

VCE Specialist Mathematics

2016–2018

Written examinations 1 and 2 – End of year

Examination specifications

Overall conditions

There will be two end-of-year examinations for VCE Specialist Mathematics – examination 1 and examination 2.

The examinations will be sat at a time and date to be set annually by the Victorian Curriculum and Assessment Authority (VCAA). VCAA examination rules will apply. Details of these rules are published annually in the *VCE and VCAL Administrative Handbook*.

Examination 1 will have 15 minutes reading time and 1 hour writing time. Students are not permitted to bring into the examination room any technology (calculators or software) or notes of any kind.

Examination 2 will have 15 minutes reading time and 2 hours writing time. Students are permitted to bring into the examination room an approved technology with numerical, graphical, symbolic and statistical functionality, as specified in the *VCAA Bulletin* and the *VCE Exams Navigator*. One bound reference may be brought into the examination room. This may be a textbook (which may be annotated), a securely bound lecture pad, a permanently bound student-constructed set of notes without fold-outs or an exercise book. Specifications for the bound reference are published annually in the *VCE Exams Navigator*.

A formula sheet will be provided with both examinations.

The examinations will be marked by a panel appointed by the VCAA.

Examination 1 will contribute 22 per cent to the study score. Examination 2 will contribute 44 per cent to the study score.

Content

The *VCE Mathematics Study Design 2016–2018* ('Specialist Mathematics Units 3 and 4') is the document for the development of the examination. All outcomes in 'Specialist Mathematics Units 3 and 4' will be examined.

All content from the areas of study, and the key knowledge and skills that underpin the outcomes in Units 3 and 4, are examinable.

Examination 1 will cover all areas of study in relation to Outcome 1. The examination is designed to assess students' knowledge of mathematical concepts, their skill in carrying out mathematical algorithms without the use of technology, and their ability to apply concepts and skills.

Examination 2 will cover all areas of study in relation to all three outcomes, with an emphasis on Outcome 2. The examination is designed to assess students' ability to understand and communicate mathematical ideas, and to interpret, analyse and solve both routine and non-routine problems.

Format

Examination 1

The examination will be in the form of a question and answer book.

The examination will consist of short-answer and extended-answer questions. All questions will be compulsory.

The total marks for the examination will be 40.

A formula sheet will be provided with the examination. The formula sheet will be the same for examinations 1 and 2.

All answers are to be recorded in the spaces provided in the question and answer book.

Examination 2

The examination will be in the form of a question and answer book.

The examination will consist of two sections.

Section A will consist of 20 multiple-choice questions worth 1 mark each and will be worth a total of 20 marks.

Section B will consist of short-answer and extended-answer questions, including multi-stage questions of increasing complexity, and will be worth a total of 60 marks.

All questions will be compulsory. The total marks for the examination will be 80.

A formula sheet will be provided with the examination. The formula sheet will be the same for examinations 1 and 2.

Answers to Section A are to be recorded on the answer sheet provided for multiple-choice questions.

Answers to Section B are to be recorded in the spaces provided in the question and answer book.

Approved materials and equipment

Examination 1

- normal stationery requirements (pens, pencils, highlighters, erasers, sharpeners and rulers)

Examination 2

- normal stationery requirements (pens, pencils, highlighters, erasers, sharpeners and rulers)
- an approved technology with numerical, graphical, symbolic and statistical functionality
- one scientific calculator
- one bound reference

Relevant references

The following publications should be referred to in relation to the VCE Specialist Mathematics examinations:

- *VCE Mathematics Study Design 2016–2018* ('Specialist Mathematics Units 3 and 4')
- *VCE Specialist Mathematics – Advice for teachers 2016–2018* (includes assessment advice)
- *VCE Exams Navigator*
- *VCAA Bulletin*

Advice

During the 2016–2018 accreditation period for VCE Specialist Mathematics, examinations will be prepared according to the examination specifications above. Each examination will conform to these specifications and will test a representative sample of the key knowledge and skills from all outcomes in Units 3 and 4.

The following sample examinations provide an indication of the types of questions teachers and students can expect until the current accreditation period is over.

Answers to multiple-choice questions are provided at the end of examination 2.

Answers to other questions are not provided.

Victorian Certificate of Education
Year

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER Letter

SPECIALIST MATHEMATICS

Written examination 1

Day Date

Reading time: *.*.* to *.*.* (15 minutes)

Writing time: *.*.* to *.*.* (1 hour)

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 11 pages.
- Formula sheet.
- Working space is provided throughout the book.

Instructions

- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ ms}^{-2}$, where $g = 9.8$

Question 1 (3 marks)

- a. Show that $\sqrt{5} - i$ is a solution of the equation $z^3 - (\sqrt{5} - i)z^2 + 4z - 4\sqrt{5} + 4i = 0$. 1 mark

- b. Find all other solutions of the equation $z^3 - (\sqrt{5} - i)z^2 + 4z - 4\sqrt{5} + 4i = 0$. 2 marks

Question 3 (5 marks)

A coffee machine dispenses volumes of coffee that are normally distributed with mean 240 mL and standard deviation 8 mL. The machine also has the option of adding milk to a cup of coffee, where the volume of milk dispensed is also normally distributed with mean 10 mL and standard deviation 2 mL.

Let the random variable X represent the volume of coffee the machine dispenses and let the random variable Y represent the volume of milk the machine dispenses. X and Y are independent random variables.

- a. Find the mean and variance of the volume of the combined drink, that is, a cup of coffee with milk. 2 marks

A second coffee machine also dispenses volumes of coffee that are normally distributed. The owner has been told that the mean volume is again 240 mL. The owner is concerned that the second coffee machine is, on average, dispensing less coffee than the first. A sample of 16 cups of coffee (with no milk) is dispensed and it is found that the mean volume of all coffees served in this sample is 235 mL. Assume that the population standard deviation of 8 mL is unchanged.

- b. i. State appropriate null and alternative hypotheses for the volume V in this situation. 1 mark

- ii. The p value for this test is given by the expression $\Pr(Z \leq a)$, where Z has the standard normal distribution.

Find the value of a and **hence** determine whether the null hypothesis should be rejected at the 0.05 level of significance. 2 marks

Question 4 (4 marks)

The region in the first quadrant enclosed by the coordinate axes, the graph with equation $y = e^{-x}$ and the straight line $x = a$ where $a > 0$, is rotated about the x -axis to form a solid of revolution.

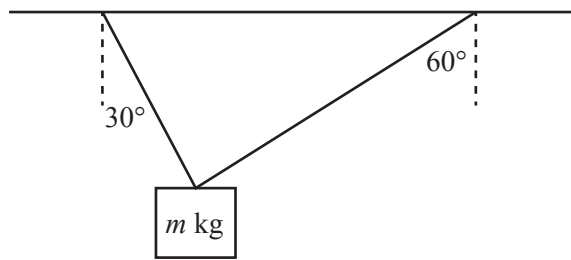
- a. Express the volume of the solid of revolution as a definite integral. 1 mark

- b. Calculate the volume of the solid of revolution in terms of a . 1 mark

- c. Find the exact value of a if the volume is $\frac{5\pi}{18}$ cubic units. 2 marks

Question 5 (4 marks)

A flowerpot of mass m kilograms is held in equilibrium by two light ropes, both of which are connected to a ceiling. The first rope makes an angle of 30° to the vertical and has tension T_1 newtons. The second rope makes an angle of 60° to the vertical and has tension T_2 newtons.



a. Show that $T_2 = \frac{T_1}{\sqrt{3}}$.

1 mark

b. The first rope is strong, but the second rope will break if the tension in it exceeds 98 N.

Find the maximum value of m for which the flowerpot will remain in equilibrium.

3 marks

TURN OVER

Question 6 (3 marks)

Evaluate $\int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} \cos^2(2x) \sin(2x) dx$.

Question 7 (4 marks)

Solve the following differential equation $\frac{dy}{dx} = \frac{y}{x^2}$ for y , given that when $x = 1$, $y = -1$.

Question 8 (4 marks)

- a. Write down a definite integral in terms of θ that gives the arc length from $\theta = 0$ to $\theta = \pi$ for the curve defined parametrically by 2 marks

$$x = \cos(2\theta) - 3$$

$$y = \sin(2\theta) + 1$$

- b. Hence find the length of this arc. 2 marks

Question 9 (5 marks)

Consider the three vectors $\underline{a} = \underline{i} - \underline{j} + 2\underline{k}$, $\underline{b} = \underline{i} + 2\underline{j} + m\underline{k}$ and $\underline{c} = \underline{i} + \underline{j} - \underline{k}$, where $m \in \mathbb{R}$.

- a. Find the value(s) of m for which $|\underline{b}| = 2\sqrt{3}$. 2 marks

- b. Find the value of m such that \underline{a} is perpendicular to \underline{b} . 1 mark

- c. i. Calculate $3\underline{c} - \underline{a}$. 1 mark

- ii. Hence find a value of m such that \underline{a} , \underline{b} and \underline{c} are linearly dependent. 1 mark

Question 10 (4 marks)

- a. Verify that $\frac{5x^3 + 12x + 4}{x^2(x^2 + 4)}$ can be written as $\frac{1}{x^2} + \frac{3}{x} + \frac{2x - 1}{x^2 + 4}$. 1 mark

- b. Find an antiderivative of $\frac{5x^3 + 12x + 4}{x^2(x^2 + 4)}$. 3 marks

Victorian Certificate of Education
Year

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STUDENT NUMBER

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Letter

SPECIALIST MATHEMATICS

Written examination 2

Day Date

Reading time: *.* to *.* (15 minutes)

Writing time: *.* to *.* (2 hours)

QUESTION AND ANSWER BOOK

Structure of book

Section	Number of questions	Number of questions to be answered	Number of marks
A	20	20	20
B	6	6	60
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 23 pages.
- Formula sheet.
- Answer sheet for multiple-choice questions.

Instructions

- Write your **student number** in the space provided above on this page.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination

- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION A – Multiple-choice questions**Instructions for Section A**

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1; an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ ms}^{-2}$, where $g = 9.8$

Question 1

A circle with centre $(a, -2)$ and radius 5 units has equation $x^2 - 6x + y^2 + 4y = b$, where a and b are real constants.

The values of a and b are respectively

- A. -3 and 38
- B. 3 and 12
- C. -3 and -8
- D. -3 and 0
- E. 3 and 18

Question 2

The maximal domain and range of the function with rule $f(x) = 3 \sin^{-1}(4x - 1) + \frac{\pi}{2}$ are respectively

- A. $[-\pi, 2\pi]$ and $\left[0, \frac{1}{2}\right]$
- B. $\left[0, \frac{1}{2}\right]$ and $[-\pi, 2\pi]$
- C. $\left[-\frac{3\pi}{2}, \frac{3\pi}{2}\right]$ and $\left[-\frac{1}{2}, 0\right]$
- D. $\left[0, \frac{1}{2}\right]$ and $[0, 3\pi]$
- E. $\left[-\frac{1}{2}, 0\right]$ and $[-\pi, 2\pi]$

Question 3

The features of the graph of the function with rule $f(x) = \frac{x^2 - 4x + 3}{x^2 - x - 6}$ include

- A. asymptotes at $x = 1$ and $x = -2$
- B. asymptotes at $x = 3$ and $x = -2$
- C. an asymptote at $x = 1$ and a point of discontinuity at $x = 3$
- D. an asymptote at $x = -2$ and a point of discontinuity at $x = 3$
- E. an asymptote at $x = 3$ and a point of discontinuity at $x = -2$

Question 4

The algebraic fraction $\frac{7x-5}{(x-4)^2(x^2+9)}$ could be expressed in partial fraction form as

- A. $\frac{A}{(x-4)^2} + \frac{B}{x^2+9}$
- B. $\frac{A}{x-4} + \frac{B}{x-3} + \frac{C}{x+3}$
- C. $\frac{A}{(x-4)^2} + \frac{Bx+C}{x^2+9}$
- D. $\frac{A}{x-4} + \frac{B}{(x-4)^2} + \frac{Cx+D}{x^2+9}$
- E. $\frac{A}{x-4} + \frac{B}{(x-4)^2} + \frac{C}{x^2+9}$

Question 5

On an Argand diagram, a set of points that lies on a circle of radius 2 centred at the origin is

- A. $\{z \in C : z\bar{z} = 2\}$
- B. $\{z \in C : z^2 = 4\}$
- C. $\{z \in C : \operatorname{Re}(z^2) + \operatorname{Im}(z^2) = 4\}$
- D. $\{z \in C : (z + \bar{z})^2 - (z - \bar{z})^2 = 16\}$
- E. $\{z \in C : (\operatorname{Re}(z))^2 + (\operatorname{Im}(z))^2 = 16\}$

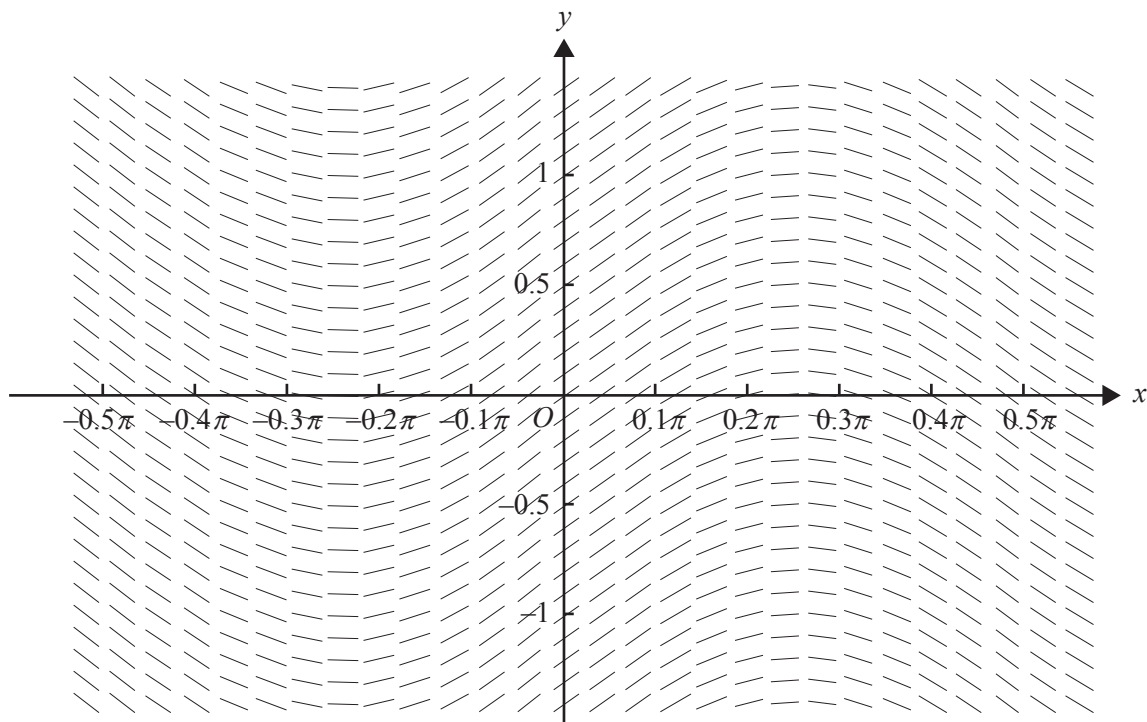
Question 6

The polynomial $P(z)$ has real coefficients. Four of the roots of the equation $P(z) = 0$ are $z = 0$, $z = 1 - 2i$, $z = 1 + 2i$ and $z = 3i$.

The **minimum** number of roots that the equation $P(z) = 0$ could have is

- A. 4
- B. 5
- C. 6
- D. 7
- E. 8

Question 7



The direction (slope) field for a certain first-order differential equation is shown above.
The differential equation could be

- A. $\frac{dy}{dx} = \sin(2x)$
- B. $\frac{dy}{dx} = \cos(2x)$
- C. $\frac{dy}{dx} = \cos\left(\frac{1}{2}y\right)$
- D. $\frac{dy}{dx} = \sin\left(\frac{1}{2}y\right)$
- E. $\frac{dy}{dx} = \cos\left(\frac{1}{2}x\right)$

Question 8

Let $f: [-\pi, 2\pi] \rightarrow \mathbb{R}$, where $f(x) = \sin^3(x)$.

Using the substitution $u = \cos(x)$, the area bounded by the graph of f and the x -axis could be found by evaluating

A. $-\int_{-\pi}^{2\pi} (1-u^2) du$

B. $3\int_{-1}^1 (1-u^2) du$

C. $-3\int_0^\pi (1-u^2) du$

D. $3\int_1^{-1} (1-u^2) du$

E. $-\int_{-1}^1 (1-u^2) du$

Question 9

Let $\frac{dy}{dx} = \frac{x+2}{x^2+2x+1}$ and $(x_0, y_0) = (0, 2)$.

Using Euler's method with a step size of 0.1, the value of y_1 , correct to two decimal places, is

A. 0.17

B. 0.20

C. 1.70

D. 2.17

E. 2.20

Question 10

The curve given by $y = \sin^{-1}(2x)$, where $0 \leq x \leq \frac{1}{2}$, is rotated about the y -axis to form a solid of revolution.

The volume of the solid may be found by evaluating

A. $\frac{\pi}{4} \int_0^{\frac{\pi}{2}} (1 - \cos(2y)) dy$

B. $\frac{\pi}{8} \int_0^{\frac{1}{2}} (1 - \cos(2y)) dy$

C. $\frac{\pi}{8} \int_0^{\frac{\pi}{2}} (1 - \cos(2y)) dy$

D. $\frac{1}{8} \int_0^{\frac{\pi}{2}} (1 - \cos(2y)) dy$

E. $\frac{\pi}{8} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (1 - \cos(2y)) dy$

Question 11

The angle between the vectors $3\mathbf{i} + 6\mathbf{j} - 2\mathbf{k}$ and $2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$, correct to the nearest tenth of a degree, is

- A. 2.0°
- B. 91.0°
- C. 112.4°
- D. 121.3°
- E. 124.9°

Question 12

The scalar resolute of $\mathbf{a} = 3\mathbf{i} - \mathbf{k}$ in the direction of $\mathbf{b} = 2\mathbf{i} - \mathbf{j} - 2\mathbf{k}$ is

- A. $\frac{8}{\sqrt{10}}$
- B. $\frac{8}{9}(2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$
- C. 8
- D. $\frac{4}{5}(3\mathbf{i} - \mathbf{k})$
- E. $\frac{8}{3}$

Question 13

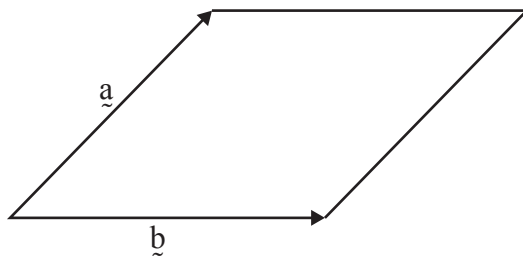
The position vector of a particle at time t seconds, $t \geq 0$, is given by $\underline{r}(t) = (3-t)\underline{i} - 6\sqrt{t}\underline{j} + 5\underline{k}$.

The direction of motion of the particle when $t = 9$ is

- A. $-6\underline{i} - 18\underline{j} + 5\underline{k}$
- B. $-\underline{i} - \underline{j}$
- C. $-6\underline{i} - \underline{j}$
- D. $-\underline{i} - \underline{j} + 5\underline{k}$
- E. $-13.5\underline{i} - 108\underline{j} + 45\underline{k}$

Question 14

The diagram below shows a rhombus, spanned by the two vectors \underline{a} and \underline{b} .



It follows that

- A. $\underline{a} \cdot \underline{b} = 0$
- B. $\underline{a} = \underline{b}$
- C. $(\underline{a} + \underline{b}) \cdot (\underline{a} - \underline{b}) = 0$
- D. $|\underline{a} + \underline{b}| = |\underline{a} - \underline{b}|$
- E. $2\underline{a} + 2\underline{b} = \underline{0}$

Question 15

A 12 kg mass moves in a straight line under the action of a variable force F , so that its velocity v ms^{-1} when it is x metres from the origin is given by $v = \sqrt{3x^2 - x^3 + 16}$.

The force F acting on the mass is given by

- A. $12\left(3x - \frac{3x^2}{2}\right)$
- B. $12(3x^2 - x^3 + 16)$
- C. $12(6x - 3x^2)$
- D. $12\sqrt{3x^2 - x^3 + 16}$
- E. $12(3 - 3x)$

Question 16

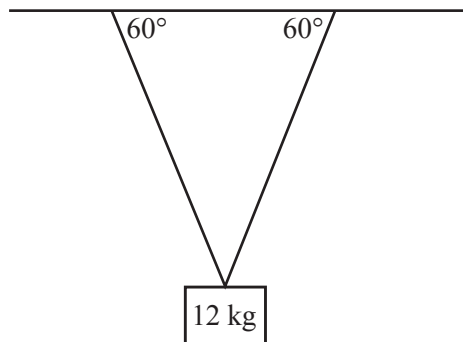
The acceleration, $a \text{ ms}^{-2}$, of a particle moving in a straight line is given by $a = \frac{v}{\log_e(v)}$, where v is the velocity of the particle in ms^{-1} at time t seconds. The initial velocity of the particle was 5 ms^{-1} .

The velocity of the particle, in terms of t , is given by

- A. $v = e^{2t}$
- B. $v = e^{2t} + 4$
- C. $v = e^{\sqrt{2t} + \log_e(5)}$
- D. $v = e^{\sqrt{2t + (\log_e 5)^2}}$
- E. $v = e^{-\sqrt{2t + (\log_e 5)^2}}$

Question 17

A 12 kg mass is suspended in equilibrium from a horizontal ceiling by two identical light strings. Each string makes an angle of 60° with the ceiling, as shown.



The magnitude, in newtons, of the tension in each string is equal to

- A. 6 g
- B. 12 g
- C. 24 g
- D. $4\sqrt{3} \text{ g}$
- E. $8\sqrt{3} \text{ g}$

Question 18

Given that X is a normal random variable with mean 10 and standard deviation 8, and that Y is a normal random variable with mean 3 and standard deviation 2, and X and Y are independent random variables, the random variable Z defined by $Z = X - 3Y$ will have mean μ and standard deviation σ given by

- A. $\mu = 1, \sigma = 28$
- B. $\mu = 19, \sigma = 2$
- C. $\mu = 1, \sigma = 2\sqrt{7}$
- D. $\mu = 19, \sigma = 14$
- E. $\mu = 1, \sigma = 10$

Question 19

The mean study score for a large VCE study is 30 with a standard deviation of 7. A class of 20 students may be considered as a random sample drawn from this cohort.

The probability that the class mean for the group of 20 exceeds 32 is

- A. 0.1007
- B. 0.3875
- C. 0.3993
- D. 0.6125
- E. 0.8993

Question 20

A type I error would occur in a statistical test where

- A. H_0 is accepted when H_0 is false.
- B. H_1 is accepted when H_1 is true.
- C. H_0 is rejected when H_0 is true.
- D. H_1 is rejected when H_1 is true.
- E. H_0 is accepted when H_0 is true.

SECTION B**Instructions for Section B**

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ ms}^{-2}$, where $g = 9.8$

Question 1 (10 marks)

Consider the function $f: [0, 3) \rightarrow \mathbb{R}$, where $f(x) = -2 + 2\sec\left(\frac{\pi x}{6}\right)$.

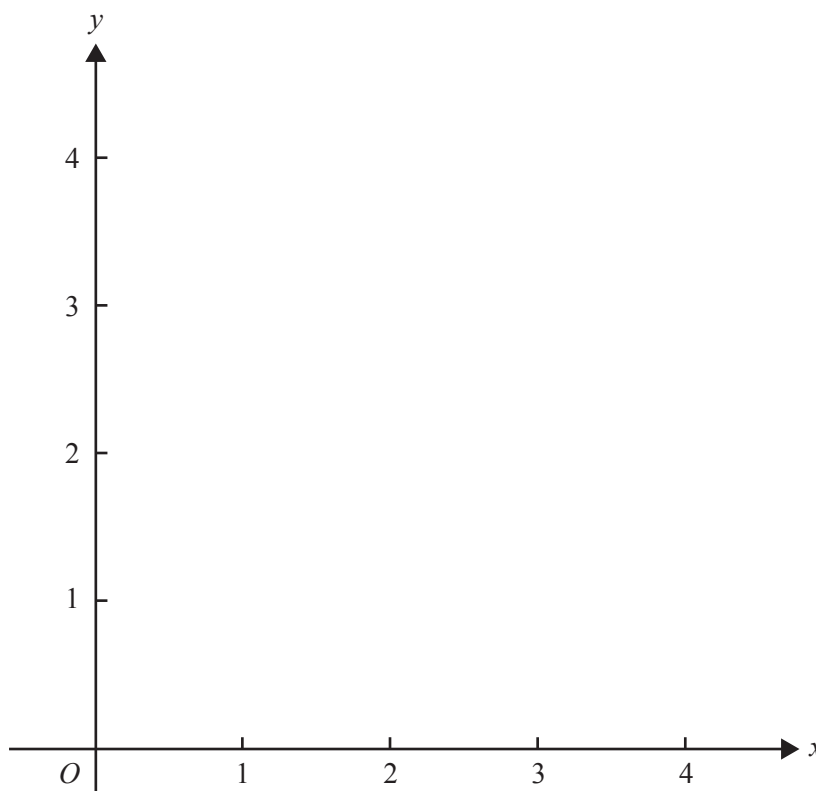
a. Evaluate $f(2)$.

1 mark

Let f^{-1} be the inverse function of f .

b. On the axes below, sketch the graphs of f and f^{-1} , showing their points of intersection.

2 marks



- c. The rule for f^{-1} can be written as $f^{-1}(x) = k \arccos\left(\frac{2}{x+2}\right)$.

Find the **exact** value of k .

2 marks

Let A be the magnitude of the area enclosed by the graphs of f and f^{-1} .

- d. Write a definite integral expression for A and evaluate it correct to three decimal places.

2 marks

- e. i. Write down a definite integral in terms of x that gives the arc length of the graph of f from $x = 0$ to $x = 2$.

2 marks

- ii. Evaluate this definite integral correct to three decimal places.

1 mark

Question 2 (9 marks)

Let $u = \frac{1}{2} + \frac{\sqrt{3}}{2}i$.

a. i. Express u in polar form.

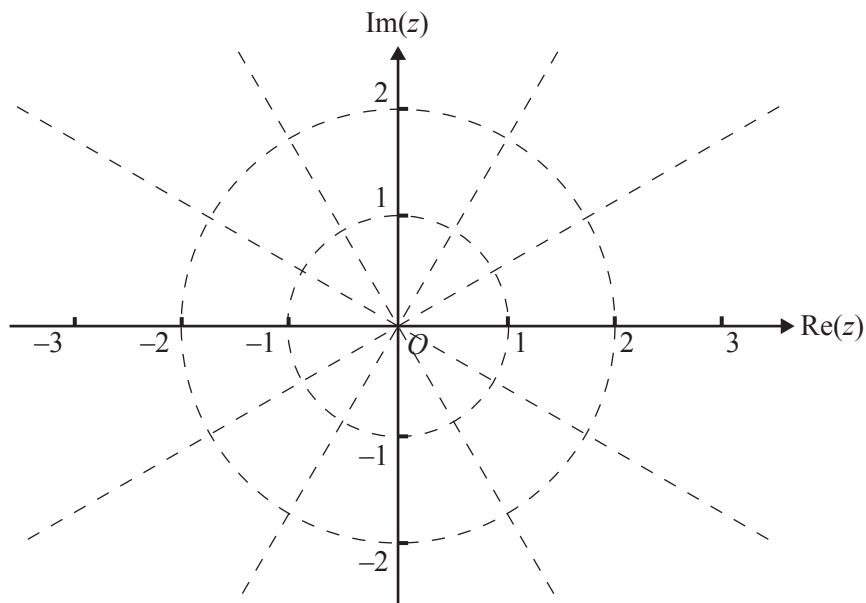
2 marks

ii. Hence show that $u^6 = 1$.

1 mark

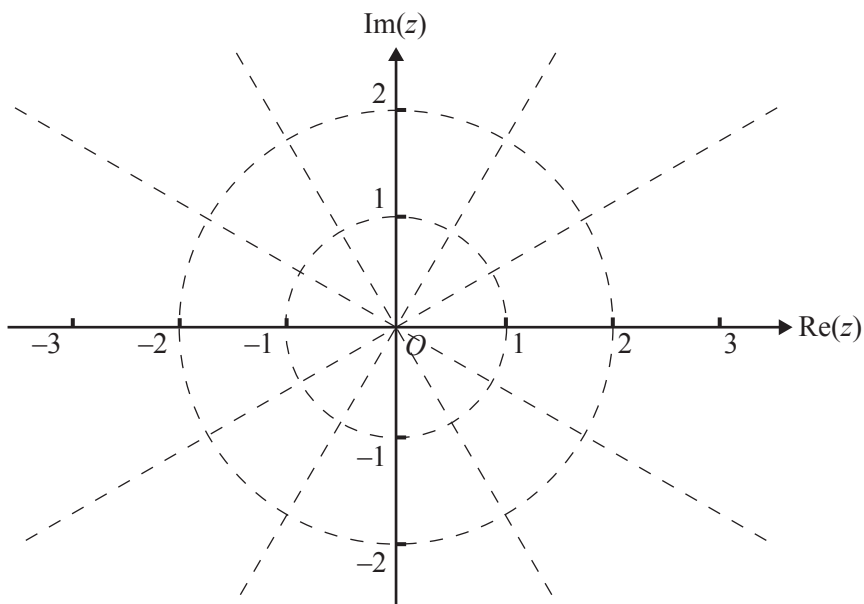
iii. Plot all roots of $z^6 - 1 = 0$ on the Argand diagram below, labelling u and w where $w = -u$.

2 marks



- b. i.** Draw and label the subset of the complex plane given by $S = \{z : |z| = 1\}$ on the Argand diagram below.

1 mark



- ii.** Draw and label the subset of the complex plane given by $T = \{z : |z - u| = |z + u|\}$ on the Argand diagram above.

1 mark

- iii.** Find the coordinates of the points of intersection of S and T .

2 marks

Question 3 (11 marks)

The number of mobile phones, N , owned in a certain community after t years may be modelled by $\log_e(N) = 6 - 3e^{-0.4t}$, $t \geq 0$.

- a. Verify by substitution that $\log_e(N) = 6 - 3e^{-0.4t}$ satisfies the differential equation

$$\frac{1}{N} \frac{dN}{dt} + 0.4 \log_e(N) - 2.4 = 0$$

2 marks

- b. Find the initial number of mobile phones owned in the community. Give your answer correct to the nearest integer.

1 mark

- c. Using this mathematical model, find the limiting number of mobile phones that would eventually be owned in the community. Give your answer correct to the nearest integer.

2 marks

The differential equation in **part a.** can also be written in the form $\frac{dN}{dt} = 0.4N(6 - \log_e(N))$.

- d. i.** Find $\frac{d^2N}{dt^2}$ in terms of N and $\log_e(N)$.

2 marks

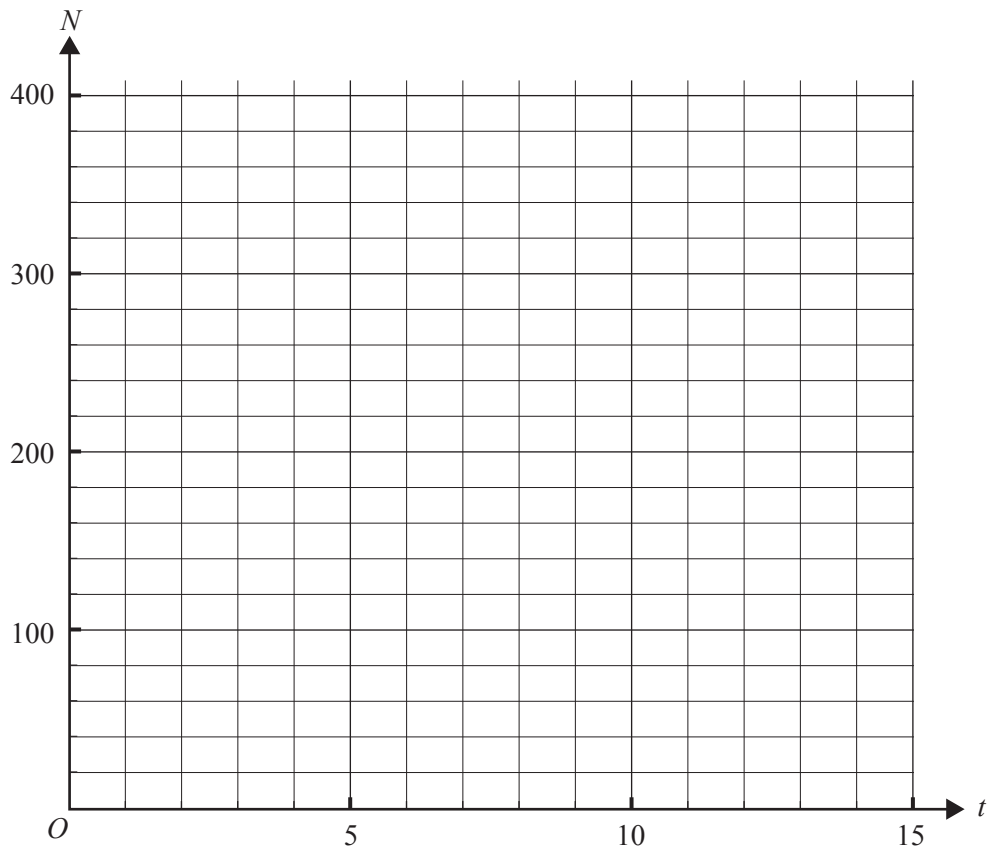
- ii.** The graph of N as a function of t has a point of inflection.

Find the values of the coordinates of this point. Give the value of t correct to one decimal place and the value of N correct to the nearest integer.

2 marks

e. Sketch the graph of N as a function of t on the axes below for $0 \leq t \leq 15$.

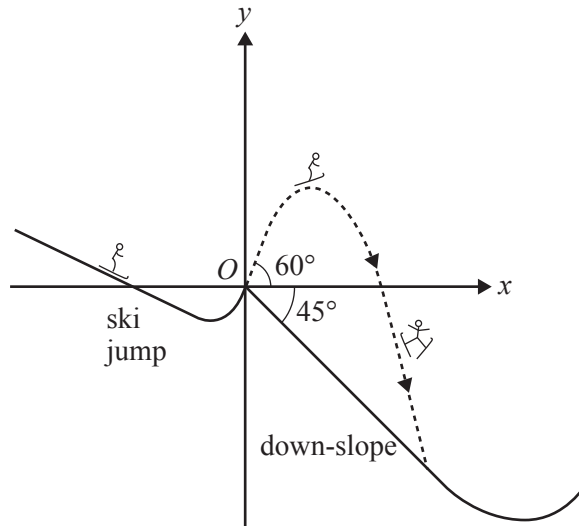
2 marks



Question 4 (10 marks)

A skier accelerates down a slope and then skis up a short ski jump, as shown below. The skier leaves the jump at a speed of 12 ms^{-1} and at an angle of 60° to the horizontal. The skier performs various gymnastic twists and lands on a straight-line section of the 45° down-slope T seconds after leaving the jump.

Let the origin O of a cartesian coordinate system be at the point where the skier leaves the jump, with \mathbf{i} a unit vector in the positive x direction and \mathbf{j} a unit vector in the positive y direction. Displacements are measured in metres and time in seconds.

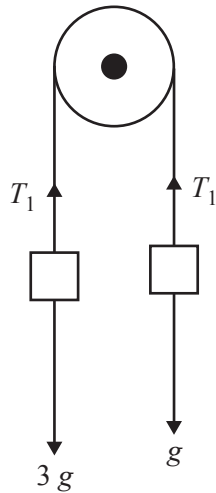


- a. Show that the initial velocity of the skier when leaving the jump is $6\mathbf{i} + 6\sqrt{3}\mathbf{j}$. 1 mark

- d.** At what speed, in metres per second, does the skier land on the down-slope? Give your answer correct to one decimal place. 3 marks

Question 5 (10 marks)

The diagram below shows particles of mass 1 kg and 3 kg connected by a light inextensible string passing over a smooth pulley. The tension in the string is T_1 newtons.



- a. Let $a \text{ ms}^{-2}$ be the acceleration of the 3 kg mass downwards.

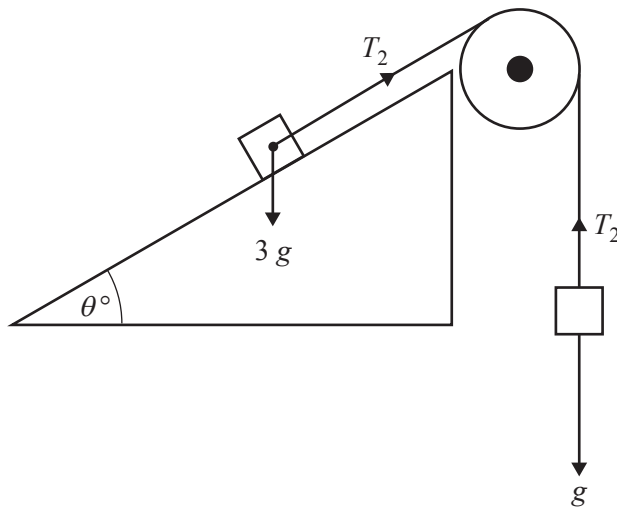
Find the value of a .

2 marks

- b. Find the value of T_1 .

1 mark

The 3 kg mass is placed on a smooth plane inclined at an angle of θ° to the horizontal. The tension in the string is now T_2 newtons.



- c. When $\theta^\circ = 30^\circ$, the acceleration of the 1 kg mass upwards is $b \text{ ms}^{-2}$.

Find the value of b .

3 marks

- d. For what angle θ° will the 3 kg mass be at rest on the plane? Give your answer correct to one decimal place.

2 marks

- e. What angle θ° will cause the 3 kg mass to accelerate up the plane at $\frac{g}{4} \left(1 - \frac{3}{\sqrt{2}}\right) \text{ ms}^{-2}$?

2 marks

Question 6 (10 marks)

A certain type of computer, once fully charged, is claimed by the manufacturer to have $\mu = 10$ hours lifetime before a recharge is needed. When checked, a random sample of $n = 25$ such computers is found to have an average lifetime of $\bar{x} = 9.7$ hours and a standard deviation of $s = 1$ hour.

To decide whether the information gained from the sample is consistent with the claim $\mu = 10$, a statistical test is to be carried out.

Assume that the distribution of lifetimes is normal and that s is a sufficiently accurate estimate of the population (of lifetimes) standard deviation σ .

- a. Write down suitable hypotheses H_0 and H_1 to test whether the mean lifetime is less than that claimed by the manufacturer. 2 marks

- b. Find the p value for this test, correct to three decimal places. 2 marks

- c. State with a reason whether H_0 should be rejected or not rejected at the 5% level of significance. 1 mark

Let the random variable \bar{X} denote the mean lifetime of a random sample of 25 computers, assuming $\mu = 10$.

- d. Find C^* such that $\Pr(\bar{X} < C^* | \mu = 10) = 0.05$. Give your answer correct to three decimal places. 2 marks

- e. i. If the mean lifetime of all computers is in fact $\mu = 9.5$ hours, find $\Pr(\bar{X} > C^* | \mu = 9.5)$, giving your answer correct to three decimal places, where C^* is your answer to **part d.** 2 marks

- ii. Does the result in **part e.i.** indicate a type I or type II error? Explain your answer. 1 mark

Answers to multiple-choice questions

Question	Answer
1	B
2	B
3	D
4	D
5	D
6	B
7	B
8	B
9	E
10	C
11	C
12	E
13	B
14	C
15	A
16	D
17	D
18	E
19	A
20	C