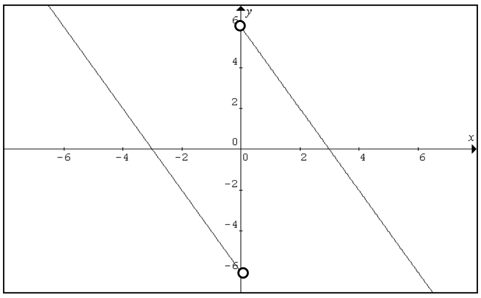
Marks	0	1	2	Average
%	13	74	13	1

$$h'(x) = \frac{df(g(x))}{dx} = \frac{6|x|}{x} - 2x \text{ or } h'(x) = \begin{cases} 6 - 2x, \text{ for } x > 0 \\ -6 - 2x, \text{ for } x < 0 \end{cases}$$

Other equivalent forms of the answer were accepted. Most students were able to get $-2x$. A common incorrect answer was $h'(x) = 6 - 2x$.

1eiii.

Marks	0	1	2	3	Average
%	54	12	17	17	1



This question was not answered well. Often the left-hand side of the graph was missing. Some students attempted to draw both sides of the graph but did not stop at $y = 6$ and $y = -6$. Clear open circles were expected at $y = 6$ and $y = -6$. Sometimes students drew the asymptote with equation $x = 0$.

Question 2

2ai.

Marks	0	1	2	3	Average
%	24	16	9	51	1.9

$$0 = \frac{1}{200}\left(8a + 4b + c\right)\ldots(1),\ -0.06 = \frac{1}{200}\left(12a + 4b\right)\ldots(2),\ 0 = \frac{1}{200}\left(48a + 8b\right)\ldots(3)$$

Equivalent forms of the equations were acceptable. Many students were able to get the first equation. Some students differentiated incorrectly; for example, $\frac{dy}{dx} = \frac{1}{200}(3ax^2 + 2b)$.

2aii.

Marks	0	1	2	Average
%	52	15	33	0.8

(3) − (2), $3a = 3$, $a = 1$. Substitute $a = 1$ into (2), $3 + b = -3, b = -6$.
Substitute $a = 1$ and $b = -6$ into (1), $0 = 8 - 24 + c$, $c = 16$.
Alternatively

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 8 & 4 & 1 \\ 12 & 4 & 0 \\ 48 & 8 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ -12 \\ 0 \end{bmatrix} = \begin{bmatrix} 8 & 4 & 1 \\ 3 & 1 & 0 \\ 6 & 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ -3 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -6 \\ 16 \end{bmatrix}$$

For 'show that' questions, students must ensure they provide sufficient relevant working using the mathematical notation.

2bi.

Marks	0	1	2	Average
%	34	15	51	1.2

$P\ (2-2\sqrt{3},0)$, $M\ (2+2\sqrt{3},0)$

Some students did not put their answers in coordinate form or they labelled the coordinates incorrectly. Exact answers were required. Students need to be careful when they change the form of the answer given by CAS, for example, $P(-2\sqrt{3}+2,\ 0) \neq P(-2(\sqrt{3}+1),\ 0)$.

2bii.

Marks	0	1	Average
%	59	41	0.4

$2\sqrt{3}$ km

Common incorrect answers were $4-2\sqrt{3}$ and $4\sqrt{3}$. An exact answer was required.

2biii.

Marks	0	1	Average
%	50	50	0.5

$\frac{2}{25} = 0.08 \text{ km} = 80 \text{ m}$

The positive value was required. Common incorrect answers were 0.8 km or 800 m. Some students gave the x coordinate (4 km) as the answer.

Many students did not attempt Questions 2c., 2d. and 2e.

2c.

Marks	0	1	Average
%	65	35	0.4

$$k = \frac{w}{\log_e\left(\frac{1}{7}\right)} = -\frac{w}{\log_e(7)}$$

Some students had trouble realising $v = w$ and $d = 0$, while others did not rearrange the equation to make k the subject.

2d.

Marks	0	1	2	Average
%	50	20	30	0.8

$$\frac{120\log_e(2)}{\log_e(7)} = \frac{w}{\log_e\left(\frac{1}{7}\right)} \times \log_e\left(\frac{3.5}{7}\right), \ w = 120$$

Many students who attempted this question were able to set up the equation with their value of k. Some students continued to solve the equation by hand but struggled with the algebra. Others left the $\log_e$ out of the expression $\log_e\left(\frac{3.5}{7}\right)$.

2e.

Marks	0	1	2	Average
%	58	18	23	0.7

$v = 0 \Rightarrow 0 = \dfrac{120}{\log_e\left(\dfrac{1}{7}\right)}$, $\log_e\left(\dfrac{d+1}{7}\right) \Rightarrow d = 6$ km

Distance from $Q = 6.2 - 6 = 0.2$ km

Students often gave the final answer as 6 km – the distance from P. Students should reread questions before moving on to the next question.

Question 3

3a.

Marks	0	1	2	Average
%	16	14	70	1.6

$X \sim \text{N}(67, 1)$, $\Pr(X < 68.5) = 0.9332$, correct to four decimal places

3b.

Marks	0	1	2	Average
%	16	14	70	1.6

$X \sim \text{N}(67, 1)$, $\Pr(65.6 < X < 68.4) = 0.8385$, correct to four decimal places

Questions 3a. and 3b. were done quite well. Some working had to be shown for students to be awarded full marks. A diagram with the correct area shaded is considered sufficient working. The use of technology syntax is to be avoided.

3ci.

Marks	0	1	Average
%	58	42	0.4

$\dfrac{0.83848...}{0.93319...} = 0.8985$, correct to four decimal places

This question was not answered well. Many students did not recognise that the question involved conditional probability. Some students gave 0.8385 as the answer.

3cii.

Marks	0	1	2	Average
%	44	52	4	0.6

$Y \sim \text{Bi}(4, 0.101486)$, $\Pr(Y \geq 1) = 0.3482$

Many students recognised that the binomial distribution was required; however, they used the wrong probability. Common incorrect answers were 0.5057 and 0.9993. Students must ensure that they work to more decimal places than what is required in the answer. The use of technology syntax is to be avoided.

3d.

Marks	0	1	2	3	Average
%	48	22	8	22	1.1

$\Pr(X < 68.4) = 0.995$, when $x = 68.4$, $z = 2.5758$

So $2.5758 = \dfrac{68.4 - 67}{\sigma}$, $\sigma = 0.54$ mm, correct to two decimal places

Some students solved $\int_{65.6}^{68.4}\left(\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-67}{\sigma}\right)^2}\right)dx = 0.99$ for σ while others used trial and error; both approaches were acceptable. Many students used 0.99 as the required probability instead of 0.995, obtaining an answer of 0.60. Some used 0.99 as their z value. Other students used 0.05 instead of 0.005, obtaining an answer of 0.85 and some used $\mu-3\sigma \le x \le \mu+3\sigma$. The use of technology syntax is to be avoided.

3e.

Marks	0	1	2	Average
%	26	3	71	1.5

$0.8 \times 0.8 \times 0.8 = 0.512 = \frac{64}{125}$

This question was done well. Some students tried to use the transition matrix.

3f.

Marks	0	1	2	3	Average
%	26	8	10	57	2

$0.8 \times 0.8 \times 0.2 + 0.8 \times 0.2 \times 0.15 + 0.2 \times 0.15 \times 0.8 = 0.176 = \frac{22}{125}$

This question was quite well answered. Some students tried to use the binomial distribution while others had the working correct but did not evaluate the expression correctly.

3g.

Marks	0	1	2	Average
%	63	14	23	0.6

$\begin{bmatrix}0.8 & 0.15\\0.2 & 0.85\end{bmatrix}^n\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}p\\\ldots\end{bmatrix}$ or $\begin{bmatrix}0.8 & 0.15\\0.2 & 0.85\end{bmatrix}^n=\begin{bmatrix}p & \ldots\\\ldots & \ldots\end{bmatrix}$, $n = 8$

Trial and error was required to answer this question. Some students had the incorrect transition matrix, using 0.75 instead of 0.85. Others used the binomial distribution. Technology syntax should be avoided; [0.8, 0.15; 0.2, 0.85] is not a standard notation for a matrix and students should use the conventional two-dimensional form.

Question 4

4ai.

Marks	0	1	Average
%	68	32	0.3

By similar triangles, $\frac{h}{r}=\frac{8}{4} \Rightarrow h = 2r$

This question was not answered well. For 'show that' questions sufficient working must be shown. Many students assumed $h = 2r$ and substituted the values into the equation, writing $h = 8$, $r = 4$, $8 = 2\times 4$.

4aii.

Marks	0	1	Average
%	60	40	0.4

$V=\frac{1}{3}\pi r^2 h,\ r=\frac{h}{2}\Rightarrow V=\frac{\pi h^3}{12}$

The wording in this question, 'at time t', confused many students and a common incorrect answer was $V = \frac{\pi h^3}{12} t$. Some students used the volume of a cylinder instead of a cone.

4b.

Marks	0	1	2	Average
%	45	15	40	1

$$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt} = \frac{9\pi}{4} \times \frac{4}{\pi h^2} = \frac{9}{h^2}$$

Many students recognised that this question was a related rates of change question. Other solutions were acceptable but would have been quite time-consuming.

4ci.

Marks	0	1	Average
%	37	63	0.7

When $h = 2, \frac{dh}{dt} = \frac{9}{4}$ m/h

This question was done quite well. Some students had the units as m/s or m. Others simplified $\frac{9}{4}$ to $\frac{3}{2}$.

4cii.

Marks	0	1	Average
%	53	47	0.5

When $\frac{dh}{dt} = \frac{9}{8}, \frac{9}{h^2} = \frac{9}{8} \Rightarrow h = 2\sqrt{2}$ m

$\frac{9}{8}$ was a common incorrect answer and some students left their answer as $h = \pm 2\sqrt{2}$. An exact answer was required.

4di.

Marks	0	1	Average
%	48	52	0.3

$$\frac{dt}{dh} = \frac{h^2}{9}$$

This question was quite well answered. Some students did not relate the question to Question 4b.

4dii.

Marks	0	1	Average
%	75	25	0.3

$t = \int \frac{h^2}{9} dh = \frac{h^3}{27} + c$, when $h = 0$, $t = 0$, so $t = \frac{h^3}{27}$, $h = 3t^{\frac{1}{3}}$

This question was not answered well. Many students left their answer as $t = \frac{h^3}{27}$.

4ei.

Marks	0	1	Average
%	66	34	0.4

Height of statue above ground level = $14 - t$

Some students were able to answer this question even though they did not attempt other parts of the question. Common incorrect answers were $6-t$ and $8-t$.

4eii.

Marks	0	1	2	Average
%	74	9	16	0.4

$14 - t = 3t^{1/3}$, $\Rightarrow t = 8$ or 5.00 pm

Some students were able to equate their answer to Question 4dii. to 4ei.