VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
Year

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER | | | | | | | | | Letter |

SPECIALIST MATHEMATICS

Written examination 1

Day Date

Reading time: *. to *.** (15 minutes)**
Writing time: *. to *.** (1 hour)**

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 11 pages.
- Formula sheet.
- Working space is provided throughout the book.

Instructions
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Version 3 – July 2016

THIS PAGE IS BLANK

Instructions

Answer **all** questions in the spaces provided.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$

Question 1 (3 marks)

a. Show that $\sqrt{5} - i$ is a solution of the equation $z^3 - (\sqrt{5} - i)z^2 + 4z - 4\sqrt{5} + 4i = 0$. 1 mark

b. Find all other solutions of the equation $z^3 - (\sqrt{5} - i)z^2 + 4z - 4\sqrt{5} + 4i = 0$. 2 marks

Question 2 (4 marks)

Given the relation $3x^2 + 2xy + y^2 = 11$, find the gradient of the **normal** to the graph of the relation at the point in the first quadrant where $x = 1$.

Question 3 (5 marks)

A coffee machine dispenses volumes of coffee that are normally distributed with mean 240 mL and standard deviation 8 mL. The machine also has the option of adding milk to a cup of coffee, where the volume of milk dispensed is also normally distributed with mean 10 mL and standard deviation 2 mL.

Let the random variable X represent the volume of coffee the machine dispenses and let the random variable Y represent the volume of milk the machine dispenses. X and Y are independent random variables.

a. Find the mean and variance of the volume of the combined drink, that is, a cup of coffee with milk. 2 marks

A second coffee machine also dispenses volumes of coffee that are normally distributed. The owner has been told that the mean volume is again 240 mL. The owner is concerned that the second coffee machine is, on average, dispensing less coffee than the first. A sample of 16 cups of coffee (with no milk) is dispensed and it is found that the mean volume of all coffees served in this sample is 235 mL. Assume that the population standard deviation of 8 mL is unchanged.

b. **i.** State appropriate null and alternative hypotheses for the volume V in this situation. 1 mark

ii. The p value for this test is given by the expression $\Pr(Z \leq a)$, where Z has the standard normal distribution.

Find the value of a and **hence** determine whether the null hypothesis should be rejected at the 0.05 level of significance. 2 marks

Question 4 (4 marks)

The region in the first quadrant enclosed by the coordinate axes, the graph with equation $y = e^{-x}$ and the straight line $x = a$ where $a > 0$, is rotated about the x-axis to form a solid of revolution.

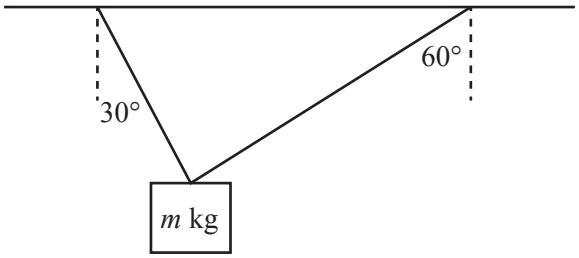
a. Express the volume of the solid of revolution as a definite integral. 1 mark

b. Calculate the volume of the solid of revolution in terms of a. 1 mark

c. Find the exact value of a if the volume is $\frac{5\pi}{18}$ cubic units. 2 marks

Question 5 (4 marks)

A flowerpot of mass m kilograms is held in equilibrium by two light ropes, both of which are connected to a ceiling. The first rope makes an angle of 30° to the vertical and has tension T_1 newtons. The second rope makes an angle of 60° to the vertical and has tension T_2 newtons.



a. Show that $T_2 = \frac{T_1}{\sqrt{3}}$. 1 mark

b. The first rope is strong, but the second rope will break if the tension in it exceeds 98 N.

Find the maximum value of m for which the flowerpot will remain in equilibrium. 3 marks

Question 6 (3 marks)

Evaluate $\int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} \cos^2(2x)\sin(2x)\,dx$.

Question 7 (4 marks)

Solve the following differential equation $\frac{dy}{dx} = \frac{y}{x^2}$ for y, given that when $x = 1$, $y = -1$.

Question 8 (4 marks)

a. Write down a definite integral in terms of θ that gives the arc length from $\theta = 0$ to $\theta = \pi$ for the curve defined parametrically by 2 marks

$$x = \cos(2\theta) - 3$$

$$y = \sin(2\theta) + 1$$

b. **Hence** find the length of this arc. 2 marks

Question 9 (5 marks)

Consider the three vectors $\underset{\sim}{a} = \underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k}$, $\underset{\sim}{b} = \underset{\sim}{i} + 2\underset{\sim}{j} + m\underset{\sim}{k}$ and $\underset{\sim}{c} = \underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k}$, where $m \in R$.

a. Find the value(s) of m for which $|\underset{\sim}{b}| = 2\sqrt{3}$. 2 marks

b. Find the value of m such that $\underset{\sim}{a}$ is perpendicular to $\underset{\sim}{b}$. 1 mark

c. **i.** Calculate $3\underset{\sim}{c} - \underset{\sim}{a}$. 1 mark

ii. **Hence** find a value of m such that $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ are **linearly dependent**. 1 mark

Question 10 (4 marks)

a. Verify that $\frac{5x^3+12x+4}{x^2(x^2+4)}$ can be written as $\frac{1}{x^2}+\frac{3}{x}+\frac{2x-1}{x^2+4}$. 1 mark

b. Find an antiderivative of $\frac{5x^3+12x+4}{x^2(x^2+4)}$. 3 marks

END OF QUESTION AND ANSWER BOOK