2008 Specialist Mathematics GA 2: Written examination 1

GENERAL COMMENTS

Examination 1 comprised 10 short answer questions worth a total of 40 marks. Students were not allowed to bring calculators or any notes into the examination.

The number of students who sat for the 2008 Specialist Maths examination 1 was 4883, which was 123 more than in 2007.

The mean score for the 2008 examination was 18.5 out of 40 (46.3%), slightly lower than the performance on the 2007 paper where the mean score was 23.5 out of 40 (58.7%). Eight out of 16 question parts had a mean score of less than 50% of the maximum possible, which was a little worse than seven out of 17 in 2007.

The overall mean and median scores were 18.5 (46.3%) out of 40 and 19 (47.5%), compared with 23.5 (58.7%) and 25 (62.5%) in 2007. About 22 per cent of students scored less than 25% of the available marks, compared to 14 per cent in 2007. At the bottom end, 81 students (1.66% of the cohort) scored zero marks and 473 students (9.7%) scored less than four marks out of 40. At the top end, 11 students (0.23%) scored full marks and 200 students (4.1%) scored more than 36 marks out of 40. These statistics indicate that more students scored close to no marks than in 2007 and fewer students scored close to full marks.

In the comments on specific questions in the next section, many common mistakes that are made year after year are highlighted. These mistakes should be brought to the attention of students so that they can try to avoid them in future. A particular concern is the need for students to read the questions carefully – responses to several questions indicated that students had not done this.

Areas of weakness included:

- not reading the question carefully enough – either not answering the question or proceeding further than required. This was common and particularly evident in Questions 1, 2, 6 and 10c.
- poor algebraic skills. This was evident in several questions, and the inability to simplify expressions often prevented students from completing a question
- showing a given result, which was required in Questions 8c. and 10a. The onus is on students to include sufficient relevant working to convince assessors that they know how to derive the result. Just as importantly, students should be reminded that they **can** use a given value in the remaining part(s) of the question, **whether or not** they were able to derive it
- recognising the need to use the chain rule when differentiating implicitly (Question 2)
- recognising the need to use the product rule when differentiating (Question 2)
- recognising the method of integration required (Questions 5b., 6 and 9)
- recognising the need for a constant when integrating (Questions 5b. and 6)
- recognising the need for a double angle formula (Questions 4 and 9b.)
- knowing the exact values for circular functions (Questions 4, 6, 7 and 9)
- ability to resolve forces into components (Question 7)
- giving answers in the required form (Questions 1, 4 and 5b.).

Students need to be reminded that the instruction ‘sketch’ (Question 1) does **not** mean that a rough and careless attempt is acceptable or that details such as a reasonable scale, correct domain, asymptotes and asymptotic behaviour can be ignored.

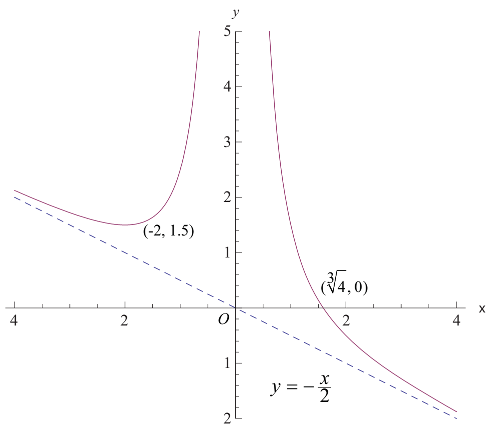
As students were not allowed to bring calculators into the examination, there was an expectation that students would be able to simplify simple arithmetic expressions. Many students were unable to do this and lost marks as a consequence.

SPECIFIC INFORMATION

Short-answer questions

Question 1

Marks	0	1	2	3	4	5	Average
%	16	16	16	17	16	19	**2.7**



Asymptotes $x=0$ and $y=-\frac{x}{2}$, x-intercept $\left(\sqrt[3]{4},\,0\right)$, turning point $\left(-2,1.5\right)$.

The attempts at this question were quite varied. Many students were able to find the equations of the asymptotes, many were able to find the *x*-intercept and many were able to find the turning point, but few were able to complete all of these successfully. Those who were able to fulfil all of these requirements often did not show asymptotic behaviour, with some graphs colliding with the asymptotes and others swerving away from them. A large number of students did not give coordinates for the *x*-intercept or the turning point, and just gave the *x*-coordinate. Sometimes there was no use of calculus to attempt to find the turning point. The vertical asymptote was often labelled as $y = 0$ and sometimes as $y = x$; often there was no equation given. Some students correctly found the oblique asymptote but then drew its graph as if it was that of $y=-x$ or $y=-2x$. Often two branches of the curve were shown but with inadequate asymptotic behaviour. On occasion the correct *x*-intercept was found but not placed between 1 and 2.

Question 2

Marks	0	1	2	3	4	Average
%	15	7	16	14	48	**2.8**

$\frac{3}{5}$

Most students did this question quite well, using implicit differentiation correctly by applying both the product rule and the chain rule. Several students made sign errors when rearranging to find $\frac{dy}{dx}$ and some failed to discard the negative *y*-value. A large number of students went beyond what was asked, finding the equation of the normal rather than its gradient. This wastes precious time and runs the risk of unnecessary errors.

Question 3

Marks	0	1	2	3	Average
%	48	30	6	16	**0.9**

$-\frac{5}{4}$

In answering this question, those who understood the question did it well. Unfortunately, many students did not fully understand the question. A large number of students did not understand the concept of linear dependence, as illustrated by their incorrect statement that linear dependence meant that $\underset{\sim}{a}+\underset{\sim}{b}+\underset{\sim}{c}=0$.

Question 4

Marks	0	1	2	3	Average
%	39	22	15	23	**1.2**

$\sqrt{5}-1$

This question was not done as well as expected, with several students not recognising the need for a double angle formula. Many attempts were let down by poor algebraic/arithmetic simplification. Many errors were seen in the expansion of $\left(\sqrt{5}-1\right)^2$. A number of students selected a double angle formula which complicated the question. Several sign errors were seen as were errors with fractions. Some students made mistakes in stating a double angle formula despite them being on the formula sheet.

Question 5a.

Marks	0	1	2	Average
%	39	13	48	**1.1**

2

Students tended to either do this question very well, or have little idea on how to proceed. A large proportion thought that acceleration was given by $a=\frac{dv}{dx}$, despite $a=v\frac{dv}{dx}$ being given on the formula sheet. A large number of students correctly found that $a=2x^3$ but then failed to substitute $x=1$, and some used constant acceleration formulas.

Question 5b.

Marks	0	1	2	3	Average
%	52	8	6	34	**1.2**

$x=\frac{1}{t+1}$

In answering this question, those who understood the question did it well. Unfortunately, many students did not fully understand the question. It was expected that students would write $v=\frac{dx}{dt}=-x^2$ and proceed from there. A common error was to start with $\frac{d^2x}{dt^2}=-x^2$ and then incorrectly state that $\frac{dt^2}{d^2x}=\frac{1}{2x^2}$. Some students gave the integral of $\frac{1}{x^2}$ as $-\frac{2}{x^3}$.

Question 6

Marks	0	1	2	3	Average
%	21	20	12	47	**1.9**

$-\frac{\sqrt{3}}{6}-\frac{1}{2}$ (other equivalent answers were accepted)

This question was well done by the majority of students with most integrating to obtain $f'(x)=-\frac{1}{2}\tan\left(2x\right)+c$ but many then made errors in their attempt to find the value of *c*. Some completely omitted *c* while others 'simplified' $-1=-\frac{1}{2}\tan\left(\frac{\pi}{4}\right)+c$ to incorrectly obtain $-2=-\tan\left(\frac{\pi}{4}\right)+c$. Several students substituted $y=2$ instead of -1. A significant number of students attempted to integrate again to find $f(x)$. Many tried to simplify their answer and introduced errors.

Question 7

Marks	0	1	2	3	Average
%	46	13	9	32	1.3

$T = 60\sqrt{2}$, $F = 60\sqrt{3} - 60$

This standard question was done quite poorly, with many students unable to resolve forces into horizontal and vertical components, or to correctly write equations given by Lami's theorem. Those who correctly used the latter approach usually could not go on to find *F*. The most common incorrect attempts involved $F = 120\cos 30^\circ$ often followed by $F = T\cos 45^\circ$.

Question 8a.

Marks	0	1	Average
%	24	76	0.8

$-2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$

Most students did this question well with most errors involving sign problems. Some thought that $\overrightarrow{AB} = \overrightarrow{OA} - \overrightarrow{OB}$.

Question 8b.

Marks	0	1	2	Average
%	58	15	27	0.7

(5, 3, 3)

By far the most common mistake was to assume that the vertices of a parallelogram did not run consecutively in a clockwise (or anti-clockwise) direction. This caused many to effectively work with the parallelogram *ABDC* instead of ABCD, which led to the incorrect statement $\overrightarrow{AB} = \overrightarrow{CD}$. Some students left the answer as $5\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$, forgetting to find the coordinates of the point.

Question 8c.

Marks	0	1	Average
%	64	36	0.4

ABCD is a rectangle. * Answer given.

This was a straightforward question but many students did not recognise what additional property would lead to a parallelogram being a rectangle. Many wasted time showing that the opposite pairs of sides had equal length or were parallel. Some students tried to show that all four angles were right angles, or that all four angles were right angles and opposite pairs of sides had equal length. A few students correctly showed an adjacent pair of sides were at right angles, but then wasted time showing that adjacent sides were unequal in length, not realising that a square is a type of rectangle. Students could also have shown that diagonals have equal length.

Question 9a.

Marks	0	1	Average
%	72	28	0.3

π

This question was well done if symmetry was noticed (i.e. the required area was simply half the area of the encompassing rectangle). If symmetry was not noticed, success was rare. Some laborious attempts at integration were seen.

Question 9b.

Marks	0	1	2	3	Average
%	28	27	8	37	1.5

$\frac{\pi^2}{2}$

Many students were unable to start this question, not realising that $x = \cos(y)$. Some attempts involved a definite integral with 'dx' instead of 'dy'. A number of students successfully obtained $\pi\int_0^\pi \cos^2(y)\,dy$ but then failed to use a double angle formula to evaluate the integral. Those who did use a double angle formula often had a wrong coefficient or sign on the $\sin(2y)$ term.

Question 10a.

Marks	0	1	Average
%	59	41	**0.4**

$\left|w^3\right| = (1+a^2)^{\frac{3}{2}}$ * Answer given.

It was surprising how many students did not know, or failed to recognise that $\left|w^3\right| = |w|^3$. Few students who attempted to expand w^3 were able to find success.

Question 10b.

Marks	0	1	Average
%	41	59	**0.6**

$\pm\sqrt{3}$

This was a straightforward question but far too many students were unable to simplify expressions involving indices. Of those who correctly obtained $a^2 = 3$, several carelessly gave the answer as $\sqrt{3}$ or ± 3.

Question 10c.

Marks	0	1	2	3	4	Average
%	61	16	13	7	3	**0.8**

$b = -4, c = 8, d = -8$ or $p(z) = z^3 - 4z^2 + 8z - 8$

This question proved to be difficult for most students. Many found that the complex conjugate pair $1+\sqrt{3}\,i$ and $1-\sqrt{3}\,i$ were two of the roots of the equation and most of these obtained the quadratic factor $z^2 - 2z + 4$. Some students then found the third root by solving $\left|z^3\right| = 8$ but the vast majority only considered the solution $z = 2$, ignoring the solution $z = -2$. These students generally gave $z - 2$ as the remaining real factor and correctly expanded to find the cubic. Very few students realised that $z + 2$ was also a possibility, but when used, led to a cubic with some zero coefficients (it gives the sum of cubes), which then had to be excluded to fully answer the question.