2007 Specialist Maths GA 3: Written examination 2

GENERAL COMMENTS

The number of students who sat the 2007 examination was 4899 compared to 5210 in 2006. This was the second year of the new structure, and students answered 22 multiple-choice questions, then five extended answer questions. The time allowed – two hours – seemed adequate, with the majority of students attempting most parts of the paper. Students generally seemed well prepared for the standard questions; however, this was not always the case as demonstrated by Questions 3a–d. Questions which were less routine, such as Question 4c., again proved challenging for the majority of students.

The mean and median scores for the paper were 43.5 and 45 out of 80 respectively, almost identical to last year's mean and median scores of 44 and 45 respectively. Five students scored full marks – the same as for the 2006 cohort. The middle 90 per cent of the 2007 cohort scored between 12 and 73 out of 80 and this compares with 14 to 72 for 2006. In Section 2 the average scores for the five questions, expressed as a percentage of the marks available for each question, were 60 per cent, 73 per cent, 49 per cent, 42 per cent and 38 per cent respectively. This compares with 53 per cent, 54 per cent, 57 per cent, 39 per cent and 45 per cent for the 2006 examination. More detailed statistical information is published on the VCAA website.

This year there were four 'show that' questions in Section 2 – Questions 1e., 1f., 2d. and 4a. It needs to be emphasised again that for this type of question students need to include adequate detail to convince the assessor that they have independently arrived at the given result. It was pleasing to see that most students who could not establish the given results in Questions 2d. and 4a. still continued on and used the results successfully to gain later marks.

Most students realised that they needed to include appropriate working where a question was worth more than one mark. Question 2f. was probably the only exception where some students merely recorded the answer of 0.15 for this two mark question.

The examination revealed both areas of strength and weakness in student performance. Areas of strength included:

- using technology to perform numerical integration
- an improved understanding as to what constitutes a definite integral
- correctly resolving forces on an inclined plane
- plotting regions in the complex plane.

Areas of weakness included:

- failing to read questions carefully – Questions 1c., and 2c. for which many students gave the answer for Question 2f. instead. They then proceeded to repeat this answer (correctly) in Question 2f.
- failing to label items correctly – Questions 1b. and 2b.
- lack of proper vector notation – Question 4
- confusion about directions when writing equations of motion related to connected particles – Question 3
- omitting to check that calculators were in the correct radian/degree mode needed for a specific question – Question 2b. and Question 5.

SPECIFIC INFORMATION

Section 1

The table below indicates the percentage of students who chose each option. The correct answer is indicated by shading.

Question	% A	% B	% C	% D	% E	% No Answer	Comments
1	10	49	20	11	9	1	Asymptotes are $y = \pm\frac{x}{2}$, hence B.
2	14	6	9	8	62	1	
3	2	6	8	78	6	0	
4	3	71	4	3	19	0	
5	12	11	19	9	48	0	Letting $x = 0$ reduces the possibilities to options B or E, but only option E displays the correct period.
6	5	57	20	9	9	1	
7	76	15	4	3	2	0	
8	4	1	83	4	8	0	
9	1	32	3	38	26	0	The factor of 3 in the denominator would be accounted for in the values of A and B. Only option D displays the correct repeated factor form.
10	20	8	62	5	3	1	
11	10	17	14	50	9	1	
12	27	16	31	17	7	1	$\frac{dh}{dt} = 500 \times \sec^2\left(\frac{\pi}{6}\right) \times 0.5$
13	11	5	9	5	70	0	
14	6	17	37	23	16	1	The number not infected is $1000 - N$. The product is then $N(1000 - N)$, hence option C is correct.
15	37	14	21	12	15	1	The gradient of the perpendicular is $\frac{2}{3}$. So the vector is $2\underset{\sim}{j} + 3\underset{\sim}{i}$. Hence option A is correct.
16	6	15	59	13	6	1	
17	4	21	15	52	8	0	
18	6	6	5	11	71	1	
19	12	61	9	9	8	1	
20	2	48	43	2	5	0	*N* must be perpendicular to the incline and friction is the only force acting to retard the motion of the box, hence option C is correct.
21	54	8	23	11	4	1	
22	24	45	8	12	10	1	mass $\times$ acceleration = thrust – **total** resistance, hence option B is correct.

The mean score for the multiple-choice section was 12.32 out of 22 and the standard deviation was 4.59. This compares to 14.26 and 4.57 respectively for 2006. There were nine questions (Questions 1, 5, 9, 11, 12, 14, 15, 20 and 22) which were answered correctly by less than 50 per cent of students. This compares with only five questions in 2006.

These statistics indicate that students probably found the multiple-choice section slightly harder than last year.

Section 2

Question 1a.

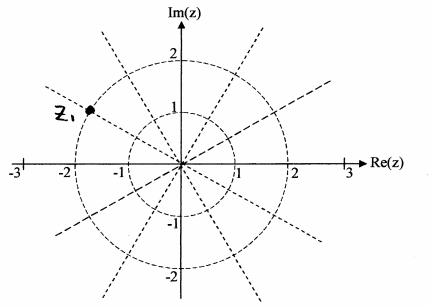
Marks	0	1	2	Average
%	6	27	67	1.7

$z_1 = 2\text{cis}\left(\dfrac{5\pi}{6}\right)$

This question was quite well done. The most common error was $\arg(z_1) = -\dfrac{\pi}{6}$.

Question 1b.

Marks	0	1	Average
%	26	74	0.8



This question was well done by most students. Apart from points in the wrong location, the most common error was neglecting to label the point z_1 or omitting the subscript.

Question 1c.

Marks	0	1	2	Average
%	25	5	70	1.5

This question was quite well done by most students, who either applied the quadratic formula or completed the square successfully.

The major error in this question was verifying the solutions by substitution, which was contrary to the explicit instruction, 'by solving…algebraically' given in the question.

Question 1d.

Marks	0	1	2	Average
%	38	14	48	1.2

$\sqrt{3} - i = -z_1$, $\sqrt{3} + i = -\overline{z_1}$ or an equivalent such as $-z_1 + 2i$

This question was fairly well done with some innovative (and usually correct) equivalents for $-\overline{z_1}$ given.

Question 1e.

Marks	0	1	2	Average
%	48	3	48	1.1

$x^2 + y^2 = \left(x + \sqrt{3}\right)^2 + \left(y - 1\right)^2$ leads to the given result.

This question was reasonably well done, with some students successfully using the perpendicular bisector approach.

Question 1f .

Marks	0	1	Average
%	72	28	**0.3**

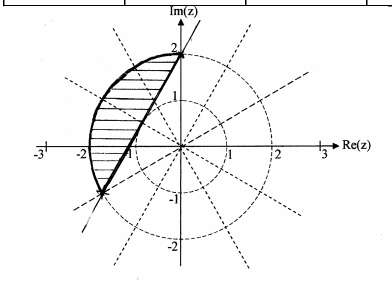
$\overline{z}_1$ has coordinates $\left(-\sqrt{3},-1\right)$

The right hand side of $y=\sqrt{3}x+2$ is $\sqrt{3}\times-\sqrt{3}+2 = -1$, which is equal to the left hand side.

This question was not very well done, with many students simply reproducing their efforts for Question 1e. Very few students worked separately on the right hand side to show that it became the left hand side.

Question 1g.

Marks	0	1	2	Average
%	36	36	28	**1.0**



This question was moderately well done, with the most common error being inaccurate placement of the lower corner point of the region. Many students simply shaded various regions inside a circle of radius 2.

Question 2a.

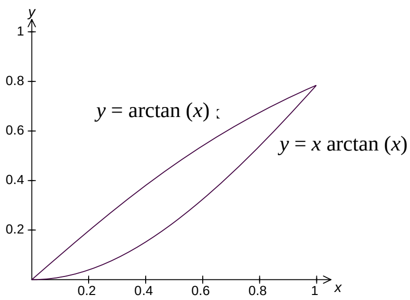
Marks	0	1	2	Average
%	16	17	67	**1.6**

$f'(x)=\dfrac{x}{1+x^2}+\arctan(x),\ f'(0)=0$

This question was well done, with a small minority of students introducing $\text{arcsec}(x)$ and $\sec^2$ terms into their answers.

Question 2b.

Marks	0	1	2	Average
%	20	11	69	**1.6**



This question was quite well done. Errors involved curves being drawn beyond the domain, incorrect concavity displayed, inaccurate location of the point of intersection and lack of labelling of the curves.

Some students used their calculators in degree mode, resulting in answers that were quite different to what was expected.

Question 2ci.

Marks	0	1	Average
%	26	74	**0.8**

$$\int_0^1 \left(x\arctan(x)\right) dx$$

This question was well done, with the major error being to misread the question. Some students gave an integral for the area between the two curves.

Question 2cii.

Marks	0	1	Average
%	33	67	**0.7**

0.285

This question was generally well done. Most students who answered Question 2ci. correctly managed the numerical integration successfully.

Question 2d.

Marks	0	1	2	Average
%	33	6	62	**1.4**

$x\arctan(x) = \int \left(\frac{x}{1+x^2} + \arctan(x)\right) dx$, from which the given result follows.

This question was fairly well done. Nearly all students showed the separation of the right side into two integrals. Some students factored out the x in $\int \left(\frac{x}{1+x^2}\right) dx$ in their attempt to get the log term.

Question 2e.

Marks	0	1	2	Average
%	23	12	66	**1.5**

$$\frac{\pi}{4} - \frac{1}{2}\log_e(2)$$

This question was quite well done. Students occasionally neglected to convert arctan(1) to $\frac{\pi}{4}$.

Question 2f.

Marks	0	1	2	Average
%	24	10	66	**1.5**

$\frac{\pi}{4} - \frac{1}{2}\log_e(2) - 0.285 = 0.15$, correct to two decimal places.

Generally this question was well done. It was surprising that a number of students obtained this answer (and integral) for Question 2c. and then reproduced it again here, without checking to see if they had misinterpreted Question 2c.

Question 3a.

Marks	0	1	2	Average
%	31	6	63	**1.4**

$$3g - T_1 = 3a,\ T_1 - g = a \ \Rightarrow\ a = \frac{g}{2} = 4.9$$

This question was fairly well done, with the major difficulty relating to signs when equations of motion were written separately for each mass. A number of students treated the masses as one system. Of those who treated the problem this way, the major error was to state $3g - g = 3a$ instead of $3g - g = 4a$.

Question 3b.

Marks	0	1	Average
%	39	61	**0.7**

$T = \frac{3g}{2} = 14.7$

This question was well done by those who obtained the correct acceleration in Question 3a.

Question 3c.

Marks	0	1	2	3	Average
%	22	26	11	42	**1.8**

$3g\sin 30° - T_2 = 3b,\ T_2 - g = b \ \Rightarrow\ b = \frac{g}{8}$

The comments made for Question 3a. also apply to this question. Most students who attempted this question did at least realise that the weight force acting down the plane on the 3 kg mass was $3g\sin 30°$.

Question 3d.

Marks	0	1	2	Average
%	42	6	52	**1.2**

19.5°

This question was reasonably well done. However, a significant number of students did not treat this as a statics problem and used either the acceleration or the tension found in Question 3c. as part of their solution.

Question 3e.

Marks	0	1	2	3	4	Average
%	49	24	12	2	14	**1.1**

$3g\sin(\theta) - 0.6g\cos(\theta) - g = 0 \ \Rightarrow\ \theta = B = 30.4°$ when about to slide down the plane.

$3g\sin(\theta) + 0.6g\cos(\theta) - g = 0 \ \Rightarrow\ \theta = A = 7.8°$ when about to slide up the plane.

This last part of Question 3 was not answered well. Only a minority of students understood that there were **two** limiting equilibrium cases to consider. Few students obtained either of the above equations (or their equivalents) and few managed to solve numerically for θ.

Question 4a.

Marks	0	1	2	3	Average
%	26	7	7	60	**2.1**

$\underset{\sim}{r}(t) = 30t\underset{\sim}{i} - 40t\underset{\sim}{j} - 4t\underset{\sim}{k} + \underset{\sim}{c}$, $-500\underset{\sim}{i} + 2500\underset{\sim}{j} + 200\underset{\sim}{k} = 300\underset{\sim}{i} - 400\underset{\sim}{j} - 40\underset{\sim}{k} + \underset{\sim}{c}$, from which the given result follows.

This question was reasonably well done, with the major errors being the omission of $\underset{\sim}{c}$, and the omission of the aircraft's altitude component $(200\underset{\sim}{k})$ from the position vector at t =10. Lack of tildes indicating what should be vectors was common.

Question 4b.

Marks	0	1	2	3	Average
%	32	9	7	52	**1.9**

$240-4t=0 \Rightarrow t=60$, $\underset{\sim}{r}(60)=1000\underset{\sim}{i}+500\underset{\sim}{j}$, Distance = $\sqrt{1000^2+500^2}$ = 1118 m.

This question was reasonably well done. A number of students incorrectly equated other position vector components to zero to find when the aircraft landed. Most seemed to understand that the magnitude of their position vector was needed to find the distance.

Question 4c.

Marks	0	1	2	Average
%	86	3	11	**0.3**

$\sin\theta=\dfrac{4}{\sqrt{30^2+40^2+4^2}}$ or equivalent gives $\theta=4.6°$

This question was handled well by only a minority of students. Most attempted to work incorrectly with position vectors and frequently answers of 80° or more were given.

Question 4d.

Marks	0	1	2	Average
%	73	7	20	**0.5**

$\underset{\sim}{r}\cdot\underset{\sim}{v}=0 \Rightarrow 30(30t-800)-40(2900-40t)-4(240-4t)=0$, which gives t = 56 seconds.

This question proved difficult for most students. The favoured method was to work with $\left|\underset{\sim}{r}(t)\right|$, but few managed to correctly minimise it to get t = 56.

Question 4e.

Marks	0	1	2	Average
%	75	8	18	**0.5**

$D=\sqrt{30^2+40^2+4^2}\times 60$, or other method, gives D = 3010 m

This question was answered competently by only a few students. A common erroneous approach was to work out $\left|\underset{\sim}{r}(60)\right|-\left|\underset{\sim}{r}(0)\right|$ instead of $\left|\underset{\sim}{r}(60)-\underset{\sim}{r}(0)\right|$, which was the favoured method used by those who attempted this question. Other students attempted to find the distance from when the aircraft flew over the beacon.

Question 5a.

Marks	0	1	Average
%	30	70	**0.7**

$17=20-2\tan^{-1}(t)$, $t=\tan(1.5)$, $t=14.1$, correct to one decimal place.

This question was quite well done. A few students used their calculators in degree mode, and a few neglected to give the answer to the required number of decimal places.

Question 5b.

Marks	0	1	Average
%	69	31	**0.3**

$\tan^{-1}(t)<\dfrac{\pi}{2}$, $V\to 20-\pi\simeq 16.858$

This question was not well done. The most common error was to attempt to solve $16=20-2\tan^{-1}(t)$.

Question 5c.

Marks	0	1	Average
%	43	57	**0.6**

$$\int_0^T \left(20 - 2\tan^{-1}(t)\right) dt$$

This question was reasonably well done. The major error involved either incorrect terminals or no terminals. Nearly all students who attempted this part did have at least *dt* present.

Question 5d.

Marks	0	1	2	3	Average
%	39	13	10	38	**1.6**

$\int_0^8 \left(20 - 2\tan^{-1}(t)\right) dt - \int_3^8 13\cos^{-1}\left(\frac{13-2t}{7}\right) dt = 60.7 \text{ m}$, correct to one decimal place.

This question was not well answered. Of those who attempted the question, the most common error was to have terminals in both integrals from 0 to 8.

Question 5e.

Marks	0	1	2	3	Average
%	70	11	5	15	**0.7**

When $t = 8,\ V_{\text{police}} = 26.178(\text{m/s})$ so $60.7 + \int_8^{T_c} \left(20 - 2\tan^{-1}(t)\right) dt = 26.178\left(T_c - 8\right)$ or equivalent.

This question was not well done. Of the students who attempted this part, most calculated the constant speed of the police car, but many simply multiplied by T_c instead of $T_c - 8$ to find the distance it travelled at constant speed.

Question 5f.

Marks	0	1	Average
%	92	8	**0.1**

$T_c = 14.6 \simeq 15$ s, correct to the nearest second.

This question was not well done. Slightly more than half of those who set up a correct equation in Question 5e. managed to arrive at the correct answer. Several methods could be used, each involving a numerical solution process. For example, using the TI 83 graphics calculator 'solver' with $0 = 60.7 + \text{fnInt}\left(20 - \tan^{-1}(X), X, 8, X\right) - 26.178(X - 8)$ and an initial estimate around 10 seconds or so, the solution is found in around 15 seconds. Using a TI 84+ calculator the solution is found in less than 10 seconds.

Alternatively, if tabulating each side of the equation in Question 5e., for example as Y_1 and Y_2 on a TI 83 graphics calculator, it takes about six seconds to display the distances travelled by each car. The solution is then readily seen to be 15 seconds, correct to the nearest second.

A small number of students realised from earlier working that at $t = 8$ the polluting car was travelling close to the asymptotic value for its speed, and assumed a constant speed of around 17 m/s which reduced the equation in Question 5e. to a linear equation in T_c. An assumed constant speed of any value from 16.9 to 17.1 m/s gave the required time, correct to the nearest second.