

Victorian Certificate of Education

# MATHEMATICS

STUDY DESIGN



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# Important information

## Accreditation period

Units 1–4: 1 January 2016 – 31 December 2022  
Implementation of this study commences in January 2016.

## Sources of information

The [VCAA Bulletin VCE, VCAL and VET](#) is the only official source of changes to regulations and accredited studies. The VCAA Bulletin also regularly includes advice on VCE studies. It is the responsibility of each VCE teacher to refer to each issue of the VCAA Bulletin. The VCAA Bulletin is available as an e-newsletter via free subscription on the VCAA's website at: [www.vcaa.vic.edu.au](http://www.vcaa.vic.edu.au).

To assist teachers in developing courses, the VCAA publishes online the *Advice for teachers*, which includes teaching and learning activities for Units 1–4, and advice on assessment tasks and performance criteria for School-assessed Coursework in Units 3 and 4.

The current [VCE and VCAL Administrative Handbook](#) contains essential information on assessment processes and other procedures.

## VCE providers

Throughout this study design the term 'school' is intended to include both schools and other VCE providers.

## Copyright

VCE schools may reproduce parts of this study design for use by teachers. The full VCAA Copyright Policy is available at: [www.vcaa.vic.edu.au/Footer/Pages/Copyright.aspx](http://www.vcaa.vic.edu.au/Footer/Pages/Copyright.aspx).

# Introduction

## Scope of study

Mathematics is the study of function and pattern in number, logic, space and structure, and of randomness, chance, variability and uncertainty in data and events. It is both a framework for thinking and a means of symbolic communication that is powerful, logical, concise and precise. Mathematics also provides a means by which people can understand and manage human and natural aspects of the world and inter-relationships between these. Essential mathematical activities include: conjecturing, hypothesising and problem posing; estimating, calculating and computing; abstracting, proving, refuting and inferring; applying, investigating, modelling and problem solving.

## Rationale

This study is designed to provide access to worthwhile and challenging mathematical learning in a way which takes into account the interests, needs, dispositions and aspirations of a wide range of students, and introduces them to key aspects of the discipline. It is also designed to promote students' awareness of the importance of mathematics in everyday life in a technological society, and to develop confidence and the disposition to make effective use of mathematical concepts, processes and skills in practical and theoretical contexts.

## Aims

This study enables students to:

- develop mathematical concepts, knowledge and skills
- apply mathematics to analyse, investigate and model a variety of contexts and solve practical and theoretical problems in situations that range from well-defined and familiar to open-ended and unfamiliar
- use technology effectively as a tool for working mathematically.

## Structure

The study is made up of the following units:

Foundation Mathematics Units 1 and 2

General Mathematics Units 1 and 2

Mathematical Methods Units 1 and 2

Specialist Mathematics Units 1 and 2

Further Mathematics Units 3 and 4

Mathematical Methods Units 3 and 4

Specialist Mathematics Units 3 and 4

Each unit deals with specific content contained in areas of study and is designed to enable students to achieve a set of three outcomes for that unit. Each outcome is described in terms of key knowledge and key skills. A glossary defining mathematical terms and notations used across Units 1 to 4 in the VCE Mathematics Study Design is included in a companion document *Advice for teachers*.

**Foundation Mathematics Units 1 and 2** are completely prescribed and provide for the continuing mathematical development of students entering VCE. In general, these students would not intend to undertake Unit 3 and 4 studies in VCE Mathematics in the following year. However, students who do well in these units and undertake some supplementary study of selected topics could proceed to Further Mathematics Units 3 and 4.

**General Mathematics Units 1 and 2** provide for a range of courses of study involving non-calculus based topics for a broad range of students and may be implemented in various ways to reflect student interests in, and applications of, mathematics. They incorporate topics that provide preparation for various combinations of studies at Units 3 and 4 and cover assumed knowledge and skills for those units.

**Mathematical Methods Units 1 and 2** are completely prescribed and provide an introductory study of simple elementary functions, algebra, calculus, probability and statistics and their applications in a variety of practical and theoretical contexts. They are designed as preparation for Mathematical Methods Units 3 and 4 and cover assumed knowledge and skills for those units.

**Specialist Mathematics Units 1 and 2** comprise a combination of prescribed and selected non-calculus based topics and provide courses of study for students interested in advanced study of mathematics, with a focus on mathematical structure and reasoning. They incorporate topics that, in conjunction with Mathematical Methods Units 1 and 2, provide preparation for Specialist Mathematics Units 3 and 4 and cover assumed knowledge and skills for those units.

**Further Mathematics Units 3 and 4** are designed to be widely accessible and comprise a combination of non-calculus based content from a prescribed core and a selection of two from four possible modules across a range of application contexts. They provide general preparation for employment or further study, in particular where data analysis, recursion and number patterns are important. The assumed knowledge and skills for the Further Mathematics Units 3 and 4 prescribed core are covered in specified topics from General Mathematics Units 1 and 2. Students who have done only Mathematical Methods Units 1 and 2 will also have had access to assumed knowledge and skills to undertake Further Mathematics but may also need to undertake some supplementary study of statistics content.

**Mathematical Methods Units 3 and 4** are completely prescribed and extend the study of simple elementary functions to include combinations of these functions, algebra, calculus, probability and statistics, and their applications in a variety of practical and theoretical contexts. They also provide background for further study in, for example, science, humanities, economics and medicine.

**Specialist Mathematics Units 3 and 4** are designed to be taken in conjunction with Mathematical Methods Units 3 and 4, or following previous completion of Mathematical Methods Units 3 and 4. The areas of study extend content from Mathematical Methods Units 3 and 4 to include rational and other quotient functions as well as other advanced mathematics topics such as complex numbers, vectors, differential equations, mechanics and statistical inference. Study of Specialist Mathematics Units 3 and 4 assumes concurrent study or previous completion of Mathematical Methods Units 3 and 4.

## Combinations of Mathematics units

Units 1 and 2	Units 3 and 4
Foundation Mathematics*	Further Mathematics
General Mathematics	Further Mathematics
Mathematical Methods	Mathematical Methods or Further Mathematics
General Mathematics and Mathematical Methods	Mathematical Methods and/or Further Mathematics
Mathematical Methods**	Mathematical Methods and Specialist Mathematics
Mathematical Methods and General Mathematics	Mathematical Methods and Specialist Mathematics
Mathematical Methods and Specialist Mathematics	Mathematical Methods and Specialist Mathematics
General Mathematics or Specialist Mathematics and Mathematical Methods	Further Mathematics, Mathematical Methods and Specialist Mathematics

\* For this combination of units students wishing to progress to Further Mathematics Units 3 and 4 will need to undertake some supplementary study with respect to assumed knowledge and skills for Area of Study 1.

\*\* For this combination of units students will need to undertake some supplementary study with respect to assumed knowledge and skills for Specialist Mathematics Units 3 and 4.

## Entry

There are no prerequisites for entry to Units 1, 2 and 3; however, students undertaking Mathematical Methods Units 1 and 2 or Specialist Mathematics Units 1 and 2 are assumed to have a sound background in number, algebra, function, geometry, probability and statistics. Students must undertake Unit 3 prior to undertaking Unit 4. Units 1 to 4 are designed to a standard equivalent to the final two years of secondary education. All VCE studies are benchmarked against comparable national and international curriculum. Enrolment in Specialist Mathematics Units 3 and 4 assumes a current enrolment in, or previous completion of, Mathematical Methods Units 3 and 4. There are no restrictions on the number of units students may obtain credit towards satisfactory completion of the VCE.

## Duration

Each unit involves at least 50 hours of scheduled classroom instruction over the duration of a semester.

## Changes to the study design

During its period of accreditation minor changes to the study will be announced in the [VCAA Bulletin VCE, VCAL and VET](#). The *VCAA Bulletin* is the only source of changes to regulations and accredited studies. It is the responsibility of each VCE teacher to monitor changes and advice about VCE studies published in the *VCAA Bulletin*.

## Monitoring for quality

As part of ongoing monitoring and quality assurance, the VCAA undertakes an audit of VCE Mathematics to ensure the study is being taught and assessed as accredited. The details of the audit procedures and requirements are published annually in the [VCE and VCAL Administrative Handbook](#). Schools will be notified if they are required to submit material to be audited.



## **Safety and wellbeing**

It is the responsibility of the school to ensure that duty of care is exercised in relation to the health and safety of all students undertaking the study.

## **Employability skills**

This study offers a number of opportunities for students to develop employability skills. The *Advice for teachers* companion document provides specific examples of how students can develop employability skills during learning activities and assessment tasks.

## **Legislative compliance**

When collecting and using information, the provisions of privacy and copyright legislation, such as the Victorian *Privacy and Data Protection Act 2014* and *Health Records Act 2001*, and the federal *Privacy Act 1988* and *Copyright Act 1968*, must be met.

# Assessment and reporting

## Satisfactory completion

The award of satisfactory completion for a unit is based on the teacher's decision that the student has demonstrated achievement of the set of outcomes specified for the unit. Demonstration of achievement of outcomes and satisfactory completion of a unit are determined by evidence gained through the assessment of a range of learning activities and tasks.

Teachers must develop courses that provide appropriate opportunities for students to demonstrate satisfactory achievement of outcomes.

The decision about satisfactory completion of a unit is distinct from the assessment of levels of achievement. Schools will report a student's result for each unit to the VCAA as S (Satisfactory) or N (Not Satisfactory).

## Levels of achievement

### Units 1 and 2

Procedures for the assessment of levels of achievement in Units 1 and 2 are a matter for school decision. Assessment of levels of achievement for these units will not be reported to the VCAA. Schools may choose to report levels of achievement using grades, descriptive statements or other indicators.

### Units 3 and 4

The VCAA specifies the assessment procedures for students undertaking scored assessment in Units 3 and 4. Designated assessment tasks are provided in the details for each unit in the VCE study designs.

The student's level of achievement in Units 3 and 4 will be determined by School-assessed Coursework (SACs) and/or School-assessed Tasks (SATs) as specified in the VCE study designs, and external assessment.

The VCAA will report the student's level of achievement on each assessment component as a grade from A+ to E or UG (ungraded). To receive a study score the student must achieve two or more graded assessments and receive S for both Units 3 and 4. The study score is reported on a scale of 0–50; it is a measure of how well the student performed in relation to all others who took the study. Teachers should refer to the current [VCE and VCAL Administrative Handbook](#) for details on graded assessment and calculation of the study score. Percentage contributions to the study score in VCE Mathematics are as follows:

#### Further Mathematics

- Unit 3 School-assessed Coursework: 20 per cent
- Unit 4 School-assessed Coursework: 14 per cent
- Units 3 and 4 Examination 1: 33 per cent
- Units 3 and 4 Examination 2: 33 per cent

#### Mathematical Methods

- Unit 3 School-assessed Coursework: 17 per cent
- Unit 4 School-assessed Coursework: 17 per cent
- Units 3 and 4 Examination 1: 22 per cent
- Units 3 and 4 Examination 2: 44 per cent

### Specialist Mathematics

- Unit 3 School-assessed Coursework: 17 per cent
- Unit 4 School-assessed Coursework: 17 per cent
- Units 3 and 4 Examination 1: 22 per cent
- Units 3 and 4 Examination 2: 44 per cent

Examination 1 for Mathematical Methods and Examination 1 for Specialist Mathematics are technology free examinations. Examinations 1 and 2 for Further Mathematics, Examination 2 for Mathematical Methods and Examination 2 for Specialist Mathematics assume student access to VCAA approved technology. Details of the assessment program are described in the sections for Units 3 and 4 in this study design.

### Authentication

Work related to the outcomes of each unit will be accepted only if the teacher can attest that, to the best of their knowledge, all unacknowledged work is the student's own. Teachers need to refer to the current [VCE and VCAL Administrative Handbook](#) for authentication procedures.

# Foundation Mathematics Units 1 and 2

Foundation Mathematics provides for the continuing mathematical development of students entering VCE and who do not necessarily intend to undertake Unit 3 and 4 studies in VCE Mathematics in the following year. This course is designed to complement General Mathematics and Mathematical Methods. Students completing this course would need to undertake additional targeted mathematical study in order to attempt Further Mathematics Units 3 and 4.

In Foundation Mathematics there is a strong emphasis on the use of mathematics in practical contexts encountered in everyday life in the community, at work and at study. The areas of study for Units 1 and 2 of Foundation Mathematics are 'Space, shape and design', 'Patterns and number', 'Data' and 'Measurement'.

All four areas of study are to be completed over the two units. The content should be developed using contexts present in students' other studies, work and personal or other familiar situations.

In undertaking these units, students are expected to be able to apply techniques, routines and processes involving rational and real arithmetic, sets, lists and tables, diagrams and geometric constructions, equations and graphs with and without the use of technology. They should have facility with relevant mental and by-hand approaches to estimation and computation. The use of numerical, graphical, geometric, symbolic and statistical functionality of technology for teaching and learning mathematics, for working mathematically, and in related assessment, is to be incorporated throughout each unit as applicable.

## Area of Study 1

### Space, shape and design

In this area of study students cover the geometric properties of lines and curves, and shapes and objects, and their graphical and diagrammatic representations with attention to scale and drawing conventions used in domestic, societal, industrial and commercial plans, maps and diagrams.

This area of study includes:

- geometric conventions and properties of shapes and objects
- interpretation and use of plans, elevations, maps, models and diagrams
- application and use of similarity and symmetry
- enlargement and reduction of diagrams and models
- interpretation and use of location, distance, direction and scale on diagrams, maps and plans
- application of Pythagoras' theorem in practical situations.

## Area of Study 2

### Patterns and number

In this area of study students cover estimation, the use and application of different forms of numbers and calculations, and the representation of patterns and generalisations in number including formulas and other algebraic expressions in everyday contexts.

This area of study includes:

- application of integers, decimals, fractions, ratios, proportions, percentages and rates to solve practical problems
- estimation, approximation and reasonableness of results

- use and interpretation of formulas and algebraic expressions to describe relationships between variables and to model patterns
- manipulation and solution of expressions and equations to solve problems including predicting a required quantity or finding a break-even point.

## Area of Study 3

### Data

In this area of study students cover collection, presentation and analysis of gathered and provided data from community, work, recreation and media contexts, including consideration of suitable forms of representation and summaries.

This area of study includes:

- features, conventions and terminology used when representing information in diagrammatic, graphical and tabular forms
- collection and representation of data in diagrammatic, tabular and graphical forms
- interpretation of diagrams, charts, tables and graphs
- use of measures of central tendency (averages) and spread to summarise and interpret data
- comparison and interpretation of data sets.

## Area of Study 4

### Measurement

In this area of study students cover the use and application of the metric system and related measurement in a variety of domestic, societal, industrial and commercial contexts, including consideration of accuracy.

This area of study includes:

- application and use of metric units and measures, including derived measures
- interpretation of scales on digital and analogue instruments
- solution of personal, societal and workplace problems involving metric measurement with consideration of error, required accuracy and tolerances
- estimation and approximation strategies
- interpretation and use of time and duration including time and date specifications, conventions, schedules, timetables and time zones.

## Outcomes

For each unit the student is required to demonstrate achievement of all three outcomes. As a set these outcomes encompass all of the selected areas of study for each unit. For each of Unit 1 and Unit 2, the outcomes apply to the content from the areas of study selected for that unit.

### Outcome 1

On completion of this unit the student should be able to use and apply a range of mathematical concepts, skills and procedures from selected areas of study to solve problems based on a range of everyday and real-life contexts.

#### Space, shape and design

To achieve this outcome the student will draw on knowledge and skills outlined in Area of Study 1.

##### Key knowledge

- the names and properties of common geometric shapes and objects
- the language, symbols and conventions for the representation of geometric objects, including point, line, ray, angle, diagonal, edge, curve, face, and vertex
- the language, symbols and labelling and drawing conventions for diagrams, maps, plans, and models, including key, scale, direction, distance, coordinates and grid reference and elevation
- transformations, symmetry and similarity
- Pythagoras' theorem.

##### Key skills

- interpret and describe objects using accurate and appropriate geometric language and conventions
- create and modify diagrams, plans, maps and designs using drawing equipment and digital drawing packages
- develop three-dimensional models for objects and produce two-dimensional representations
- interpret diagrams, plans, maps and models and evaluate their accuracy
- interpret information on maps to plan and describe travel routes, including use of navigational software and tools
- apply similarity, symmetry and Pythagoras' theorem to problems in art, design and measurement.

#### Patterns and number

To achieve this outcome the student will draw on knowledge and skills outlined in Area of Study 2.

##### Key knowledge

- integers, decimals, fractions, ratios, proportions, percentages and rates
- place value, rounding, leading digit approximation and order of magnitude as powers of 10
- numerals and symbols, number facts and operations and strategies for calculation
- concepts of constant, variable and formula.

##### Key skills

- form estimates and carry out relevant calculations using mental and by-hand methods
- use technology effectively for accurate, reliable and efficient calculation
- solve practical problems which require the use and application of a range of numerical computations involving integers, decimals, fractions, ratios, proportions, percentages, rates, powers and roots
- solve practical problems using constants, variables, formulas and algebra
- check for accuracy and reasonableness of results.

**Data**

To achieve this outcome the student will draw on knowledge and skills outlined in Area of Study 3.

**Key knowledge**

- categorical and numerical data
- methods of data collection and organisation
- the key terminology, features and conventions of diagrams, charts, tables and graphs
- the purposes for using different forms of data representation and types of data scales (categorical and numerical)
- the common measures of central tendency (averages) and spread
- characteristics and properties of data sets and their distributions
- the terminology for comparison and analysis of data sets, graphs and summary statistics.

**Key skills**

- collect, organise, collate and represent categorical and numerical data
- accurately read and interpret diagrams, charts, tables and graphs
- summarise statistical data and calculate commonly used measures of central tendency and spread
- describe, compare and analyse data sets and any limitations.

**Measurement**

To achieve this outcome the student will draw on knowledge and skills outlined in Area of Study 4.

**Key knowledge**

- the metric units for length, area, volume, capacity, time, mass, temperature and common derived units
- the meaning and conventions of different metric units, relative scale and conversions, including International System of Units (SI)
- digital and analogue tools and instruments and scales
- the formulas for calculating length, area, surface area, volume and capacity
- concepts of tolerance and error.

**Key skills**

- identify and use common metric and other relevant measurements
- convert between a range of metric and other relevant units
- estimate and accurately measure different quantities using appropriate tools
- calculate and interpret length, area, surface area, volume, capacity and duration
- solve a broad range of personal, societal or workplace measurement problems with consideration of error, required accuracy and tolerances, estimation, rounding and approximation strategies.

## Outcome 2

On completion of this unit the student should be able to apply mathematical procedures to solve practical problems in both familiar and new contexts, and communicate their results.

To achieve this outcome the student will draw on knowledge and skills outlined in all the areas of study.

### Key knowledge

- the common uses and applications of mathematics in aspects of everyday life
- relevant and appropriate mathematics in areas relating to student's study, work, social or personal contexts
- the common methods of presenting and communicating mathematics in everyday life, for example charts, graphs, maps, plans, tables, algebraic expressions and diagrams.

### Key skills

- identify and recognise how mathematics is used in everyday situations and contexts, making connections between mathematics and the real world
- extract the mathematics embedded in everyday situations and contexts and formulate what mathematics can be used to solve practical problems in both familiar and new contexts
- undertake a range of mathematical tasks, applications and processes to solve practical problems, such as drawing, measuring, counting, estimating, calculating, generalising and modelling
- interpret results and outcomes of the application of mathematics in a context, including how appropriately and accurately they fit the situation
- represent, communicate and discuss the results and outcomes of the application of mathematics in a range of contexts.

## Outcome 3

On completion of this unit the student should be able to select and use technology to solve problems in practical contexts.

To achieve this outcome the student will draw on knowledge and skills outlined in all the areas of study.

### Key knowledge

- the conventions for representations of mathematical information, objects and operations, including order of operation and use of brackets
- the numerical, graphical, symbolic, geometric and statistical functionalities of a range of technologies
- the conditions and settings for effective application of a given technology and its functionality.

### Key skills

- use technology to carry out computations and analysis, and produce diagrams, tables, charts and graphs which model situations and solve practical problems
- interpret, evaluate and discuss the outputs of technology
- use technology to communicate the results of working mathematically.



## Assessment

The award of satisfactory completion for a unit is based on whether the student has demonstrated the set of outcomes specified for the unit. Teachers should use a variety of learning activities and assessment tasks that provide a range of opportunities for students to demonstrate the key knowledge and key skills in the outcomes.

The areas of study, including the key knowledge and key skills listed for the outcomes, should be used for course design and the development of learning activities and assessment tasks. Assessment must be a part of the regular teaching and learning program and should be completed mainly in class and within a limited timeframe.

All assessments at Units 1 and 2 are school-based. Procedures for assessment of levels of achievement in Units 1 and 2 are a matter for school decision.

Assessment tasks must include components to be completed with and without the use of technology as applicable to the outcomes.

Demonstration of achievement of Outcomes 1 and 2 should be based on the student's performance on a selection of the following assessment tasks:

- investigations and projects; for example, a report on an application of mathematics such as costing of a birthday party, budgeting for a holiday, a survey of types of television programs or design of a car park
- assignments, summary or review notes of mathematics that students have encountered in their work or study; for example, a written or a multimedia or an oral presentation of wages calculations, materials estimation for a task, personal budgeting
- tests of mathematical skills developed across application contexts.

Demonstration of achievement of Outcome 3 should be based on the student's performance on aspects of tasks completed in demonstrating achievement of Outcomes 1 and 2 that incorporate opportunity for the effective and appropriate use of technology.

Where teachers allow students to choose between tasks they must ensure that the tasks they set are of comparable scope and demand.

# General Mathematics Units 1 and 2

General Mathematics provides for different combinations of student interests and preparation for study of VCE Mathematics at the Unit 3 and 4 level. The areas of study for General Mathematics Unit 1 and Unit 2 are ‘Algebra and structure’, ‘Arithmetic and number’, ‘Discrete mathematics’, ‘Geometry, measurement and trigonometry’, ‘Graphs of linear and non-linear relations’ and ‘Statistics’.

For Units 1 and 2, to suit the range of students entering the study, content must be selected from the six areas of study using the following rules:

- for each unit, content covers four or more topics in their entirety, selected from at least three different areas of study
- courses intended as preparation for study at the Units 3 and 4 level should include a selection of topics from areas of study that provide a suitable background for these studies
- topics can also be selected from those available for Specialist Mathematics Units 1 and 2
- content covered from an area of study provides a clear progression in knowledge and skills from Unit 1 to Unit 2.

In undertaking these units, students are expected to be able to apply techniques, routines and processes involving rational and real arithmetic, sets, lists and tables, diagrams and geometric constructions, algebraic manipulation, equations and graphs with and without the use of technology. They should have facility with relevant mental and by-hand approaches to estimation and computation. The use of numerical, graphical, geometric, symbolic, financial and statistical functionality of technology for teaching and learning mathematics, for working mathematically, and in related assessment, is to be incorporated throughout each unit as applicable.

## Area of Study 1

### Algebra and structure

In this area of study students cover representation and manipulation of linear relations and equations, including simultaneous linear equations, and their applications in a range of contexts.

#### Linear relations and equations

This topic includes:

- substitution into, and transposition of linear relations, such as scale conversion
- solution of linear equations, including literal linear equations
- developing formulas from word descriptions and substitution of values into formulas and evaluation
- construction of tables of values from a given formula
- linear relations defined recursively and simple applications of these relations
- numerical, graphical and algebraic solutions of simultaneous linear equations in two variables
- use of linear equations, including simultaneous linear equations in two variables, and their application to solve practical problems.

## Area of Study 2

### Arithmetic and number

In this area of study students cover mental, by-hand and technology assisted computation with rational numbers, practical arithmetic and financial arithmetic, including estimation, order of magnitude and accuracy.

#### Computation and practical arithmetic

This topic includes:

- review of computation: order of operations, directed numbers, scientific notation, estimation, exact and approximate answers, rounding correct to a given number of decimal places or significant figures
- efficient mental and by-hand estimation and computation in relevant contexts
- effective use of technology for computation
- orders of magnitude, units of measure that range over multiple orders of magnitude and their use and interpretation, and the use and interpretation of log to base 10 scales, such as the Richter scale
- use of ratios and proportions, and percentages and percentage change to solve practical problems
- the unitary method and its use to make comparisons and solve practical problems involving ratio and proportion.

#### Financial arithmetic

This topic includes:

- percentage increase and decrease applied to various financial contexts such as the price to earnings ratios of shares and percentage dividends, determining the impact of inflation on costs and the spending power of money over time, calculating percentage mark-ups and discounts, and calculating GST
- applications of simple interest and compound interest
- cash flow in common savings and credit accounts including interest calculation
- compound interest investments and loans
- comparison of purchase options including cash, credit and debit cards, personal loans, and time payments (hire purchase).

## Area of Study 3

### Discrete mathematics

In this area of study students cover matrices, graphs and networks, and number patterns and recursion, and their use to model practical situations and solve a range of related problems.

#### Matrices

This topic includes:

- use of matrices to store and display information that can be presented in a rectangular array of rows and columns such as databases and links in social and road networks
- types of matrices (row, column, square, zero and identity) and the order of a matrix
- matrix addition, subtraction, multiplication by a scalar, and matrix multiplication including determining the power of a square matrix using technology as applicable
- use of matrices, including matrix products and powers of matrices, to model and solve problems, for example costing or pricing problems, and squaring a matrix to determine the number of ways pairs of people in a network can communicate with each other via a third person
- inverse matrices and their applications including solving a system of simultaneous linear equations.

## Graphs and networks

This topic includes:

- introduction to the notations, conventions and representations of types and properties of graphs, including edge, loop, vertex, the degree of a vertex, isomorphic and connected graphs and the adjacency matrix
- description of graphs in terms of faces (regions), vertices and edges and the application of Euler's formula for planar graphs
- connected graphs: walks, trails, paths, cycles and circuits with practical applications
- weighted graphs and networks, and an introduction to the shortest path problem (solution by inspection only) and its practical application
- trees and minimum spanning trees, Prim's algorithm, and their use to solve practical problems.

## Number patterns and recursion

This topic includes:

### *Number patterns and sequences*

- the concept of a sequence as a function
- use of a first-order linear recurrence relation to generate the terms of a number sequence
- tabular and graphical display of sequences.

### *The arithmetic sequence*

- generation of an arithmetic sequence using a recurrence relation, tabular and graphical display; and the rule for the  $n^{\text{th}}$  term of an arithmetic sequence and its evaluation
- use of a recurrence relation to model and analyse practical situations involving discrete linear growth or decay such as a simple interest loan or investment, the depreciating value of an asset using the unit cost method; and the rule for the value of a quantity after  $n$  periods of linear growth or decay and its use.

### *The geometric sequence*

- generation of a geometric sequence using a recurrence relation and its tabular or graphical display; and the rule for the  $n^{\text{th}}$  term and its evaluation
- use of a recurrence relation to model and analyse practical situations involving geometric growth or decay such as the growth of a compound interest loan, the reducing height of a bouncing ball, reducing balance depreciation; and the rule for the value of a quantity after  $n$  periods of geometric growth or decay and its use.

### *The Fibonacci sequence*

- generation of the Fibonacci and similar sequences using a recurrence relation, tabular and graphical display
- use of Fibonacci and similar sequences to model and analyse practical situations.

## Area of Study 4

### Geometry, measurement and trigonometry

In this area of study students cover shape, measurement and trigonometry and their application to formulating and solving two- and three-dimensional problems involving length, angle, area and surface area, volume and capacity, and similarity and the application of linear scale factors to measurement.

#### Shape and measurement

This topic includes:

- review of units of measurement of length, angle, area, volume and capacity
- Pythagoras' theorem in two dimensions, and simple examples in three dimensions, and application to practical problems

- perimeter and areas of triangles (including the use of Heron's formula), quadrilaterals, circles and composite shapes and practical applications
- volumes and surface areas of solids (spheres, cylinders, pyramids and prisms and their composites) and practical applications
- similar figures including the mathematical conditions for similarity of two-dimensional shapes, and the linear scale factor and its extension to areas and volumes
- similarity of solids and the application of linear scale factor  $k > 0$  to scale lengths, surface areas and volumes with practical applications.

### Applications of trigonometry

This topic includes:

- review of the use of trigonometric ratios for sine, cosine and tangent to find the length of an unknown side or the size of an unknown angle in a right-angled triangle
- application of the trigonometry of right-angled triangles to solve practical problems including the use of angles of elevation and depression, and the use of three figure (true) bearings in navigation
- extension of sine and cosine to angles of up to  $180^\circ$
- area of a triangle using the rule  $Area = \frac{1}{2}ab \sin(C)$
- the sine rule (including the ambiguous case) and cosine rule (as a generalisation of Pythagoras' theorem) and their application to solving practical problems requiring the solution of non-right angled triangles
- sets of sufficient information to determine a triangle.

## Area of Study 5

### Graphs of linear and non-linear relations

In this area study students cover continuous models involving linear and non-linear relations and their graphs, linear inequalities and programming, and variation.

#### Linear graphs and models

This topic includes:

- review of linear functions and graphs
- the concept of a linear model and its specification
- the construction of a linear model to represent a practical situation including domain of application
- the interpretation of the parameters of a linear model and its use to make predictions, including the issues of interpolation and extrapolation
- fitting a linear model to data by using the equation of a line fitted by eye
- use of piecewise linear (line segment) graphs to model and analyse practical situations.

#### Inequalities and linear programming

This topic includes:

- linear inequalities in one and two variables and their graphical representation
- the linear programming problem and its purpose
- the concepts of feasible region, constraint and objective function in the context of solving a linear programming problem
- use of the corner-point principle to determine the optimal solution/s of a linear programming problem
- formulation and graphical solution of linear programming problems involving two variables.

## Variation

This topic includes:

- numerical, graphical and algebraic approaches to direct, inverse and joint variation
- transformation of data to linearity to establish relationships between variables, for example  $y$  and  $x^2$ , or  $y$  and  $\frac{1}{x}$
- modelling of given non-linear data using the relationships  $y = kx^2 + c$  and  $y = \frac{k}{x} + c$  where  $k > 0$
- modelling of data using the logarithmic function  $y = a \log_{10}(x) + c$  where  $a > 0$ .

## Area of Study 6

### Statistics

In this area of study students cover representing, analysing and comparing data distributions and investigating relationships between two numerical variables, including an introduction to correlation.

#### Investigating and comparing data distributions

This topic includes:

- types of data, including categorical (nominal or ordinal) or numerical (discrete or continuous)
- display and description of categorical data distributions using frequency tables and bar charts; and the mode and its interpretation
- display and description of numerical data distributions in terms of shape, centre and spread using histograms, stem plots (including back-to-back stem plots) and dot plots and choosing between plots
- measures of centre and spread and their use in summarising numerical data distributions, including use of and calculation of the sample summary statistics, median, mean, range, interquartile range (IQR) and standard deviation; and choosing between the measures of centre and spread
- the five-number summary and the boxplot as its graphical representation and display, including the use of the lower fence ( $Q_1 - 1.5 \times \text{IQR}$ ) and upper fence ( $Q_3 + 1.5 \times \text{IQR}$ ) to identify possible outliers
- use of back-to-back stem plots or parallel boxplots, as appropriate, to compare the distributions of a single numerical variable across two or more groups in terms of centre (median) and spread (IQR and range), and the interpretation of any differences observed in the context of the data.

#### Investigating relationships between two numerical variables

This topic includes:

- response and explanatory variables
- scatterplots and their use in identifying and qualitatively describing the association between two numerical variables in terms of direction, form and strength
- the Pearson correlation coefficient  $r$ , calculation and interpretation, and correlation and causation
- use of the least squares line to model an observed linear association and the interpretation of its intercept and slope in the context of the data
- use of the model to make predictions and identify limitations of extrapolation.

## Outcomes

For each unit the student is required to demonstrate achievement of three outcomes. As a set these outcomes encompass all of the selected areas of study for each unit. For each of Unit 1 and Unit 2, the outcomes apply to the content from the areas of study selected for that unit.

### Outcome 1

On completion of this unit the student should be able to define and explain key concepts as specified in the selected content from the areas of study, and apply a range of related mathematical routines and procedures.

To achieve this outcome the student will draw on knowledge and skills outlined in the areas of study.

## Area of Study 1

### Algebra and structure

#### Linear relations and equations

##### Key knowledge

- the forms of linear relations and equations including literal linear equations
- the rules of linear functions and tables of values
- the forms of simultaneous linear equations in two variables and their solution.

##### Key skills

- solve linear equations including literal linear equations
- construct tables of values from a given formula
- solve, algebraically and/or graphically, simultaneous linear equations in two variables
- solve word problems that involve the setting up and solving of a linear equation or a pair of simultaneous linear equations.

## Area of Study 2

### Arithmetic and number

#### Computation and practical arithmetic

##### Key knowledge

- the order of operations, directed numbers, scientific notation, exact and approximate answers, significant figures and rounding
- orders of magnitude, units of measure that range over multiple orders of magnitude, and the concept of a log scale (base 10)
- concepts of ratio, proportion, percentage, percentage change and rate
- the unitary method.

**Key skills**

- distinguish between exact and approximate answers and write approximate answers correct to a given number of decimal places or significant figures
- use efficient mental and by-hand estimation and computation
- use technology effectively for computation
- use a log scale (base 10) to represent quantities that range over several orders of magnitude
- solve practical problems involving the use of ratios, proportions, percentages, percentage change, rates and the unitary method.

**Financial arithmetic****Key knowledge**

- concepts of simple and compound interest and their application
- cash flow in common savings and credit accounts including interest calculations
- compound interest investments and debts.

**Key skills**

- apply ratio and proportion, and percentage and percentage change, to solve problems in a range of financial contexts
- apply simple interest to analyse cash flow in common savings and credit accounts
- apply compound interest to solve problems involving compound interest investments and loans
- compare the costs of a range of purchase options such as cash, credit and debit cards, personal loans, and time payments (hire purchase).

## Area of Study 3

### Discrete mathematics

**Matrices****Key knowledge**

- the concept of a matrix and its use to store, display and manipulate information
- types of matrices (row, column, square, zero, identity) and the order of a matrix
- matrix arithmetic: the definition of addition, subtraction, multiplication by a scalar, multiplication, the power of a square matrix, and the conditions for their use
- determinant and inverse of a matrix.

**Key skills**

- use matrices to store and display information that can be presented in rows and columns
- identify row, column, square, zero, and identity matrices and determine their order
- add and subtract matrices, multiply a matrix by a scalar or another matrix, raise a matrix to a power and determine its inverse, using technology as applicable
- use matrix sums, difference, products, powers and inverses to model and solve practical problems.



## Graphs and networks

### Key knowledge

- the language, properties and types of graphs, including edge, face, loop, vertex, the degree of a vertex, isomorphic and connected graphs, and the adjacency matrix, Euler's formula for planar graphs, and walks, trails, paths, circuits, bridges and cycles in the context of traversing a graph
- weighted graphs and networks, and the shortest path problem
- trees, minimum spanning trees and Prim's algorithm.

### Key skills

- describe a planar graph in terms of the number of faces (regions), vertices and edges and apply Euler's formula to solve associated problems
- apply the concepts of connected graphs: trails, paths, circuits, bridges and cycles to model and solve practical problems related to traversing a graph
- find the shortest path in a weighted graph (solution by inspection only)
- apply the concepts of trees and minimum spanning trees to solve practical problems using Prim's algorithm when appropriate.

## Number patterns and recursion

### Key knowledge

- the concept of sequence as a function and its recursive specification
- the use of a first-order linear recurrence relation to generate the terms of a number sequence including the special cases of arithmetic and geometric sequences; and the rule for the  $n^{\text{th}}$  term,  $t_n$ , of an arithmetic sequence and a geometric sequence and their evaluation
- the use of a first-order linear recurrence relation to model linear growth and decay, including the rule for evaluating the term after  $n$  periods of linear growth or decay
- the use of a first-order linear recurrence relation to model geometric growth and decay, including the use of the rule for evaluating the term after  $n$  periods of geometric growth or decay
- Fibonacci and related sequences and their recursive specification.

### Key skills

- use a given recurrence relation to generate an arithmetic or a geometric sequence, deduce the rule for the  $n^{\text{th}}$  term from the recursion relation and evaluate
- use a recurrence relation to model and analyse practical situations involving discrete linear and geometric growth or decay
- formulate the recurrence relation to generate the Fibonacci sequence and use this sequence to model and analyse practical situations.

## Area of Study 4

### Geometry, measurement and trigonometry

#### Shape and measurement

##### Key knowledge

- the measures of length, area, volume and capacity and their units of measurement
- Pythagoras' theorem and its application
- the perimeter and areas of triangles, quadrilaterals, circles and composites
- the volumes and surface areas of solids (spheres, cylinders, pyramids, prisms and their composites)
- similarity and scaling, and the linear scale factor  $k$  and its extension to areas and volumes.

##### Key skills

- solve practical problems involving the use of Pythagoras' theorem in two and three dimensions
- calculate the perimeter and areas of triangles, quadrilaterals, circles and composites in practical situations
- calculate the volumes and surface areas of solids (spheres, cylinders, pyramids and prisms and their composites) in practical situations
- use a linear scale factor to scale lengths, areas and volumes of similar figures and shapes in practical situations.

#### Applications of trigonometry

##### Key knowledge

- trigonometric ratios sine, cosine and tangent
- angles of elevation and depression and three figure bearings
- the definition of sine and cosine for angles up to  $180^\circ$
- the sine rule (including the ambiguous case) and cosine rule.

##### Key skills

- use trigonometric ratios sine, cosine and tangent to find the length of an unknown side or the size of an unknown angle, in a right-angled triangle
- solve practical problems involving right-angled triangles including the use of angles of elevation and depression, and the use of three-figure (true) bearings in navigation
- calculate the areas of triangles in practical situations using the rule  $Area = \frac{1}{2}ab\sin(C)$  and Heron's formula
- solve practical problems requiring the calculation of side lengths or angles in non-right angled triangles using the sine rule or the cosine rule as appropriate
- identify sufficient sets of information to determine a triangle.

## Area of Study 5

### Graphs of linear and non-linear relations

#### Linear graphs and models

##### Key knowledge

- the properties of linear functions and their graphs
- the concept of a linear model and its properties
- the concepts of interpolation and extrapolation
- situations that can be modelled by piecewise linear (line-segment) graphs.

**Key skills**

- develop a linear model to represent and analyse a practical situation and specify its domain of application
- interpret the slope and the intercept of a straight-line graph in terms of its context and use the equation to make predictions with consideration of limitations of extrapolation
- fit a linear model to data by finding a line fitted by eye and use piecewise linear (line-segment) graphs to model and analyse practical situations.

**Inequalities and linear programming****Key knowledge**

- the concept of a linear inequality and its graphical representation
- the linear programming problem and its purpose
- the concepts of feasible region, constraint, the objective function and the corner-point principle defined in the context of solving a linear programming problem
- linear inequalities and their use in specifying the constraints and defining the feasible region of a linear programming problem.

**Key skills**

- graph linear inequalities in one and two variables and use to solve practical problems
- construct the constraints of a linear programming problem (with two decision variables) as a set of linear inequalities
- construct the feasible region of a linear programming problem by graphing its constraints
- determine the optimum value of the objective function using the corner-point principle.

**Variation****Key knowledge**

- the concepts of direct, inverse and joint variation
- the methods of transforming data
- the use of log (base 10) and other scales.

**Key skills**

- solve problems which involve the use of direct, inverse or joint variation
- model non-linear data by using suitable transformations
- apply log (base 10) and other scales to solve variation problems.

## Area of Study 6

### Statistics

**Investigating and comparing data distributions****Key knowledge**

- types of data, including categorical (nominal or ordinal) or numerical (discrete and continuous)
- the concept of a data distribution and its display using a statistical plot
- the five number summary
- mean  $\bar{x}$  and standard deviation  $s = \sqrt{\frac{\sum(x - \bar{x})^2}{n - 1}}$ .

**Key skills**

- construct and interpret graphical displays of data, and describe the distributions of the variables involved and interpret in the context of the data
- calculate the values of appropriate summary statistics to represent the centre and spread of the distribution of a numerical variable and interpret in the context of the data
- construct and use parallel boxplots or back-to-back stem plots (as appropriate) to compare the distribution of a numerical variable across two or more groups in terms of centre (median), spread (IQR and range) and outliers, interpreting any observed differences in the context of the data.

**Investigating relationships between two numerical variables****Key knowledge**

- the response and explanatory variables and their role in modelling associations between two numerical variables
- scatterplots and their use in identifying and describing the association between two numerical variables
- the correlation coefficient  $r$  as a measure of the strength of a linear association, and the concepts of correlation and causation
- the equation of a fitted line.

**Key skills**

- use a scatterplot to describe an observed association between two numerical variables in terms of direction, strength and form
- estimate the value of the correlation coefficient  $r$  from a scatterplot and calculate its value from the data using technology
- identify the explanatory variable and use the equation of the least squares line fitted to the data to model an observed linear association
- calculate the intercept and slope correct to a specified number of decimal places or significant figures, and interpret the slope and intercept of the model in the context of data
- use the model to make predictions, being aware of the limitations of extrapolation.

**Outcome 2**

On completion of each unit the student should be able to select and apply mathematical facts, concepts, models and techniques from the topics covered in the unit to investigate and analyse extended application problems in a range of contexts.

To achieve this outcome the student will draw on knowledge and skills outlined in the areas of study.

**Key knowledge**

- the facts, concepts, and techniques associated with the topics studied
- the standard mathematical models used in the topics studied
- the facts, concepts, techniques and/or models suitable to solve extended application problems or to conduct a structured investigation in context
- assumptions and conditions underlying the facts, concepts, techniques, and models used when solving a problem or conducting an investigation.

**Key skills**

- identify, recall and select the mathematical facts, concepts and techniques needed to solve an extended problem or conduct an investigation in a variety of contexts
- recall, select and use standard mathematical models to represent practical situations
- use specific models to comment on particular situations being analysed and to make predictions
- interpret and report the results of applying these models in terms of the context of the problem being solved, including discussing the assumptions applying to the application of such models.

**Outcome 3**

On completion of this unit the student should be able to select and use numerical, graphical, symbolic and statistical functionalities of technology to develop mathematical ideas, produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches.

To achieve this outcome the student will draw on knowledge and skills outlined in all the areas of study.

**Key knowledge**

- the difference between exact numerical and approximate numerical answers when using technology to perform computation, and rounding to a given number of decimal places or significant figures
- domain and range requirements for specification of graphs, and the role of parameters in specifying general forms of models, relations and equations
- the relation between numerical, graphical and symbolic forms of information about models, relations and equations and the corresponding features of those functions, relations and equations
- similarities and differences between formal mathematical expressions and their representation by technology
- the selection of an appropriate functionality of technology in a variety of mathematical contexts.

**Key skills**

- distinguish between exact and approximate presentations of mathematical results produced by technology, and interpret these results to a specified degree of accuracy
- use technology to carry out numerical, graphical and symbolic computation as applicable
- produce results using a technology which identifies examples or counter-examples for propositions
- produce tables of values, families of graphs and collections of other results using technology, which support general analysis in problem-solving, investigative and modelling contexts
- use appropriate domain and range specifications to illustrate key features of graphs of models and relations
- identify the connection between numerical, graphical and symbolic forms of information about functions, relations and equations and the corresponding features of those models, relations and equations
- specify the similarities and differences between formal mathematical expressions and their representation by technology
- select an appropriate functionality of technology in a variety of mathematical contexts and provide a rationale for these selections
- apply suitable constraints and conditions, as applicable, to carry out required computations
- relate the results from a particular technology application to the nature of a particular mathematical task (investigative, problem solving or modelling) and verify these results
- specify the process used to develop a solution to a problem using technology and communicate the key stages of mathematical reasoning (formulation, solution, interpretation) used in this process.

## Assessment

The award of satisfactory completion for a unit is based on whether the student has demonstrated the set of outcomes specified for the unit. Teachers should use a variety of learning activities and assessment tasks that provide a range of opportunities for students to demonstrate the key knowledge and key skills in the outcomes.

The areas of study, including the key knowledge and key skills listed for the outcomes, should be used for course design and the development of learning activities and assessment tasks. Assessment must be a part of the regular teaching and learning program and should be completed mainly in class and within a limited timeframe.

All assessments at Units 1 and 2 are school-based. Procedures for assessment of levels of achievement in Units 1 and 2 are a matter for school decision.

Assessment tasks must include components to be completed with and without the use of technology as applicable to the outcomes.

Demonstration of achievement of Outcome 1 should be based on the student's performance on a selection of the following assessment tasks:

- assignments
- tests
- summary or review notes.

Demonstration of achievement of Outcome 2 should be based on the student's performance on a selection of the following assessment tasks:

- modelling tasks
- problem-solving tasks
- mathematical investigations.

Demonstration of achievement of Outcome 3 should be based on the student's performance on aspects of tasks completed in demonstrating achievement of Outcomes 1 and 2 that incorporate opportunity for the effective and appropriate use of technology.

Where teachers allow students to choose between tasks they must ensure that the tasks they set are of comparable scope and demand.

# Mathematical Methods Unit 1

Mathematical Methods Units 1 and 2 provide an introductory study of simple elementary functions of a single real variable, algebra, calculus, probability and statistics and their applications in a variety of practical and theoretical contexts. They are designed as preparation for Mathematical Methods Units 3 and 4 and contain assumed knowledge and skills for these units. The focus of Unit 1 is the study of simple algebraic functions, and the areas of study are 'Functions and graphs', 'Algebra', 'Calculus' and 'Probability and statistics'. At the end of Unit 1, students are expected to have covered the content outlined in each area of study, with the exception of 'Algebra' which extends across Units 1 and 2. This content should be presented so that there is a balanced and progressive development of skills and knowledge from each of the four areas of study with connections between and across the areas of study being developed consistently throughout both Units 1 and 2.

In undertaking this unit, students are expected to be able to apply techniques, routines and processes involving rational and real arithmetic, sets, lists and tables, diagrams and geometric constructions, algebraic manipulation, equations, graphs and differentiation with and without the use of technology. They should have facility with relevant mental and by-hand approaches to estimation and computation. The use of numerical, graphical, geometric, symbolic and statistical functionality of technology for teaching and learning mathematics, for working mathematically, and in related assessment, is to be incorporated throughout the unit as applicable.

## Area of Study 1

### Functions and graphs

In this area of study students cover the graphical representation of simple algebraic functions (polynomial and power functions) of a single real variable and the key features of functions and their graphs such as axis intercepts, domain (including the concept of maximal, natural or implied domain), co-domain and range, stationary points, asymptotic behaviour and symmetry. The behaviour of functions and their graphs is explored in a variety of modelling contexts and theoretical investigations.

This area of study includes:

- review of coordinate geometry
- functions and function notation, domain, co-domain and range, representation of a function by rule, graph and table
- use of the vertical line test to determine whether a relation is a function or not, including examples of relations that are not functions and their graphs such as  $x = k$ ,  $x = ay^2$  and circles in the form  $(x - h)^2 + (y - k)^2 = r^2$
- qualitative interpretation of features of graphs of functions, including those of real data not explicitly represented by a rule, with approximate location of stationary points
- graphs of power functions  $f(x) = x^n$  for  $n \in \mathbb{N}$  and  $n \in \{-2, -1, \frac{1}{3}, \frac{1}{2}\}$ , and transformations of these graphs to the form  $y = a(x + b)^n + c$  where  $a, b, c \in \mathbb{R}$  and  $a \neq 0$
- graphs of polynomial functions to degree 4 and other polynomials of higher degree such as  $g(x) = (x + 2)^2(x - 1)^3 + 10$
- graphs of inverse functions.

## Area of Study 2

### Algebra

This area of study supports students' work in the 'Functions and graphs', 'Calculus' and 'Probability and statistics' areas of study, and content is to be distributed between Units 1 and 2. In Unit 1 the focus is on the algebra of polynomial functions of low degree and transformations of the plane.

This area of study includes:

- use of symbolic notation to develop algebraic expressions and represent functions, relations, equations and systems of simultaneous equations
- substitution into and manipulation of these expressions
- recognition of equivalent expressions and simplification of algebraic expressions involving different forms of polynomial and power functions, the use of distributive and exponent laws applied to these functions, and manipulation from one form of expression to an equivalent form, including expansion of  $(x + a)^n$  where  $n \in \mathbb{N}$
- use of parameters to represent families of functions and determination of rules of simple functions and relations from given information
- transformations of the plane and application to basic functions and relations by simple combinations of dilations (students should be familiar with both 'parallel to an axis' and 'from an axis' descriptions), reflections in an axis and translations, including the use of matrices for transformations
- the connection between the roots of a polynomial function, its factors and the horizontal axis intercepts of its graph, including the remainder, factor and rational root theorems
- solution of polynomial equations of low degree, numerically (including numerical approximation of roots of simple polynomial functions using bisection), graphically and algebraically
- solution of a set of simultaneous linear equations (geometric interpretation only required for two variables) and equations of the form  $f(x) = g(x)$  numerically, graphically and algebraically.

## Area of Study 3

### Calculus

In this area of study students cover constant and average rates of change and an introduction to instantaneous rate of change of a function in familiar contexts, including graphical and numerical approaches to estimating and approximating these rates of change.

This area of study includes:

- average and instantaneous rates of change in a variety of practical contexts and informal treatment of instantaneous rate of change as a limiting case of the average rate of change
- interpretation of graphs of empirical data with respect to rate of change such as temperature or pollution levels over time, motion graphs and the height of water in containers of different shapes that are being filled at a constant rate, with informal consideration of continuity and smoothness
- use of gradient of a tangent at a point on the graph of a function to describe and measure instantaneous rate of change of the function, including consideration of where the rate of change is positive, negative, or zero, and the relationship of the gradient function to features of the graph of the original function.



## Area of Study 4

### Probability and statistics

In this area of study students cover the concepts of event, frequency, probability and representation of finite sample spaces and events using various forms such as lists, grids, venn diagrams, karnaugh maps, tables and tree diagrams. This includes consideration of impossible, certain, complementary, mutually exclusive, conditional and independent events involving one, two or three events (as applicable), including rules for computation of probabilities for compound events.

This area of study includes:

- random experiments, sample spaces, outcomes, elementary and compound events
- simulation using simple random generators such as coins, dice, spinners and pseudo-random generators using technology, and the display and interpretation of results, including informal consideration of proportions in samples
- probability of elementary and compound events and their representation as lists, grids, venn diagrams, karnaugh maps, tables and tree diagrams
- the addition rule for probabilities,  $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$ , and the relation that for mutually exclusive events  $\Pr(A \cap B) = 0$ , hence  $\Pr(A \cup B) = \Pr(A) + \Pr(B)$
- conditional probability in terms of reduced sample space, the relations  $\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$  and  $\Pr(A \cap B) = \Pr(A|B) \times \Pr(B)$
- the law of total probability for two events  $\Pr(A) = \Pr(A|B)\Pr(B) + \Pr(A|B')\Pr(B')$
- the relations that for pairwise independent events  $A$  and  $B$ ,  $\Pr(A|B) = \Pr(A)$ ,  $\Pr(B|A) = \Pr(B)$  and  $\Pr(A \cap B) = \Pr(A) \times \Pr(B)$ .

## Outcomes

For this unit the student is required to demonstrate achievement of three outcomes. As a set these outcomes encompass all of the areas of study for the unit.

### Outcome 1

On completion of this unit the student should be able to define and explain key concepts as specified in the content from the areas of study, and apply a range of related mathematical routines and procedures.

To achieve this outcome the student will draw on knowledge and skills outlined in all the areas of study.

#### Key knowledge

- the equation of a straight line, gradient and axis intercepts, midpoint of a line segment, distance between two points, and parallel and perpendicular lines
- the equation of a circle with specified radius and centre
- the definition of a function, the concepts of domain, co-domain and range, notation for specification of the domain (including the concept of maximal, natural or implied domain), co-domain and range and rule of a function
- the key features and properties of power and polynomial functions and their graphs
- the effect of transformations of the plane, dilation, reflection in axes, translation and simple combinations of these transformations, on the graphs of linear and power functions
- the relation between the graph of a one-to-one function, its inverse function and reflection in the line  $y = x$
- the matrix representation of points and transformations

- factorisation patterns, the quadratic formula and discriminant, the remainder, factor and rational root theorems and the null factor law
- the index (exponent) laws
- average and instantaneous rates of change and their interpretation with respect to the graphs of functions
- forms of representation of sample spaces and events
- that probabilities for a given sample space are non-negative and the sum of these probabilities is one.

**Key skills**

- determine by hand the length of a line segment and the coordinates of its midpoint, the equation of a straight line given two points or one point and gradient, and the gradient and equation of lines parallel and perpendicular to a given line through some other point
- specify the rule, domain (including maximal, natural or implied domain), co-domain, and range of a relation and identify whether or not a relation is a function
- substitute integer, simple rational and irrational numbers in exact form into expressions, including rules of functions and relations, and evaluate these by hand
- re-arrange and solve simple algebraic equations and inequalities by hand
- expand and factorise linear and simple quadratic expressions with integer coefficients by hand
- express  $ax^2 + bx + c$  in completed square form where  $a, b, c \in \mathbb{Z}$  and  $a \neq 0$ , by hand
- express a cubic polynomial  $p(x)$ , with integer coefficients, in the form  $p(x) = (x - a)q(x) + r$  and determine  $\frac{p(x)}{x - a}$ , by hand
- use algebraic, graphical and numerical approaches, including the factor theorem and the bisection method, to determine and verify solutions to equations over a specified interval
- apply distributive and index (exponent) laws to manipulate and simplify expressions involving polynomial and power function, by hand in simple cases
- set up and solve systems of simultaneous linear equations involving up to four unknowns, including by hand for a system of two equations in two unknowns
- sketch by hand graphs of linear, quadratic and cubic polynomial functions, and quartic polynomial functions in factored form (approximate location of stationary points only for cubic and quartic functions), including cases where an  $x$ -axis intercept is a touch point or a stationary point of inflection
- sketch by hand graphs of power functions  $f(x) = x^n$  where  $n \in \{-2, -1, \frac{1}{3}, \frac{1}{2}, 1, 2, 3, 4\}$  and simple transformations of these, and graphs of circles
- draw graphs of polynomial functions of low degree, simple power functions and simple relations that are not functions
- describe the effect of transformations on the graphs of relations and functions and apply matrix transformations, by hand in simple cases
- use graphical, numerical and algebraic approaches to find an approximate value or the exact value (as appropriate) for the gradient of a secant or tangent to a curve at a given point
- set up probability simulations, and describe the notion of randomness, variability and its relation to events
- calculate probabilities for compound events using rules and tree diagrams, by hand in simple cases
- solve probability problems involving karnaugh maps and tree diagrams, by hand in simple cases.

## Outcome 2

On completion of this unit the student should be able to apply mathematical processes in non-routine contexts, including situations requiring problem-solving, modelling or investigative techniques or approaches, and analyse and discuss these applications of mathematics.

To achieve this outcome the student will draw on knowledge and skills outlined in one or more areas of study.

### Key knowledge

- key mathematical content from one or more areas of study related to a given context
- specific and general formulations of concepts used to derive results for analysis within a given context
- the role of examples, counter-examples and general cases in working mathematically
- inferences from analysis and their use to draw valid conclusions related to a given context.

### Key skills

- specify the relevance of key mathematical content from one or more areas of study to the investigation of various questions in a given context
- develop mathematical formulations of specific and general cases used to derive results for analysis within a given context
- use a variety of techniques to verify results
- make inferences from analysis and use these to draw valid conclusions related to a given context
- communicate conclusions using both mathematical expression and everyday language, in particular, the interpretation of mathematics with respect to the context.

## Outcome 3

On completion of this unit the student should be able to use numerical, graphical, symbolic and statistical functionalities of technology to develop mathematical ideas, produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches.

To achieve this outcome the student will draw on knowledge and skills outlined in all the areas of study.

### Key knowledge

- exact and approximate specification of mathematical information such as numerical data, graphical forms and general or specific forms of solutions of equations produced by use of technology
- domain and range requirements for specification of graphs of functions and relations when using technology
- the role of parameters in specifying general forms of functions and equations
- the relation between numerical, graphical and symbolic forms of information about functions and equations and the corresponding features of those functions and equations
- similarities and differences between formal mathematical expressions and their representation by technology
- the selection of an appropriate functionality of technology in a variety of mathematical contexts.

### Key skills

- distinguish between exact and approximate presentations of mathematical results produced by technology, and interpret these results to a specified degree of accuracy
- use technology to carry out numerical, graphical and symbolic computation as applicable
- produce results, using technology, which identify examples or counter-examples for propositions
- produce tables of values, families of graphs and collections of other results using technology, which support general analysis in problem-solving, investigative and modelling contexts

- use appropriate domain and range specifications to illustrate key features of graphs of functions and relations
- identify the relation between numerical, graphical and symbolic forms of information about functions and equations and the corresponding features of those functions and equations
- specify the similarities and differences between formal mathematical expressions and their representation by technology, in particular, equivalent forms of symbolic expressions
- select an appropriate functionality of technology in a variety of mathematical contexts and provide a rationale for these selections
- apply suitable constraints and conditions, as applicable, to carry out required computations
- relate the results from a particular technology application to the nature of a particular mathematical task (investigative, problem solving or modelling) and verify these results
- specify the process used to develop a solution to a problem using technology and communicate the key stages of mathematical reasoning (formulation, solution, interpretation) used in this process.

## Assessment

The award of satisfactory completion for a unit is based on whether the student has demonstrated the set of outcomes specified for the unit. Teachers should use a variety of learning activities and assessment tasks that provide a range of opportunities for students to demonstrate the key knowledge and key skills in the outcomes.

The areas of study, including the key knowledge and key skills listed for the outcomes, should be used for course design and the development of learning activities and assessment tasks. Assessment must be a part of the regular teaching and learning program and should be completed mainly in class and within a limited timeframe.

All assessments at Units 1 and 2 are school-based. Procedures for assessment of levels of achievement in Units 1 and 2 are a matter for school decision.

Assessment tasks must include components to be completed with and without the use of technology as applicable to the outcomes.

Demonstration of achievement of Outcome 1 should be based on the student's performance on a selection of the following assessment tasks:

- assignments
- tests
- summary or review notes.

Demonstration of achievement of Outcome 2 should be based on the student's performance on a selection of the following assessment tasks:

- modelling tasks
- problem-solving tasks
- mathematical investigations.

Demonstration of achievement of Outcome 3 should be based on the student's performance on aspects of tasks completed in demonstrating achievement of Outcomes 1 and 2 that incorporate opportunity for the effective and appropriate use of technology.

Where teachers allow students to choose between tasks they must ensure that the tasks they set are of comparable scope and demand.

# Mathematical Methods Unit 2

In Unit 2 students focus on the study of simple transcendental functions and the calculus of simple algebraic functions. The areas of study are ‘Functions and graphs’, ‘Algebra’, ‘Calculus’, and ‘Probability and statistics’. At the end of Unit 2, students are expected to have covered the material outlined in each area of study. Material from the ‘Functions and graphs’, ‘Algebra’, ‘Calculus’, and ‘Probability and statistics’ areas of study should be organised so that there is a clear progression of skills and knowledge from Unit 1 to Unit 2 in each area of study.

In undertaking this unit, students are expected to be able to apply techniques, routines and processes involving rational and real arithmetic, sets, lists and tables, diagrams and geometric constructions, algebraic manipulation, equations, graphs, differentiation and anti-differentiation with and without the use of technology. They should have facility with relevant mental and by-hand approaches to estimation and computation. The use of numerical, graphical, geometric, symbolic and statistical functionality of technology for teaching and learning mathematics, for working mathematically, and in related assessment, is to be incorporated throughout the unit as applicable.

## Area of Study 1

### Functions and graphs

In this area of study students cover graphical representation of functions of a single real variable and the key features of graphs of functions such as axis intercepts, domain (including maximal, natural or implied domain), co-domain and range, asymptotic behaviour, periodicity and symmetry.

This area of study includes:

- review of trigonometry (sine and cosine rules not required)
- the unit circle, radians, arc length and conversion between radian and degree measures of angle
- sine, cosine and tangent as functions of a real variable, and the relationships  $\sin(x) \approx x$  for small values of  $x$ ,  $\sin^2(x) + \cos^2(x) = 1$  and  $\tan(x) = \frac{\sin(x)}{\cos(x)}$
- exact values for sine, cosine and tangent of  $\frac{n\pi}{6}$  and  $\frac{n\pi}{4}$ ,  $n \in \mathbb{Z}$
- symmetry properties, complementary relations and periodicity properties for sine, cosine and tangent functions
- circular functions of the form  $y = af(bx) + c$  and their graphs, where  $f$  is the sine, cosine or tangent function, and  $a, b, c \in \mathbb{R}$  with  $a, b \neq 0$
- simple applications of sine and cosine functions of the above form, with examples from various modelling contexts, the interpretation of period, amplitude and mean value in these contexts and their relationship to the parameters  $a, b$  and  $c$
- exponential functions of the form  $f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = Aa^{kx} + C$  and their graphs, where  $a \in \mathbb{R}^+, A, k, C \in \mathbb{R}, A \neq 0$
- logarithmic functions of the form  $f: \mathbb{R}^+ \rightarrow \mathbb{R}, f(x) = \log_a(x)$ , where  $a > 1$ , and their graphs, as the inverse function of  $y = a^x$ , including the relationships  $a^{\log_a(x)} = x$  and  $\log_a(a^x) = x$
- simple applications of exponential functions of the above form, with examples from various modelling contexts, and the interpretation of initial value, rate of growth or decay, half-life and long run value in these contexts and their relationship to the parameters  $A, k$  and  $C$ .

## Area of Study 2

### Algebra

This area of study supports students' work in the 'Functions and graphs', 'Calculus' and 'Probability and statistics' areas of study. In Unit 2 the focus is on the algebra of some simple transcendental functions and transformations of the plane. This area of study provides an opportunity for the revision, further development and application of content prescribed in Unit 1, as well as the study of additional algebra material introduced in the other areas of study in Unit 2 as follows:

- use of inverse functions and transformations to solve equations of the form  $Af(bx) + c = k$ , where  $A, b, c, k \in R$  and  $A, b \neq 0$  and  $f$  is sine, cosine, tangent or  $a^x$ , using exact or approximate values on a given domain
- index (exponent) laws and logarithm laws, including their application to the solution of simple exponential equations
- numerical approximation of roots of cubic polynomial functions using Newton's method.

## Area of Study 3

### Calculus

In this area of study students cover first principles approach to differentiation, differentiation and anti-differentiation of polynomial functions and power functions by rule, and related applications including the analysis of graphs.

This area of study includes:

- graphical and numerical approaches to approximating the value of the gradient function for simple polynomial functions and power functions at points in the domain of the function
- the derivative as the gradient of the graph of a function at a point and its representation by a gradient function
- notations for the derivative of a function:  $f'(x)$ ,  $\frac{dy}{dx}$ ,  $\frac{d}{dx}(f(x))$ ,  $D_x(f)$
- first principles approach to differentiation of  $f(x) = x^n$ ,  $n \in Z$ , and simple polynomial functions
- derivatives of simple power functions and polynomial functions by rule
- applications of differentiation, including finding instantaneous rates of change, stationary values of functions, local maxima or minima, points of inflection, analysing graphs of functions, solving maximum and minimum problems and solving simple problems involving straight-line motion
- notations for an anti-derivative, primitive or indefinite integral of a function:  $F(x)$ ,  $\int f(x) dx$
- use of a boundary condition to determine a specific anti-derivative of a given function
- anti-differentiation as the inverse process of differentiation and identification of families of curves with the same gradient function, including application of anti-differentiation to solving simple problems involving straight-line motion.

## Area of Study 4

### Probability and statistics

In this area of study students cover introductory counting principles and techniques and their application to probability and the law of total probability in the case of two events.

This area of study includes:

- addition and multiplication principles for counting
- combinations: concept of a selection and computation of  ${}^nC_r$ , application of counting techniques to probability.

## Outcomes

For this unit the student is required to demonstrate achievement of three outcomes. As a set these outcomes encompass all of the areas of study for the unit.

### Outcome 1

On completion of this unit the student should be able to define and explain key concepts as specified in the content from the areas of study, and apply a range of related mathematical routines and procedures.

To achieve this outcome the student will draw on knowledge and skills outlined in all the areas of study.

#### Key knowledge

- the exact values of sine, cosine and tangent for  $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$  (and their degree equivalents) and integer multiples of these
- $\sin(x) \approx x$  for small values of  $x$
- the key features and properties of the circular functions sine, cosine and tangent, and their graphs
- the effect of transformations of the plane on the graphs of sine, cosine, tangent and exponential functions
- characteristics of data which suggest the use of sine, cosine, exponential or logarithmic functions as an appropriate type of model for a given context
- the key features of the exponential and logarithmic functions and their graphs
- the relationship between an exponential function to a given base and the logarithmic function to the same base as inverse functions
- limit definition of the derivative of a function, and the derivative as the rate of change or gradient function of a given function
- informal concepts of limit, continuity and differentiability
- the sign of the gradient at and near a point and its interpretation in terms of key features of a graph of simple polynomial functions
- the rules for finding derivatives and anti-derivatives of simple power functions and polynomial functions
- counting techniques and their application to probability.

#### Key skills

- sketch by hand graphs of the sine, cosine and exponential functions, and simple transformations of these to the form  $Af(bx) + c$ , and sketch by hand graphs of  $\log_a(x)$  and the tangent function
- draw graphs of circular, exponential and simple logarithmic functions over a given domain and identify and discuss key features and properties of these graphs
- describe the effect of transformations of the plane on the graphs of the sine, cosine, tangent and exponential functions, and apply matrices to transformations, by hand in simple cases
- solve simple equations over a specified interval related to circular, exponential and simple logarithmic functions using graphical, numerical and analytical approaches
- use Newton's method to find a numerical approximation to a root of a cubic polynomial function
- recognise characteristics of data which suggest that a circular or exponential function is an appropriate model for the data
- evaluate limiting values of a function
- use a variety of approaches (numerical, graphical, first principles and by rule) to find the value of the derivative of a function at a given point
- use first principles to find by hand the derivative of simple polynomial functions up to degree 3
- find by hand the derivative function and an anti-derivative function for a simple power function, or a polynomial function of low degree

- use derivatives to assist in the sketching of graphs of simple polynomial functions and to solve simple maximum and minimum optimisation problems
- find a family of anti-derivative functions for a given power or polynomial function, and determine a specific anti-derivative given a boundary condition
- apply counting techniques to solve probability problems and calculate probabilities for compound events, by hand in simple cases.

## Outcome 2

On completion of this unit the student should be able to apply mathematical processes in non-routine contexts, including situations requiring problem-solving, modelling or investigative techniques or approaches, and analyse and discuss these applications of mathematics.

To achieve this outcome the student will draw on knowledge and skills outlined in one or more areas of study.

### Key knowledge

- the key mathematical content from one or more areas of study relating to a given context for investigation
- specific and general formulations of concepts used to derive results for analysis within a given context for investigation
- the role of examples, counter-examples and general cases in developing mathematical analysis
- inferences from analysis and their use to draw valid conclusions related to a given context for investigation
- the representation of conditional probabilities using karnaugh maps and tree diagrams.

### Key skills

- specify the relevance of key mathematical content from one or more areas of study to the investigation of various questions in a given context
- develop mathematical formulations of specific and general cases used to derive results for analysis within a given context for investigation
- use a variety of techniques to verify results
- make inferences from analysis and use these to draw valid conclusions related to a given context for investigation
- communicate conclusions using both mathematical expression and everyday language, in particular, the interpretation of mathematics with respect to the context for investigation.

## Outcome 3

On completion of this unit the student should be able to select and use numerical, graphical, symbolic and statistical functionalities of technology to develop mathematical ideas, produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches.

To achieve this outcome the student will draw on knowledge and skills outlined in all the areas of study.

### Key knowledge

- exact and approximate specification of mathematical information such as numerical data, graphical forms and general or specific forms of solutions of equations produced by use of technology
- domain and range requirements for specification of graphs of functions and relations, when using technology
- the role of parameters in specifying general forms of functions and equations
- the relation between numerical, graphical and symbolic forms of information about functions and equations and the corresponding features of those functions and equations
- similarities and differences between formal mathematical expressions and their representation by technology
- the selection of an appropriate functionality of technology in a variety of mathematical contexts.



**Key skills**

- distinguish between exact and approximate presentations of mathematical results produced by technology, and interpret these results to a specified degree of accuracy
- use technology to carry out numerical, graphical and symbolic computation as applicable
- produce results using a technology which identify examples or counter-examples for propositions
- produce tables of values, families of graphs and collections of other results using technology, which support general analysis in problem-solving, investigative and modelling contexts
- use appropriate domain and range specifications to illustrate key features of graphs of functions and relations
- identify the relation between numerical, graphical and symbolic forms of information about functions and equations and the corresponding features of those functions and equations
- specify the similarities and differences between formal mathematical expressions and their representation by technology, in particular, equivalent forms of symbolic expressions
- select an appropriate functionality of technology in a variety of mathematical contexts, and provide a rationale for these selections
- apply suitable constraints and conditions, as applicable, to carry out required computations
- relate the results from a particular technology application to the nature of a particular mathematical task (investigative, problem solving or modelling) and verify these results
- specify the process used to develop a solution to a problem using technology, and communicate the key stages of mathematical reasoning (formulation, solution, interpretation) used in this process.

**Assessment**

The award of satisfactory completion for a unit is based on whether the student has demonstrated the set of outcomes specified for the unit. Teachers should use a variety of learning activities and assessment tasks that provide a range of opportunities for students to demonstrate the key knowledge and key skills in the outcomes.

The areas of study, including the key knowledge and key skills listed for the outcomes, should be used for course design and the development of learning activities and assessment tasks. Assessment must be a part of the regular teaching and learning program and should be completed mainly in class and within a limited timeframe.

All assessments at Units 1 and 2 are school-based. Procedures for assessment of levels of achievement in Units 1 and 2 are a matter for school decision.

Assessment tasks must include components to be completed with and without the use of technology as applicable to the outcomes.

Demonstration of achievement of Outcome 1 should be based on the student's performance on a selection of the following assessment tasks:

- assignments
- tests
- summary or review notes.

Demonstration of achievement of Outcome 2 should be based on the student's performance on a selection of the following assessment tasks:

- modelling tasks
- problem-solving tasks
- mathematical investigations.

Demonstration of achievement of Outcome 3 should be based on the student's performance on aspects of tasks completed in demonstrating achievement of Outcomes 1 and 2 that incorporate opportunity for the effective and appropriate use of technology.

Where teachers allow students to choose between tasks they must ensure that the tasks they set are of comparable scope and demand.

# Specialist Mathematics Units 1 and 2

Specialist Mathematics Units 1 and 2 provide a course of study for students who wish to undertake an in-depth study of mathematics, with an emphasis on concepts, skills and processes related to mathematical structure, modelling, problem solving and reasoning. This study has a focus on interest in the discipline of mathematics in its own right and investigation of a broad range of applications, as well as development of a sound background for further studies in mathematics and mathematics related fields.

Mathematical Methods Units 1 and 2 and Specialist Mathematics Units 1 and 2, taken in conjunction, provide a comprehensive preparation for Specialist Mathematics Units 3 and 4. The areas of study for Units 1 and 2 of Specialist Mathematics are 'Algebra and structure', 'Arithmetic and number', 'Discrete mathematics', 'Geometry, measurement and trigonometry', 'Graphs of linear and non-linear relations' and 'Statistics'.

For Units 1 and 2, to suit the range of students entering the study, and cover the four prescribed topics, content must be selected from the six areas of study using the following rules:

- for each unit, content covers four or more topics in their entirety, selected from at least three different areas of study
- each unit must include two of the prescribed topics: Number systems and recursion; Vectors in the plane; Geometry in the plane and proof; and Graphs of non-linear relations
- other topics can be selected from those included in the areas of study for Specialist Mathematics Units 1 and 2 and/or General Mathematics Units 1 and 2
- courses intended as preparation for study at the Units 3 and 4 level should include selection of content from areas of study that provide a suitable background for these studies
- content from an area of study provides a clear progression in knowledge and skills from Unit 1 to Unit 2.

In undertaking these units, students are expected to be able to apply techniques, routines and processes involving rational, real and complex arithmetic, sets, lists and tables, diagrams and geometric constructions, algebraic manipulation, equations and graphs with and without the use of technology. They should have facility with relevant mental and by-hand approaches to estimation and computation. The use of numerical, graphical, geometric, symbolic and statistical functionality of technology for teaching and learning mathematics, for working mathematically, and in related assessment, is to be incorporated throughout each unit as applicable.

## Prescribed topics

The prescribed topics are detailed below and are included in Areas of Study 2, 4 and 5. Two of these prescribed topics must be covered in their entirety in Unit 1 and the other two prescribed topics must be covered in their entirety in Unit 2.

## Area of Study 2

### Arithmetic and number

#### Number systems and recursion

This topic includes:

- definition and properties of the natural numbers  $N$ , arithmetic, order, primes, divisibility and related proofs, including the infinitude of primes
- definition and properties of the rational numbers  $Q$ , arithmetic, order and the equivalence between fraction and decimal forms

- sequences and series as maps between the natural numbers and the real numbers, the use of technology to generate sequences and series and their graphs, and sequences generated by recursion, including arithmetic and geometric sequences
- proof by mathematical induction, for example, the Tower of Hanoi, formula for the sum of the first  $n$  square numbers
- limiting behaviour as  $n \rightarrow \infty$  of the terms  $t_n$  in a geometric sequence, the sum of the first  $n$  terms  $S_n$  and their dependence on the value of the common ratio  $r$
- definition and properties of the real numbers including the absolute value of a real number and their one-to-one correspondence with points on a line to produce the real number line
- proof of irrationality for some real numbers such as surds of the form  $\sqrt{n}$  where  $n$  is not a perfect square, the golden ratio  $\phi$  and logarithms such as  $\log_2(5)$
- definition and properties of the complex numbers  $C$ , arithmetic, modulus of a complex number, the representation of complex numbers as points on an argand diagram, general solution of quadratic equations, with real coefficients, of a single variable over  $C$  and conjugate roots.

## Area of Study 4

### Geometry, measurement and trigonometry

#### Geometry in the plane and proof

This topic includes:

- geometric objects and relations: point, line, parallel, perpendicular, plane, angle, polygons, circles and semi-circles, arcs, chords, segments, sectors, secants, tangents, similarity and congruence
- straight edge and compass and dynamic geometry construction of these objects and illustration of these relations, including exact angles multiples of  $30^\circ$  and  $45^\circ$
- principles of proof including propositions and quantifiers, examples and counter-examples, direct proof, proof by contradiction, and proof using contrapositive; and the role of diagrams in geometric proof
- proofs of Pythagoras' theorem, properties of quadrilaterals, interior angles and angle sums of polygons
- congruence of triangles and the sine and cosine rules including applications
- proof of circle theorems such as:
  - the angle at the centre subtended by an arc (chord) of a circle is twice the angle at the circumference subtended by the same arc (chord), including the case of semi-circle and right angle
  - angles at the circumference of a circle subtended by the same arc (chord) are equal
  - the opposite angles of a cyclic quadrilateral are supplementary
  - chords of equal length subtend equal angles at the centre and conversely chords subtending equal angles at the centre of a circle have the same length
  - the alternate segment theorem
  - results about intersecting chords where the chords intersect inside or outside the circle (as secants) including the limiting case, where one of the lines is a tangent
  - converses of some of the above results.

#### Vectors in the plane

This topic includes:

- representation of plane vectors as directed lines segments, examples involving position, displacement and velocity
- magnitude and direction of a plane vector, and unit vectors

- geometric representation of addition, subtraction (triangle and/or parallelogram rules) scalar multiple and linear combination of plane vectors
- representation of a plane vector as an ordered pair  $(a, b)$  and as a column matrix  $\begin{pmatrix} a \\ b \end{pmatrix}$
- representation of a vector  $(a, b)$  in the form  $a\hat{i} + b\hat{j} = a\begin{pmatrix} 1 \\ 0 \end{pmatrix} + b\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$  where  $\hat{i}$  and  $\hat{j}$  are the standard orthogonal unit vectors, and direction cosines
- simple vector algebra (addition, subtraction, multiplication by a scalar, linear combination) using these forms
- a scalar product of two plane vectors, perpendicular and parallel vectors, projection of one vector onto another, and angle between two vectors
- application of vectors to geometric proofs, orienteering, navigation, and statics.

## Area of Study 5

### Graphs of linear and non-linear relations

#### Graphs of non-linear relations

This topic includes:

- interpreting graphical representations of data such as daily UV levels or water storage levels over time
- graphs of simple reciprocal functions, including those for sine, cosine and tangent
- locus definition and construction in the plane of lines, parabolas, circles, ellipses and hyperbolas
- cartesian, polar and parametric forms and graphs of lines, parabolas, circles, ellipses and hyperbolas
- polar form and graphs of other relations in the plane such as limaçons, cardioids, roses, lemniscates and spirals
- parametric form and graphs of other relations in the plane such as spirals, cycloids, lissajous figures and epicycles.

### Other topics

Other topics are to be selected from the following additional advanced mathematics topics and/or topics from General Mathematics Units 1 and 2. Two or more of these other topics must also be covered in their entirety in each of Unit 1 and Unit 2 as well as covering the two prescribed topics in their entirety.

As two of the prescribed topics are from Area of Study 4, there are no additional advanced mathematics topics for this area of study.

## Area of Study 1

### Algebra and structure

#### Logic and algebra

This topic includes:

- atomic and compound propositions, connectives, truth values, karnaugh maps and truth tables
- tautologies, validity and proof patterns and the application of these to proofs in natural language and in mathematics
- boolean algebra, the algebra of sets and propositional logic
- electronic gates and circuits and circuit simplification
- boolean operators and their use in search engines and databases.

## Transformations, trigonometry and matrices

This topic includes:

### *Linear transformations of the plane*

- points in the plane, coordinates and their representation as  $2 \times 1$  matrices (column vectors)
- linear transformations of the plane  $(x, y) \rightarrow (ax + by, cx + dy)$  as a map of the plane onto itself, dilations (students should be familiar with both ‘parallel to an axis’ and ‘from an axis’ descriptions), rotations about the origin and reflection in a line through the origin and their representation as  $2 \times 2$  matrices
- effect of these linear transformations and their inverse transformations, and compositions of these transformations on subsets of the plane such as points, lines, shapes and graphs
- invariance of properties under transformation and the relationship between the determinant of a transformation matrix and the effect of the linear transformation on area
- use of matrix multiplication to obtain mathematical results, such as  $\sin(x + y) = \sin(x) \cos(y) + \sin(y) \cos(x)$  and the equivalence between a rotation about the origin and composition of two reflections in lines through the origin.

### *Identities*

- proof and application of the Pythagorean identities; the angle sum, difference and double angle identities and the identities for products of sines and cosines expressed as sums and differences
- identities between  $a \sin(x) + b \cos(x)$  and  $r \sin(x \pm \alpha)$  or  $r \cos(x \pm \alpha)$  where  $\alpha$  is in the first quadrant, and their application to sketching graphs, solving equations and other problems
- proof and application of other trigonometric identities.

## Area of Study 2

### Arithmetic and number

#### Principles of counting

This topic includes:

- one-to-one correspondence between sets, countable and uncountable subsets of  $\mathbb{R}$
- pigeon-hole principle: solve problems and prove results using this principle
- inclusion–exclusion principle for the union of two sets and three sets: determination and use of formulas for finding the number of elements in the union of two and the union of three sets
- permutations and solution of problems involving permutations and restrictions with or without repeated objects
- combinations and the relationship between permutations and combinations and problems involving restrictions
- derivation and use of simple identities associated with Pascal's triangle.

## Area of Study 3

### Discrete mathematics

#### Graph theory

This topic includes:

- vertices and edges for undirected graphs and their representation using lists, diagrams and matrices (including multiple edges and loops) with examples from a range of contexts such as molecular structure, electrical circuits, social networks and utility connections
- use of examples to discuss types of problems in graph theory including existence problems, construction problems, counting problems and optimisation problems
- degree of a vertex and the result that the sum of all the vertex degrees is equal to twice the number of edges (handshaking lemma)
- simple graphs, sub-graphs, connectedness, complete graphs and the complement of a graph, and isomorphism of graphs
- bi-partite graphs, trees, regular graphs (including the platonic graphs), planar graphs and related proofs and applications such as:
  - Euler's formula  $v + f - e = 2$  for simple connected planar graphs
  - the complete graph  $K_n$  on  $n$  vertices has  $\frac{n(n-1)}{2}$  edges
  - a regular graph with  $n$  vertices each of degree  $r$  has  $\frac{nr}{2}$  edges
  - the planarity of various types of graphs, including all trees,  $K_n$  the complete graph with  $n$  vertices, if  $n \leq 4$ , and  $K_{m,n}$  the complete bipartite graph with  $m$  and  $n$  vertices, if  $m \leq 2$  or  $n \leq 2$
  - equivalent conditions for a simple graph with  $n$  vertices to be a tree
- walks, trails, paths, cycles and circuits, eulerian circuits and eulerian trails, hamiltonian cycles and paths.

## Area of Study 5

### Graphs of linear and non-linear relations

#### Kinematics

This topic includes:

- diagrammatic and graphical representation of empirical position-time data for a single particle in rectilinear motion, including examples with variable velocity
- graphical modelling and numerical analysis of position-time and velocity-time including consideration of average velocity and distance travelled over an interval
- modelling and analysis of rectilinear motion under constant acceleration, including use of constant acceleration formulas
- qualitative graphical analysis of the relationship between position-time, velocity-time and acceleration-time graphs for simple cases of rectilinear motion involving variable acceleration
- numerical approximation to instantaneous rate of change of a function  $f$  at time  $t = a$  by evaluation of the central difference  $\frac{f(a+h) - f(a-h)}{2h}$  for small values of  $h$ ; and its application to approximate evaluation of instantaneous velocity and instantaneous acceleration in simple cases of rectilinear motion
- approximation of velocity-time relationships by step functions; and its application to approximate evaluation of distance travelled in simple cases of rectilinear motion involving variable velocity and variable acceleration, as a sum of areas of rectangles.

## Area of Study 6

### Statistics

#### Simulation, sampling and sampling distributions

This topic includes:

##### *Simulation*

- random experiments, events and event spaces
- use of simulation to generate a random sample.

##### *Sampling distributions*

- simple random sampling from a finite population and the probability of obtaining a particular sample
- introduction to random variables for discrete distributions
- distinction between a *population parameter* and a *sample statistic* and use of the sample statistics mean  $\bar{x}$  and proportion  $\hat{p}$  as an *estimate* of the associated population parameter mean  $\mu$  and proportion  $p$
- concept of a sampling distribution and its random variable
- distribution of sample means and proportions considered empirically, including comparing the distributions of different size samples from the same population in terms of centre and spread
- display of variation in sample proportions and means through dot plots and other displays and considering the centre and spread of these distributions
- consideration of the mean and standard deviation of both the distribution of sample means and the distribution of sample proportions and consideration of the effect of taking larger samples.

## Outcomes

For each unit the student is required to demonstrate achievement of three outcomes. As a set these outcomes encompass all of the selected areas of study for each unit. For each of Unit 1 and Unit 2 the outcomes as a set apply to the content from the areas of study and topics selected for that unit.

### Outcome 1

On completion of this unit the student should be able to define and explain key concepts in relation to the topics from the selected areas of study, and apply a range of related mathematical routines and procedures.

To achieve this outcome the student will draw on knowledge and skills outlined in the areas of study.

## Area of Study 1

### Algebra and structure

#### Logic and algebra

##### **Key knowledge**

- the concepts of proposition, truth value, tautology, set and set operations
- the concepts of validity, argument and proof
- boolean operators, and axioms of boolean algebra
- gates and circuit diagrams.

**Key skills**

- represent and test the truth of propositions and validity of arguments using karnaugh maps and truth tables
- develop proofs of propositions in natural language and mathematics
- represent circuits using gates and simplify these circuits
- use boolean operators for searches in databases and by search engines.

**Transformations, trigonometry and matrices****Key knowledge**

- coordinate and matrix representation of points and transformations
- dilation, rotations and reflections and invariance properties
- the inverse transformations and composition of transformations
- trigonometric identities.

**Key skills**

- define and apply transformations to the plane and specify their effect on subsets of the plane
- identify the set of points that are invariant under a given transformation
- find and apply inverse transformations and composite transformations, and interpret their effects on subsets of the plane
- prove trigonometric identities and apply them to solve problems.

## Area of Study 2

### Arithmetic and number

**Principles of counting****Key knowledge**

- the concept of one-to-one correspondence of sets and its application to consideration of countability
- the pigeon-hole principle as a problem-solving technique
- techniques of counting such as permutations and combinations and the inclusion–exclusion principle
- identities involving Pascal’s triangle.

**Key skills**

- use one-to-one correspondence to demonstrate the countability of certain subsets of  $R$
- solve problems which involve techniques of counting
- use deductive reasoning to solve problems involving counting techniques, the pigeon-hole principle and Pascal’s triangle.

**Number systems and recursion****Key knowledge**

- the representation of natural, integer, rational and irrational real numbers in various structures and contexts including arithmetic and geometric sequences and series
- the concepts of identity, inverse, conjugate and limit
- operations on number, order properties, and algorithms for computation in a variety of contexts
- the representation of complex numbers and the conventions for arithmetic of complex numbers in cartesian form.



**Key skills**

- define and represent number in various structures and contexts such as integer, rational, real and complex number systems, ordered sets of numbers such as sequences and series
- identify and determine special forms such as identity, inverse, conjugate, and limit value
- perform exact and approximate computations and apply algorithms in various structures and contexts, including sequences and series, and interpret results
- apply deductive reasoning, including mathematical induction, and use appropriate language in the construction of mathematical arguments and proofs involving number and algebra.

## Area of Study 3

### Discrete mathematics

**Graph theory****Key knowledge**

- the notation, definition and representation of graphs
- the types of graphs and their properties
- applications of graph theory and constructions and graphs
- elementary theorems including Euler's formula.

**Key skills**

- construct graphs and use them to model situations
- use algorithms to construct subsets of graphs according to conditions and solve related problems
- develop and understand results on areas including planar graphs, trails and circuits
- solve problems and prove theorems involving graphs.

## Area of Study 4

### Geometry, measurement and trigonometry

**Geometry in the plane and proof****Key knowledge**

- standard geometric conventions and notation for points, lines and angles, and the definitions of parallel lines, transversals and related angles
- the definitions and properties of common polygons, circle and related geometric constructs
- notions of congruence and similarity and conditions for congruence and similarity, and the sine and cosine rules and conditions for their application
- geometric theorems involving lines, polygons and circles, including Pythagoras' theorem and its converse.

**Key skills**

- identify assumptions, give definitions and provide examples and counter-examples using appropriate mathematical language, diagrams and models
- complete geometric constructions using compass and straight edge and dynamic geometry technology
- model situations and solve problems involving geometry
- prove theorems involving lines, polygons and circles.

## Vectors in the plane

### Key knowledge

- the standard notation and representations for vectors in the plane, including arrows, coordinates and matrices
- the definition of arithmetical operations for vectors in the plane
- the geometric and coordinate definition of the scalar product of two vectors
- geometrical representation of vectors.

### Key skills

- use vectors to model situations involving direction and magnitude
- apply vector operations of scalar multiples, addition, subtraction and scalar product
- use vectors as an alternative geometric tool and apply to problems in describing position, displacement, velocity and force
- use vectors to solve geometric problems and prove theorems.

## Area of Study 5

### Graphs of linear and non-linear relations

#### Kinematics

##### Key knowledge

- the concepts of position, time, average and instantaneous speed, velocity and acceleration, displacement and distance travelled
- formulas for rectilinear motion involving constant acceleration
- central difference, step functions, numerical approximation and limiting value.

##### Key skills

- construct continuous position-graphs, velocity-time and acceleration-time graphs based on empirical data, and interpret these and given graphs in context
- determine position, time, speed, displacement, distance travelled, velocity and acceleration in contexts involving rectilinear motion, and solve related problems
- apply the formulas for rectilinear motion involving constant acceleration to solve problems.

#### Non-linear relations and functions

##### Key knowledge

- the definition of relation and function, independent and dependent variables, domain, co-domain and range, graphs and their features
- reciprocal functions and their properties
- distance formula and locus definitions of curves in the plane
- cartesian, polar and parametric coordinate systems and graphs, including exact values for circular functions.

##### Key skills

- construct graphs from empirical data and form continuous interpolations and extrapolations, and identify and interpret key features of these graphs and given graphs
- construct the graph of a reciprocal function from the graph of the original function
- use the distance formula and locus definitions to obtain the rule of a relation and draw the corresponding curve
- graph non-linear relations in the plane from their cartesian, polar and parametric representations, and identify and interpret their key features.

## Area of Study 6

### Statistics

#### Simulation, sampling and sampling distributions

##### Key knowledge

- methods of simulation
- sampling distributions and how to describe the distribution through central tendency and spread
- the effect of taking larger samples from a fixed population.

##### Key skills

- simulate the sampling process from a population
- display the results of taking multiple samples of the same size from a fixed population
- consider measures of central tendency and spread of the distribution of sample means and sample proportions.

### Outcome 2

On completion of each unit the student should be able to apply mathematical processes in non-routine contexts, and analyse and discuss these applications of mathematics in at least three areas of study.

To achieve this outcome the student will draw on knowledge and skills outlined in at least three areas of study.

##### Key knowledge

- the application of mathematical content from one or more areas of study in a given context for investigation
- specific and general formulations of concepts used to derive results for analysis within a given context for investigation
- the role of examples, counter-examples and general cases in developing mathematical analysis
- inferences from analysis and their use to draw valid conclusions related to a given context for investigation.

##### Key skills

- specify the relevance of key mathematical content from one or more areas of study to the investigation of various questions in a given context
- develop mathematical formulations of specific and general cases used to derive results for analysis within a given context for investigation
- make inferences from analysis and use these to draw valid conclusions related to a given context for investigation
- construct mathematical proofs
- communicate conclusions using both mathematical expression and everyday language; in particular, the interpretation of mathematics with respect to the context for investigation.

### Outcome 3

On completion of this unit the student should be able to use technology to produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches in at least three areas of study.

To achieve this outcome the student will draw on knowledge and related skills outlined in at least three areas of study.

**Key knowledge**

- exact and approximate technological specification of mathematical information such as numerical data, graphical forms and the solutions of equations
- domain and range requirements for the technological specification of graphs of functions and relations
- the relation between numerical, graphical and symbolic forms of information about functions and equations and the corresponding features of those functions and equations
- similarities and differences between formal mathematical expressions and their representation in various technology applications
- the selection of an appropriate technology application in a variety of mathematical contexts.

**Key skills**

- distinguish between exact and approximate presentations of mathematical results, and interpret these results to a specified degree of accuracy
- use technology to carry out numerical, graphical and symbolic computation as applicable
- produce results using technology which identify examples or counter-examples for propositions
- produce tables of values, families of graphs and collections of other results which support general analysis in problem-solving, investigative or modelling contexts
- apply analysis in problem-solving, investigative or modelling contexts
- use appropriate domain and range technological specifications which illustrate key features of graphs of functions and relations
- identify the relation between numerical, graphical and symbolic forms of information about functions and equations, and the corresponding features of those functions and equations
- specify the similarities and differences between formal mathematical expressions and their representation in various technology applications
- make appropriate selections for technology applications in a variety of mathematical contexts, and provide a rationale for these selections
- relate the results from a particular application to a mathematical task (investigative, problem solving or modelling).

**Assessment**

The award of satisfactory completion for a unit is based on whether the student has demonstrated the set of outcomes specified for the unit. Teachers should use a variety of learning activities and assessment tasks that provide a range of opportunities for students to demonstrate the key knowledge and key skills in the outcomes.

The areas of study, including the key knowledge and key skills listed for the outcomes, should be used for course design and the development of learning activities and assessment tasks. Assessment must be a part of the regular teaching and learning program and should be completed mainly in class and within a limited timeframe.

All assessments at Units 1 and 2 are school-based. Procedures for assessment of levels of achievement in Units 1 and 2 are a matter for school decision.

Assessment tasks must include components to be completed with and without the use of technology as applicable to the outcomes.

Demonstration of achievement of Outcome 1 should be based on the student's performance on a selection of the following assessment tasks:

- assignments
- tests
- summary or review notes.

Demonstration of achievement of Outcome 2 should be based on the student's performance on a selection of the following assessment tasks:

- modelling tasks
- problem-solving tasks
- mathematical investigations.

Demonstration of achievement of Outcome 3 should be based on the student's performance on aspects of tasks completed in demonstrating achievement of Outcomes 1 and 2 that incorporate opportunity for the effective and appropriate use of technology.

Where teachers allow students to choose between tasks they must ensure that the tasks they set are of comparable scope and demand.

# Further Mathematics Units 3 and 4

Further Mathematics consists of two areas of study, a compulsory Core area of study to be completed in Unit 3 and an Applications area of study to be completed in Unit 4. The Core comprises ‘Data analysis’ and ‘Recursion and financial modelling’. The Applications comprises two modules to be completed in their entirety, from a selection of four possible modules: ‘Matrices’, ‘Networks and decision mathematics’, ‘Geometry and measurement’ and ‘Graphs and relations’. ‘Data analysis’ comprises 40 per cent of the content to be covered, ‘Recursion and financial modelling’ comprises 20 per cent of the content to be covered, and each selected module comprises 20 per cent of the content to be covered. Assumed knowledge and skills for the Core are contained in the General Mathematics Units 1 and 2 topics: ‘Computation and practical arithmetic’, ‘Investigating and comparing data distributions’, ‘Investigating relationships between two numerical variables’, ‘Linear graphs and modelling’, ‘Linear relations and equations’, and ‘Number patterns and recursion’. For each module there are related topics in General Mathematics Units 1 and 2.

In undertaking these units, students are expected to be able to apply techniques, routines and processes involving rational and real arithmetic, sets, lists and tables, diagrams and geometric constructions, algebraic manipulation, equations, and graphs. They should have a facility with relevant mental and by-hand approaches to estimation and computation. The use of numerical, graphical, geometric, symbolic, financial and statistical functionality of technology for teaching and learning mathematics, for working mathematically, and in related assessment, is to be incorporated throughout each unit as applicable.

## Area of Study 1 – Unit 3

### Core

#### Data analysis

*Investigating data distributions, including:*

- review of types of data
- review of representation, display and description of the distributions of categorical variables: data tables, two-way frequency tables and their associated segmented bar charts
- use of the distribution/s of one or more categorical variables to answer statistical questions
- review of representation, display and description of the distributions of numerical variables: dot plots, stem plots, histograms; the use of a log (base 10) scale to display data ranging over several orders of magnitude and their interpretation in powers of ten
- summary of the distributions of numerical variables; the five-number summary and boxplots (including the use of the lower fence ( $Q_1 - 1.5 \times \text{IQR}$ ) and upper fence ( $Q_3 + 1.5 \times \text{IQR}$ ) to identify and display possible outliers); the sample mean and standard deviation and their use in comparing data distributions in terms of centre and spread
- use of the distribution/s of one or more numerical variables to answer statistical questions
- the normal model for bell-shaped distributions and the use of the 68–95–99.7% rule to estimate percentages and to give meaning to the standard deviation; standardised values (z-scores) and their use in comparing data values across distributions
- population and sample, random numbers and their use to draw simple random samples from a population or randomly allocate subjects to groups, the difference between population parameters (e.g.,  $\mu$  and  $\sigma$ ), sample statistics (e.g.,  $\bar{x}$  and  $s$ ).

*Investigating associations between two variables, including:*

- response and explanatory variables and their role in investigating associations between variables
- contingency (two-way) frequency tables, two-way frequency tables and their associated bar charts (including percentaged segmented bar charts) and their use in identifying and describing associations between two categorical variables
- back-to-back stem plots, parallel dot plots and boxplots and their use in identifying and describing associations between a numerical and a categorical variable
- scatterplots and their use in identifying and qualitatively describing the association between two numerical variables in terms of direction (positive/negative), form (linear/non-linear) and strength (strong/moderate/weak)
- answering statistical questions that require a knowledge of the associations between pairs of variables
- Pearson correlation coefficient,  $r$ , its calculation and interpretation
- cause and effect; the difference between observation and experimentation when collecting data and the need for experimentation to definitively determine cause and effect
- non-causal explanations for an observed association including common response, confounding, and coincidence; discussion and communication of these explanations in a particular situation in a systematic and concise manner.

*Investigating and modelling linear associations, including:*

- least squares line of best fit  $y = a + bx$ , where  $x$  represents the explanatory variable and  $y$  represents the response variable; the determination of the coefficients  $a$  and  $b$  using technology, and the formulas  $b = r \frac{S_y}{S_x}$  and  $a = \bar{y} - b\bar{x}$
- modelling linear association between two numerical variables, including the:
  - identification of the explanatory and response variables
  - use of the least squares method to fit a linear model to the data
- interpretation of the slope and intercepts of the least squares line in the context of the situation being modelled, including:
  - use of the rule of the fitted line to make predictions being aware of the limitations of extrapolation
  - use of the coefficient of determination,  $r^2$ , to assess the strength of the association in terms of explained variation
  - use of residual analysis to check quality of fit
- data transformation and its use in transforming some forms of non-linear data to linearity using a square, log or reciprocal transformation (on one axis only)
- interpretation and use of the equation of the least squares line fitted to the transformed data to make predictions.

*Investigating and modelling time series data, including:*

- qualitative features of time series plots; recognition of features such as trend (long-term direction), seasonality (systematic, calendar related movements) and irregular fluctuations (unsystematic, short-term fluctuations); possible outliers and their sources, including one-off real world events, and signs of structural change such as a discontinuity in the time series
- numerical smoothing of time series data using moving means with consideration of the number of terms required (using centring when appropriate) to help identify trends in time series plot with large fluctuations
- graphical smoothing of time series plots using moving medians (involving an odd number of points only) to help identify long-term trends in time series with large fluctuations
- seasonal adjustment including the use and interpretation of seasonal indices and their calculation using seasonal and yearly means
- modelling trend by fitting a least squares line to a time series with time as the explanatory variable (data de-seasonalised where necessary), and the use of the model to make forecasts (with re-seasonalisation where necessary) including consideration of the possible limitations of fitting a linear model and the limitations of extending into the future.

## Recursion and financial modelling

This topic covers the use of first-order linear recurrence relations and technology to model and analyse a range of financial situations, and solve related problems involving interest, appreciation and depreciation, loans, annuities and perpetuities.

*Depreciation of assets, including:*

- review of the use of a first-order linear recurrence relation to generate the terms of a sequence
- use of a recurrence relation to model and compare (numerically and graphically) flat rate, unit cost and reducing balance depreciation of the value of an asset with time, including the use of a recurrence relation to determine the depreciating value of an asset after  $n$  depreciation periods, including from first principles for  $n \leq 5$
- use of the rules for the future value of an asset after  $n$  depreciation periods for flat rate, unit cost and reducing balance depreciation and their application.

*Compound interest investments and loans, including:*

- review of the concepts of simple and compound interest
- use of a recurrence relation to model and analyse (numerically and graphically) a compound interest investment or loan, including the use of a recurrence relation to determine the value of the compound interest loan or investment after  $n$  compounding periods, including from first principles for  $n \leq 5$
- difference between nominal and effective interest rates and the use of effective interest rates to compare investment returns and the cost of loans when interest is paid or charged, for example, daily, monthly, quarterly
- rule for the future value of a compound interest investment or loan after  $n$  compounding periods and its use to solve practical problems.

*Reducing balance loans (compound interest loans with periodic repayments), including:*

- use of a first-order linear recurrence relation to model and analyse (numerically and graphically) the amortisation of a reducing balance loan, including the use of a recurrence relation to determine the value of the loan or investment after  $n$  payments, including from first principles for  $n \leq 5$
- use of a table to investigate and analyse the amortisation of a reducing balance loan on a step-by-step basis, the payment made, the amount of interest paid, the reduction in the principal and the balance of the loan
- use of technology with financial modelling functionality to solve problems involving reducing balance loans, such as repaying a personal loan or a mortgage, including the impact of a change in interest rate on repayment amount, time to repay the loan, total interest paid and the total cost of the loan.

*Annuities and perpetuities (compound interest investments with periodic payments made from the investment), including:*

- use of a first-order linear recurrence relation to model and analyse (numerically and graphically) the amortisation of an annuity, including the use of a recurrence relation to determine the value of the annuity after  $n$  payments, including from first principles for  $n \leq 5$
- use of a table to investigate and analyse the amortisation of an annuity on a step-by-step basis, the payment made, the interest earned, the reduction in the principal and the balance of the annuity
- use of technology to solve problems involving annuities including determining the amount to be invested in an annuity to provide a regular income paid, for example, monthly, quarterly
- simple perpetuity as a special case of an annuity that lasts indefinitely.

*Compound interest investment with periodic and equal additions to the principal (an annuity investment), including:*

- use of a first-order linear recurrence relation to model and analyse (numerically and graphically) annuity investment, including the use of a recurrence relation to determine the value of the investment after  $n$  payments have been made, including from first principles for  $n \leq 5$
- use of a table to investigate and analyse the growth of an annuity investment on a step-by-step basis after each payment is made, the payment made, the interest earned and the balance of the investment
- use of technology with financial modelling functionality to solve problems involving annuity investments, including determining the future value of an investment after a number of compounding periods, the number of compounding periods for the investment to exceed a given value and the interest rate or payment amount needed for an investment to exceed a given value in a given time.



## Outcomes – Unit 3

For this unit the student is required to demonstrate achievement of three outcomes. As a set these outcomes encompass Area of Study 1.

### Outcome 1

On completion of this unit the student should be able to define and explain key concepts and apply related mathematical techniques and models as specified in Area of Study 1 in routine contexts.

To achieve this outcome the student will draw on knowledge and skills outlined in Area of Study 1.

#### Data analysis

##### Key knowledge

- types of data: categorical (nominal and ordinal) and numerical (discrete and continuous)
- frequency tables, bar charts including segmented bar charts, histograms, stem plots, dot plots, and their application in the context of displaying and describing distributions
- log (base 10) scales, and their purpose and application
- five-number summary and boxplots (including the designation and display of possible outliers)
- mean  $\bar{x}$  and standard deviation  $s = \sqrt{\frac{\sum(x - \bar{x})^2}{n - 1}}$
- normal model and the 68–95–99.7% rule, and standardised values (z-scores)
- response and explanatory variables
- two-way frequency tables, segmented bar charts, back-to-back stem plots, parallel boxplots, and scatterplots, and their application in the context of identifying and describing associations
- correlation coefficient,  $r$ , its interpretation, the issue of correlation and cause and effect
- least squares line and its use in modelling linear associations
- data transformation and its purpose
- time series data and its analysis.

##### Key skills

- construct frequency tables and bar charts and use them to describe and interpret the distributions of categorical variables
- answer statistical questions that require a knowledge of the distribution/s of one or more categorical variables
- construct stem and dot plots, boxplots, histograms and appropriate summary statistics and use them to describe and interpret the distributions of numerical variables
- answer statistical questions that require a knowledge of the distribution/s of one or more numerical variables
- solve problems using the z-scores and the 68–95–99.7% rule
- construct two-way tables and use them to identify and describe associations between two categorical variables
- construct parallel boxplots and use them to identify and describe associations between a numerical variable and a categorical variable
- construct scatterplots and use them to identify and describe associations between two numerical variables
- calculate the correlation coefficient,  $r$ , and interpret it in the context of the data
- answer statistical questions that require a knowledge of the associations between pairs of variables
- determine the equation of the least squares line giving the coefficients correct to a required number of decimal places or significant figures as specified

- distinguish between correlation and causation
- use the least squares line of best fit to model and analyse the linear association between two numerical variables and interpret the model in the context of the association being modelled
- calculate the coefficient of determination,  $r^2$ , and interpret in the context of the association being modelled and use the model to make predictions, being aware of the problem of extrapolation
- construct a residual analysis to test the assumption of linearity and, in the case of clear non-linearity, transform the data to achieve linearity and repeat the modelling process using the transformed data
- identify key qualitative features of a time series plot including trend (using smoothing if necessary), seasonality, irregular fluctuations and outliers, and interpret these in the context of the data
- calculate, interpret and apply seasonal indices
- model linear trends using the least squares line of best fit, interpret the model in the context of the trend being modelled, use the model to make forecasts being aware of the limitations of extending forecasts too far into the future.

## Recursion and financial modelling

### Key knowledge

- the concept of a first-order linear recurrence relation and its use in generating the terms in a sequence
- uses of first-order linear recurrence relations to model growth and decay problems in financial contexts
- the use of first-order linear recurrence relations to model flat rate and unit cost, and reduce balance depreciation of an asset over time, including the rule for the future value of the asset after  $n$  depreciation periods
- the concepts of financial mathematics including simple and compound interest, nominal and effective interest rates, the present and future value of an investment, loan or asset, amortisation of a reducing balance loan or annuity and amortisation tables
- the use of first-order linear recurrence relations to model compound interest investments and loans, and the flat rate, unit cost and reducing balance methods for depreciating assets, reducing balance loans, annuities, perpetuities and annuity investments.

### Key skills

- use a given first-order linear recurrence relation to generate the terms of a sequence
- model and analyse growth and decay in financial contexts using a first-order linear recurrence relation of the form  $u_0 = a, u_{n+1} = bu_n + c$
- demonstrate the use of a recurrence relation to determine the depreciating value of an asset or the future value of an investment or a loan after  $n$  time periods, including from first principles for  $n \leq 5$
- use a rule for the future value of a compound interest investment or loan, or a depreciating asset, to solve practical problems
- use a table to investigate and analyse on a step-by-step basis the amortisation of a reducing balance loan or an annuity, and interpret amortisation tables
- with the aid of technology with financial mathematics capabilities, solve practical problems associated with compound interest investments and loans, reducing balance loans, annuities and perpetuities, and annuity investments.

## Outcome 2

On completion of this unit the student should be able to select and apply the mathematical concepts, models and techniques as specified in Area of Study 1 in a range of contexts of increasing complexity.

To achieve this outcome the student will draw on knowledge and skills outlined in Area of Study 1.

**Key knowledge**

- the facts, concepts and techniques associated with data analysis and recursion and financial modelling
- standard models studied in data analysis and recursion and financial modelling and their area of application
- general formulation of the concepts, techniques and models studied in data analysis and recursion and financial modelling
- assumptions and conditions underlying the use of the concepts, techniques, and models associated with data analysis and recursion and financial modelling.

**Key skills**

- identify, recall and select facts, concepts, models and techniques needed to investigate and analyse statistical features of a data set with several variables that can include time series data
- select and implement standard financial models to investigate and analyse a financial or mathematically equivalent non-financial situation that requires the use of increasingly sophisticated models to complete the analysis
- interpret and report the results of a statistical investigation or of completing a modelling or problem-solving task in terms of the context under consideration, including discussing the assumptions in application of these models.

**Outcome 3**

On completion of this unit the student should be able to select and appropriately use numerical, graphical, symbolic and statistical functionalities of technology to develop mathematical ideas, produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches.

To achieve this outcome the student will draw on knowledge and related skills outlined in Area of Study 1.

**Key knowledge**

- the difference between exact numerical and approximate numerical answers when using technology to perform computation, and rounding to a given number of decimal places or significant figures
- domain and range requirements for specification of graphs of models and relations, when using technology
- the role of parameters in specifying general forms of models and equations
- the relation between numerical, graphical and symbolic forms of information about models and equations and the corresponding features of those functions and equations
- similarities and differences between formal mathematical expressions and their representation by technology
- the selection of an appropriate functionality of technology in a variety of mathematical contexts.

**Key skills**

- distinguish between exact and approximate presentations of mathematical results produced by technology, and interpret these results to a specified degree of accuracy in terms of a given number of decimal places or significant figures
- use technology to carry out numerical, graphical and symbolic computation as applicable
- produce results using a technology which identify examples or counter-examples for propositions
- produce tables of values, families of graphs and collections of other results using technology, which support general analysis in problem-solving, investigative and modelling contexts
- use appropriate domain and range specifications to illustrate key features of graphs
- identify the relation between numerical, graphical and symbolic forms of information about models and equations and the corresponding features of those models and equations
- specify the similarities and differences between formal mathematical expressions and their representation by technology

- select an appropriate functionality of technology in a variety of mathematical contexts, related to data analysis, recurrence relations and financial modelling, and provide a rationale for these selections
- apply suitable constraints and conditions, as applicable, to carry out required computations
- relate the results from a particular technology application to the nature of a particular mathematical task (investigative, problem solving or modelling) and verify these results
- specify the process used to develop a solution to a problem using technology, and communicate the key stages of mathematical reasoning (formulation, solution, interpretation) used in this process.

## Area of Study 2 – Unit 4

### Applications

Students must complete two modules selected from the following four modules.

#### Matrices

This module covers definition of matrices, different types of matrices, matrix operations, transition matrices and the use of first-order linear matrix recurrence relations to model a range of situations and solve related problems.

*Matrices and their applications, including:*

- review of matrix arithmetic: the order of a matrix, types of matrices (row, column, square, diagonal, symmetric, triangular, zero, binary and identity), the transpose of a matrix, elementary matrix operations (sum, difference, multiplication of a scalar, product and power)
- inverse of a matrix, its determinant, and the condition for a matrix to have an inverse
- use of matrices to represent numerical information presented in tabular form, and the use of a rule for the  $a_{ij}$ th element of a matrix to construct the matrix
- binary and permutation matrices, and their properties and applications
- communication and dominance matrices and their use in analysing communication systems and ranking players in round-robin tournaments
- use of matrices to represent systems of linear equations and the solution of these equations as an application of the inverse matrix; the concepts of dependent systems of equations and inconsistent systems of equations in the context of solving pairs of simultaneous equations in two variables; the formulation of practical problems in terms of a system of linear equations and their solution using the matrix inverse method.

*Transition matrices, including:*

- use of the matrix recurrence relation:  $S_0$  = initial state matrix,  $S_{n+1} = TS_n$  where  $T$  is a transition matrix and  $S_n$  is a column state matrix, to generate a sequence of state matrices, including in the case of regular transition matrices an informal identification of the equilibrium state matrix (recognised by no noticeable change from one state matrix to the next)
- use of transition diagrams, their associated transition matrices and state matrices to model the transitions between states in discrete dynamical situations and their application to model and analyse practical situations such as the modelling and analysis of an insect population comprising eggs, juveniles and adults
- use of the matrix recurrence relation  $S_0$  = initial state matrix,  $S_{n+1} = TS_n + B$  to extend the modelling to populations that include culling and restocking.

## Networks and decision mathematics

This module covers definition and representation of different kinds of undirected and directed graphs, eulerian trails, eulerian circuits, bridges, hamiltonian paths and cycles, and the use of networks to model and solve problems involving travel, connection, flow, matching, allocation and scheduling.

*Graphs and networks, including:*

- a review of the concepts, conventions and terminology of graphs including planar graphs and Euler's rule, and directed (digraphs) and networks
- use of matrices to represent graphs, digraphs and networks and their application.

*Exploring and travelling problems, including:*

- review of the concepts, conventions and notations of walks, trails, paths, cycles and circuits
- eulerian trails and eulerian circuits: the conditions for a graph to have an eulerian trail or an eulerian circuit, properties and applications
- hamiltonian paths and cycles: properties and applications.

*Trees and minimum connector problems, including:*

- review of the basic concepts of trees and spanning trees
- minimum spanning trees in a weighted connected graph and their determination either by inspection or by using Prim's algorithm for larger scale problems
- use of minimal spanning trees to solve minimal connector problems.

*Flow problems, including:*

- use of networks to model flow problems: capacity, sinks and sources
- solution of small-scale network flow problems by inspection and the use of the 'maximum-flow minimum-cut' theorem to aid the solution of larger scale problems.

*Shortest path problems, including:*

- determination of the shortest path between two specified vertices in a graph, digraph or network by inspection
- Dijkstra's algorithm and its use to determine the shortest path between a given vertex and each of the other vertices in a weighted graph or network.

*Matching problems, including:*

- use of a bipartite graph and its tabular or matrix form to represent a matching problem
- determination of the optimum assignment/s of people or machines to tasks by inspection or by use of the hungarian algorithm for larger scale problems.

*The scheduling problem and critical path analysis, including:*

- construction of an activity network from a precedence table (or equivalent) including the use of dummy activities where necessary
- use of forward and backward scanning to determine the earliest starting times (EST) and latest starting times (LST) for each activity
- use of ESTs and LSTs to identify the critical path in the network and determine the float times for non-critical activities
- use of crashing to reduce the completion time of the project or task being modelled.

## Geometry and measurement

This module covers the use of measurement, geometry and trigonometry to formulate and solve problems involving angle, length, area and volume in two and three dimensions, with respect to objects, the plane and the surface of the earth.

*Measurement and trigonometry, including:*

- calculation of surface area and volume of spheres, cylinders, cones, pyramids and prisms, and their composites
- application of linear scale factor  $k > 0$  of similar figures and shapes to scale lengths, areas and volumes with practical applications
- review of the methods for solving right and non-right-angled triangles, including the ambiguous case of the sine rule, and their application to solving practical problems in two and three dimensions
- specification of location (distance and direction) in two dimensions using three-figure bearings with applications such as navigation and orienteering, including situations involving the solution of non-right-angled triangles.

*Spherical geometry, including:*

- circle mensuration; arc length using the rule  $s = r \times \frac{\pi}{180} \times \theta^\circ$  with practical applications
- arc length of a sector of a circle, and the areas of sectors and segments with practical applications
- use of trigonometry and Pythagoras' theorem in two and three dimensions to solve problems involving the solution of right-angled triangles within a sphere
- use of a sphere of radius 6400 km as a model of the earth, and meridians and parallels and their use in locating points on the surface of the earth in terms of latitude and longitude (specified in decimal degrees) using the Greenwich meridian and the equator as reference
- use of meridians to determine the shortest distance from any point on the earth to a pole or the equator
- use of a great circle to determine the shortest distance between two points on the surface of the earth that have the same longitude
- use of  $15^\circ$  of longitude as equating to a 1 hour time difference to identify time zones, and determining travel times of journeys that cross two or more time zones from departure and arrival times.

## Graphs and relations

This module covers the use of linear relations, including piecewise defined relations, and non-linear relations to model a range of practical situations and solve related problems, including optimisation problems by linear programming.

*Construction and interpretation of graphs, including:*

- straight-line graphs, line segment graphs and step graphs and their use to model and analyse practical situations
- simultaneous linear equations in two unknowns and their use to model and analyse practical situations including break-even analysis, where cost and revenue functions are linear
- non-linear graphs and their use to model and analyse practical and familiar situations including the practical significance and interpretation of intercepts, slope, maximum/minimum points and the average rate of change when interpreting the graph
- non-linear graphs, either constructed from a table of data or given, the use of interpolation and extrapolation to predict values, estimation of maximum/minimum values and location; and coordinates of points of intersection for applications such as break-even analysis with non-linear cost and revenue functions
- graphical representation of relations of the form of  $y = kx^n$  for  $x \geq 0$ , where  $n \in \{-2, -1, 1, 2, 3\}$ , and their use in modelling practical situations including the determination of the constant of proportionality  $k$  by substitution of known values or by plotting  $y$  against  $x^n$  to linearise a given set of data, and the use of linearisation to test the validity of a proposed model.

*Linear programming, including:*

- review of linear inequalities in one and two variables and their graphical representation
- graphs of systems of linear inequalities (no more than five including those involving only one variable) and the use of shading-in to identify a feasible region
- linear programming and its purpose
- formulation of a linear programming problem including the identification of the decision variables, the construction of a system of linear inequalities to represent the constraints, and the expression of the quantity to be optimised (the objective function) in terms of the decision variables
- use of the graphical method to solve simple linear programming problems with two decision variables, and the sliding-line method and the corner-point principle as alternative methods for identifying optimal solutions
- extension of the linear programming method to include problems where integer solutions are required (for feasible regions containing only a small number of possible integer solutions only).

## Outcomes – Unit 4

For this unit the student is required to demonstrate achievement of three outcomes. As a set these outcomes encompass the two selected modules from Area of Study 2, Applications.

### Outcome 1

On completion of this unit the student should be able to define and explain key concepts as specified in the content from the two selected modules, and apply related mathematical techniques and models in routine contexts.

To achieve this outcome the student will draw on knowledge and skills outlined in the two modules selected from Area of Study 2.

#### Matrices

##### Key knowledge

- the order of a matrix, types of matrices (row, column, square, diagonal, symmetric, triangular, zero, binary, permutation and identity), the transpose of a matrix, elementary matrix operations (sum, difference, multiplication of a scalar, product and power)
- the inverse of a matrix and the condition for a matrix to have an inverse, including determinant
- communication and dominance matrices and their application
- the use of matrices to represent and solve a system of linear equations
- transition diagrams and transition matrices and regular transition matrices and their identification.

##### Key skills

- use the matrix recurrence relation:  $S_0 =$  initial state matrix,  $S_{n+1} = TS_n$  to generate a sequence of state matrices, including an informal identification of the equilibrium or steady state matrix in the case of regular state matrices
- construct a transition matrix from a transition diagram or a written description and vice versa
- construct a transition matrix to model the transitions in a population with an equilibrium state
- use the matrix recurrence relation  $S_0 =$  initial state matrix,  $S_{n+1} = TS_n + B$  to extend the modelling to populations that include culling and restocking.

## Networks and decision mathematics

### Key knowledge

- the conventions, terminology, properties and types of graphs; edge, face, loop, vertex, the degree of a vertex, isomorphic and connected graphs, and the adjacency matrix, and Euler's formula for planar graphs and its application
- the exploring and travelling problem, walks, trails, paths, eulerian trails and circuits, and hamiltonian cycles
- the minimum connector problem, trees, spanning trees and minimum spanning trees
- the flow problem, and the minimum cut/maximum flow theorem
- the shortest path problem and Dijkstra's algorithm
- the matching problem and the hungarian algorithm
- the scheduling problem and critical path analysis.

### Key skills

- construct graphs, digraphs and networks and their matrix equivalents to model and analyse practical situations
- recognise that a problem is an example of the exploring and travelling problem and to solve it by utilising the concepts of walks, trails, paths, eulerian trails and circuits, and hamiltonian paths and cycles
- recognise that a problem is an example of the minimum connector problem and solve it by utilising the properties of trees, spanning trees and by determining a minimum spanning tree by inspection or using Prim's algorithm for larger scale problems
- recognise that a problem is an example of the flow problem, use networks to model flow problems and determine the minimum flow problem by inspection, or by using the minimum cut/maximum flow theorem for larger scale problems
- recognise that a problem is an example of the shortest path problem and solve it by inspection or using Dijkstra's algorithm for larger scale problems
- recognise that a problem is an example of the matching problem and solve it by inspection or using the hungarian algorithm for larger scale problems
- recognise that a problem is an example of the scheduling problem and solve it by using critical path analysis.

## Geometry and measurement

### Key knowledge

- angle properties of triangles, Pythagoras' theorem, the trigonometric ratios sine, cosine and tangent, methods for solving non right-angled triangles (including the ambiguous case of the sine rule) and their application
- the rules for calculating the surface area and volume of spheres, cylinders, cones, pyramids and prisms, and their composites
- three figure bearings and their application
- the rule  $s = r \times \frac{\pi}{180} \times \theta^\circ$  for determining the length of an arc of a circle
- a sphere of radius 6400 km as a model of the earth, great circles (meridians) and small circles (parallels), latitude and longitude (specified in decimal degrees).

### Key skills

- solve practical problems involving the calculation of the side lengths, angles and areas of triangles, including the construction of diagrams based on word descriptions
- solve practical problems involving the calculation of the surface area and volume of spheres, cylinders, prisms and their composites
- solve practical problems involving the use of a linear scale factor to scale lengths, areas and volumes of similar figures and shapes



- use a sphere of radius 6400 km as a model of the earth to solve practical problems involving meridians and parallels and latitude and longitude (specified in decimal degrees)
- use the concept of a great circle to find the shortest distance between two points on the earth's surface with the same longitude
- solve time zone problems.

## Graphs and relations

### Key knowledge

- intercepts and slope of a straight-line graph and their interpretation in practical situations
- graphical and algebraic solutions of simultaneous linear equations in two unknowns
- intercepts, slope, maximum/minimum points and average rate of change of non-linear graphs and their interpretation in practical situations
- graphical representation of relations of the form  $y = kx^n$  for  $x \geq 0$ , where  $n \in \{-2, -1, 1, 2, 3\}$
- properties of graphs used to model situations involving two independent variables
- linear inequalities in one and two variables and their interpretation in practical situations
- linear programming and its application
- the terms constraint, feasible region and objective function in the context of linear programming
- the sliding-line method and the corner-point principle for identifying the optimal solution of a linear programming problem.

### Key skills

- construct and interpret straight-line graphs, line segment graphs and step graphs used to model practical situations
- construct from a table of values and interpret non-linear graphs used to model practical situations
- solve practical problems involving finding the point of intersection of two straight-line graphs
- solve graphically practical problems involving finding the point of intersection of a linear graph with a non-linear graph
- use relations of the form  $y = kx^n$  for  $x \geq 0$ , where  $n \in \{-2, -1, 1, 2, 3\}$  to model and analyse practical situations
- interpret graphs used to model situations involving two independent variables
- graph linear inequalities in one or two variables and interpret them when used in practical situations
- formulate a linear programming problem with two decision variables and solve graphically
- extend the linear programming method of solution to include only integer solutions where required.

## Outcome 2

On completion of this unit the student should be able to select and apply the mathematical concepts, models and techniques from the two selected modules in a range of contexts of increasing complexity.

To achieve this outcome the student will draw on knowledge and skills outlined in the two modules selected from Area of Study 2.

### Key knowledge

- the facts, concepts and techniques associated with the modules studied
- the standard models studied and their area of application
- the general formulation of the concepts, techniques and models introduced in the applications modules studied
- assumptions and conditions underlying the use of the facts, concepts, techniques, and models introduced in the modules studied.

**Key skills**

- identify, recall and select the mathematical concepts, models and techniques needed to solve an extended problem or conduct an investigation in a variety of contexts
- solve and analyse an extended investigation or practical problem in an unfamiliar context
- implement standard models to analyse a practical context that requires the use of increasingly sophisticated variations of the original model to answer the questions posed
- interpret and report the results of modelling or problem-solving activities in terms of the context of the situation being analysed, including discussing assumptions made.

**Outcome 3**

On completion of this unit the student should be able to select and appropriately use numerical, graphical, symbolic and statistical functionalities of technology to develop mathematical ideas, produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches.

To achieve this outcome the student will draw on knowledge and skills in the two modules selected from Area of Study 2.

**Key knowledge**

- the difference between exact numerical and approximate numerical answers when using technology to perform computation, and rounding to a given number of decimal places or significant figures
- domain requirements for specification of graphs of models and relations, when using technology
- the role of parameters in specifying general forms of models, relations and equations
- the relation between numerical, graphical and symbolic forms of information about models, relations and equations and the corresponding features of those models, relations and equations
- the selection of an appropriate functionality of technology in a variety of mathematical contexts.

**Key skills**

- distinguish between exact and approximate presentations of mathematical results produced by technology, and interpret these results to a specified degree of accuracy
- use technology to carry out numerical, graphical and symbolic computation as applicable
- produce results using a technology which identify examples or counter-examples for propositions
- produce tables of values, families of graphs and collections of other results using technology that supports general analysis in problem-solving, investigative and modelling contexts
- use appropriate domain and range specifications to illustrate key features of graphs of models and relations
- identify the relation between numerical, graphical and symbolic forms of information about functions and equations and the corresponding features of those models and equations
- specify the similarities and differences between formal mathematical expressions and their representation by technology
- select an appropriate functionality of technology in a variety of mathematical contexts related to matrices, networks, geometry and measurement, and graphs and relations as applicable, and provide a rationale for these selections
- apply suitable constraints and conditions, as applicable, to carry out required computations related to matrices, networks, geometry and measurement, and graphs and relations as applicable
- relate the results from a particular technology application to the nature of a particular mathematical task (investigative, problem solving or modelling) and verify these results
- specify the process used to develop a solution to a problem using technology, and communicate the key stages of mathematical reasoning (formulation, solution, interpretation) used in this process.

## School-based assessment

### Satisfactory completion

The award of satisfactory completion for a unit is based on whether the student has demonstrated the set of outcomes specified for the unit. Teachers should use a variety of learning activities and assessment tasks to provide a range of opportunities for students to demonstrate the key knowledge and key skills in the outcomes.

The areas of study and key knowledge and key skills listed for the outcomes should be used for course design and the development of learning activities and assessment tasks.

### Assessment of levels of achievement

The student's level of achievement for Units 3 and 4 will be determined by School-assessed Coursework. School-assessed Coursework tasks must be a part of the regular teaching and learning program and must not unduly add to the workload associated with that program. They must be completed mainly in class and within a limited timeframe.

Where teachers provide a range of options for the same School-assessed Coursework task, they should ensure that the options are of comparable scope and demand.

The types and range of forms of School-assessed Coursework for the outcomes are prescribed within the study design. The VCAA publishes *Advice for teachers* for this study, which includes advice on the design of assessment tasks and the assessment of student work for a level of achievement.

Teachers will provide to the VCAA a numerical score representing an assessment of the student's level of achievement. The score must be based on the teacher's assessment of the performance of each student on the tasks set out in the following tables.

### Contribution to final assessment

School-assessed Coursework for Unit 3 and Unit 4 will contribute 20 and 14 per cent respectively to the study score.

### Unit 3

Outcomes	Marks allocated*	Assessment tasks				
		<p>The <b>Application task</b> is a guided investigation of a given data set with several variables. The task has three components of increasing complexity:</p> <ul style="list-style-type: none"> <li>the construction, description and interpretation of data plots, including smoothed plots where time series data is used</li> <li>the calculation and interpretation of summary statistics, including seasonal indices and their application where time series data is used</li> <li>the modelling of linear associations, or trends where time series data is used, including the use of data transformation as appropriate.</li> </ul> <p>The application task is to be of 4–6 hours duration over a period of 1–2 weeks.</p> <p><b>Modelling or problem-solving task 1</b> is to relate to Recursion and financial modelling. This task is to be of 2–3 hours duration over a period of 1 week.</p>				
<p><b>Outcome 1</b> Define and explain key concepts and apply related mathematical techniques and models as specified in Area of Study 1 in routine contexts.</p>	15	<table border="1"> <tr> <td>10</td> <td>Application task</td> </tr> <tr> <td>5</td> <td>Modelling or problem-solving task 1</td> </tr> </table>	10	Application task	5	Modelling or problem-solving task 1
10	Application task					
5	Modelling or problem-solving task 1					
<p><b>Outcome 2</b> Select and apply the mathematical concepts, models and techniques as specified in Area of Study 1 in a range of contexts of increasing complexity.</p>	30	<table border="1"> <tr> <td>20</td> <td>Application task</td> </tr> <tr> <td>10</td> <td>Modelling or problem-solving task 1</td> </tr> </table>	20	Application task	10	Modelling or problem-solving task 1
20	Application task					
10	Modelling or problem-solving task 1					
<p><b>Outcome 3</b> Select and appropriately use numerical, graphical, symbolic and statistical functionalities of technology to develop mathematical ideas, produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches.</p>	15	<table border="1"> <tr> <td>10</td> <td>Application task</td> </tr> <tr> <td>5</td> <td>Modelling or problem-solving task 1</td> </tr> </table>	10	Application task	5	Modelling or problem-solving task 1
10	Application task					
5	Modelling or problem-solving task 1					
<b>Total marks</b>	<b>60</b>					

\*School-assessed coursework for Unit 3 contributes 20 per cent.

## Unit 4

Outcomes	Marks allocated*	Assessment tasks
<b>Outcome 1</b> Define and explain key concepts as specified in the content from the two selected modules, and apply related mathematical techniques and models in routine contexts.	10	5 Modelling or problem-solving task 2
		5 Modelling or problem-solving task 3
<b>Outcome 2</b> Select and apply the mathematical concepts, models and techniques from the two selected modules in a range of contexts of increasing complexity.	20	10 Modelling or problem-solving task 2
		10 Modelling or problem-solving task 3
<b>Outcome 3</b> Select and appropriately use numerical, graphical, symbolic and statistical functionalities of technology to develop mathematical ideas, produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches.	10	5 Modelling or problem-solving task 2
		5 Modelling or problem-solving task 3
<b>Total marks</b>	<b>40</b>	<b>Modelling or problem-solving task 2</b> is related to the first selected module; <b>Modelling or problem-solving task 3</b> is related to the second selected module. The modelling or problem-solving tasks are to be of 2–3 hours duration over a period of 1 week.

\*School-assessed coursework for Unit 4 contributes 14 per cent.

## External assessment

The level of achievement for Units 3 and 4 will also be assessed by two end-of-year examinations.

### Contribution to final assessment

The examinations will each contribute 33 per cent.

## End-of-year examinations

All of the content from the areas of study and the key knowledge and key skills that underpin the outcomes in Units 3 and 4 are examinable.

### Examination 1

#### Description

This examination comprises multiple-choice questions covering both Areas of Study 1 and 2. The examination is designed to assess students' knowledge of mathematical concepts, models and techniques and their ability to reason, interpret, and apply this knowledge in a range of contexts.

**Conditions**

The examination will be of one and a half hours duration and student access to an approved technology with numerical, graphical, symbolic, financial and statistical functionality will be assumed. One bound reference, text (which may be annotated) or lecture pad, may be brought into the examination. VCAA examination rules will apply.

**Examination 2****Description**

This examination comprises written response questions covering both Areas of Study 1 and 2. The examination will be designed to assess students' ability to select and apply mathematical facts, concepts, models and techniques to solve extended application problems in a range of contexts.

**Conditions**

The examination will be of one and a half hours duration and student access to an approved technology with numerical, graphical, symbolic, financial and statistical functionality will be assumed. One bound reference, text (which may be annotated) or lecture pad, may be brought into the examination. VCAA examination rules will apply.

**Further advice**

The VCAA publishes specifications for all VCE examinations on the VCAA website. Examination specifications include details about the sections of the examination, their weighting, the question format/s and any other essential information. The specifications are published in the first year of implementation of the revised Units 3 and 4 sequence together with any sample material.

# Mathematical Methods Units 3 and 4

Mathematical Methods Units 3 and 4 are completely prescribed and extend the introductory study of simple elementary functions of a single real variable, to include combinations of these functions, algebra, calculus, probability and statistics, and their applications in a variety of practical and theoretical contexts. Units 3 and 4 consist of the areas of study 'Functions and graphs', 'Calculus', 'Algebra' and 'Probability and statistics', which must be covered in progression from Unit 3 to Unit 4, with an appropriate selection of content for each of Unit 3 and Unit 4. Assumed knowledge and skills for Mathematical Methods Units 3 and 4 are contained in Mathematical Methods Units 1 and 2, and will be drawn on, as applicable, in the development of related content from the areas of study, and key knowledge and skills for the outcomes of Mathematical Methods Units 3 and 4.

For Unit 3 a selection of content would typically include the areas of study 'Functions and graphs' and 'Algebra', and applications of derivatives and differentiation, and identifying and analysing key features of the functions and their graphs from the 'Calculus' area of study. For Unit 4, this selection would typically consist of remaining content from the areas of study: 'Functions and graphs', 'Calculus' and 'Algebra', and the study of random variables and discrete and continuous probability distributions and the distribution of sample proportions. For Unit 4, the content from the 'Calculus' area of study would be likely to include the treatment of anti-differentiation, integration, the relation between integration and the area of regions specified by lines or curves described by the rules of functions, and simple applications of this content.

The selection of content from the areas of study should be constructed so that there is a development in the complexity and sophistication of problem types and mathematical processes used (modelling, transformations, graph sketching and equation solving) in application to contexts related to these areas of study. There should be a clear progression of skills and knowledge from Unit 3 to Unit 4 in each area of study.

In undertaking these units, students are expected to be able to apply techniques, routines and processes involving rational and real arithmetic, sets, lists and tables, diagrams and geometric constructions, algebraic manipulation, equations, graphs, differentiation, anti-differentiation, integration and inference with and without the use of technology. They should have facility with relevant mental and by-hand approaches to estimation and computation. The use of numerical, graphical, geometric, symbolic and statistical functionality of technology for teaching and learning mathematics, for working mathematically, and in related assessment, is to be incorporated throughout each unit as applicable.

## Area of Study 1

### Functions and graphs

In this area of study students cover transformations of the plane and the behaviour of some elementary functions of a single real variable, including key features of their graphs such as axis intercepts, stationary points, points of inflection, domain (including maximal, implied or natural domain), co-domain and range, asymptotic behaviour and symmetry. The behaviour of these functions and their graphs is to be linked to applications in practical situations.

This area of study includes:

- graphs and identification of key features of graphs of the following functions:
  - power functions,  $y = x^n$ ,  $n \in \mathcal{Q}$
  - exponential functions,  $y = a^x$ ,  $a \in \mathcal{R}^+$ , in particular  $y = e^x$ , and logarithmic functions,  $y = \log_e(x)$  and  $y = \log_{10}(x)$
  - circular functions,  $y = \sin(x)$ ,  $y = \cos(x)$  and  $y = \tan(x)$
- graphs of polynomial functions
- transformation from  $y = f(x)$  to  $y = Af(n(x+b)) + c$ , where  $A$ ,  $n$ ,  $b$  and  $c \in \mathcal{R}$ ,  $A$ ,  $n \neq 0$ , and  $f$  is one of the functions specified above, and the inverse transformation

- the relation between the graph of an original function and the graph of a corresponding transformed function (including families of transformed functions for a single transformation parameter)
- graphs of sum, difference, product and composite functions where  $f$  and  $g$  are functions of the types specified above (not including composite functions that result in reciprocal or quotient functions), use of polynomial, power, circular, exponential and logarithmic functions, simple transformation and combinations of these functions, including simple piecewise (hybrid) functions, to model practical situations.

## Area of Study 2

### Algebra

In this area of study students cover the algebra of functions, including composition of functions, simple functional relations, inverse functions and the solution of equations. They also study the identification of appropriate solution processes for solving equations, and systems of simultaneous equations, presented in various forms. Students also cover recognition of equations and systems of equations that are solvable using inverse operations or factorisation, and the use of graphical and numerical approaches for problems involving equations where exact value solutions are not required or which are not solvable by other methods. This content is to be incorporated as applicable to the other areas of study.

This area of study includes:

- review of algebra of polynomials, equating coefficients and solution of polynomial equations with real coefficients of degree  $n$  having up to  $n$  real solutions
- use of simple functional relations such as  $f(x + k) = f(x)$ ,  $f(x^n) = nf(x)$ ,  $f(x) + f(-x) = 0$ ,  $f(xy) = f(x)f(y)$ , to characterise properties of functions including periodicity and symmetry, and to specify algebraic equivalence, including the exponent and logarithm laws
- functions and their inverses, including conditions for the existence of an inverse function, and use of inverse functions to solve equations involving exponential, logarithmic, circular and power functions
- composition of functions, where  $f$  composition  $g$  is defined by  $f(g(x))$ , given  $r_g \subseteq d_f$  (the notation  $f \circ g$  may be used, but is not required)
- solution of equations of the form  $f(x) = g(x)$  over a specified interval, where  $f$  and  $g$  are functions of the type specified in the 'Functions and graphs' area of study, by graphical, numerical and algebraic methods, as applicable
- solution of literal equations and general solution of equations involving a single parameter
- solution of simple systems of simultaneous linear equations, including consideration of cases where no solution or an infinite number of possible solutions exist (geometric interpretation only required for two equations in two variables).

## Area of Study 3

### Calculus

In this area of study students cover graphical treatment of limits, continuity and differentiability of functions of a single real variable, and differentiation, anti-differentiation and integration of these functions. This material is to be linked to applications in practical situations.

This area of study includes:

- review of average and instantaneous rates of change, tangents to the graph of a given function and the derivative function
- deducing the graph of the derivative function from the graph of a given function and deducing the graph of an anti-derivative function from the graph of a given function



- derivatives of  $x^n$ , for  $n \in \mathcal{Q}$ ,  $e^x$ ,  $\log_e(x)$ ,  $\sin(x)$ ,  $\cos(x)$  and  $\tan(x)$
- derivatives of  $f(x) \pm g(x)$ ,  $f(x) \times g(x)$ ,  $\frac{f(x)}{g(x)}$  and  $f(g(x))$  where  $f$  and  $g$  are polynomial functions, exponential, circular, logarithmic or power functions and transformations or simple combinations of these functions
- application of differentiation to graph sketching and identification of key features of graphs, identification of intervals over which a function is constant, stationary, strictly increasing or strictly decreasing, identification of the maximum rate of increase or decrease in a given application context (consideration of the second derivative is not required), identification of local maximum/minimum values over an interval and application to solving problems, and identification of interval endpoint maximum and minimum values
- anti-derivatives of polynomial functions and functions of the form  $f(ax + b)$  where  $f$  is  $x^n$ , for  $n \in \mathcal{Q}$ ,  $e^x$ ,  $\sin(x)$ ,  $\cos(x)$  and linear combinations of these
- informal consideration of the definite integral as a limiting value of a sum involving quantities such as area under a curve, including examples such as distance travelled in a straight line and cumulative effects of growth such as inflation
- anti-differentiation by recognition that  $F'(x) = f(x)$  implies  $\int f(x)dx = F(x) + c$
- informal treatment of the fundamental theorem of calculus,  $\int_a^b f(x)dx = F(b) - F(a)$
- properties of anti-derivatives and definite integrals
- application of integration to problems involving finding a function from a known rate of change given a boundary condition, calculation of the area of a region under a curve and simple cases of areas between curves, distance travelled in a straight line, average value of a function and other situations.

## Area of Study 4

### Probability and statistics

In this area of study students cover discrete and continuous random variables, their representation using tables, probability functions (specified by rule and defining parameters as appropriate); the calculation and interpretation of central measures and measures of spread; and statistical inference for sample proportions. The focus is on understanding the notion of a random variable, related parameters, properties and application and interpretation in context for a given probability distribution.

This area of study includes:

- random variables, including the concept of a random variable as a real function defined on a sample space and examples of discrete and continuous random variables
- discrete random variables:
  - specification of probability distributions for discrete random variables using graphs, tables and probability mass functions
  - calculation and interpretation and use of mean ( $\mu$ ), variance ( $\sigma^2$ ) and standard deviation of a discrete random variable and their use
  - bernoulli trials and the binomial distribution,  $\text{Bi}(n, p)$ , as an example of a probability distribution for a discrete random variable
  - effect of variation in the value/s of defining parameters on the graph of a given probability mass function for a discrete random variable
  - calculation of probabilities for specific values of a random variable and intervals defined in terms of a random variable, including conditional probability

- continuous random variables:
  - construction of probability density functions from non-negative functions of a real variable
  - specification of probability distributions for continuous random variables using probability density functions
  - calculation and interpretation of mean ( $\mu$ ), median, variance ( $\sigma^2$ ) and standard deviation of a continuous random variable and their use
  - standard normal distribution,  $N(0, 1)$ , and transformed normal distributions,  $N(\mu, \sigma^2)$ , as examples of a probability distribution for a continuous random variable
  - effect of variation in the value/s of defining parameters on the graph of a given probability density function for a continuous random variable
  - calculation of probabilities for intervals defined in terms of a random variable, including conditional probability (the cumulative distribution function may be used but is not required)
- Statistical inference, including definition and distribution of sample proportions, simulations and confidence intervals:
  - distinction between a population *parameter* and a sample *statistic* and the use of the sample statistic to estimate the population parameter
  - concept of the sample proportion  $\hat{P} = \frac{X}{n}$  as a random variable whose value varies between samples, where  $X$  is a binomial random variable which is associated with the number of items that have a particular characteristic and  $n$  is the sample size
  - approximate normality of the distribution of  $\hat{P}$  for large samples and, for such a situation, the mean  $p$ , (the population proportion) and standard deviation,  $\sqrt{\frac{p(1-p)}{n}}$
  - simulation of random sampling, for a variety of values of  $p$  and a range of sample sizes, to illustrate the distribution of  $\hat{P}$
  - determination of, from a large sample, an approximate confidence interval  $\left( \hat{p} - z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \right)$  for a population proportion where  $z$  is the appropriate quantile for the standard normal distribution, in particular the 95% confidence interval as an example of such an interval where  $z \approx 1.96$  (the term standard error may be used but is not required).

## Outcomes

For each unit the student is required to demonstrate achievement of three outcomes. As a set these outcomes encompass all of the selected areas of study for each unit. For each of Unit 3 and Unit 4 the outcomes as a set apply to the content from the areas of study covered in that unit.

### Outcome 1

On completion of each unit the student should be able to define and explain key concepts as specified in the content from the areas of study, and apply a range of related mathematical routines and procedures.

To achieve this outcome the student will draw on knowledge and skills outlined in all the areas of study.

#### Key knowledge

- the key features and properties of a function or relation and its graph and of families of functions and relations and their graphs
- the effect of transformations on the graphs of a function or relation
- the matrix representation of points and transformations of the plane

- the concepts of domain, maximal domain, range and asymptotic behaviour of functions
- functional relations that describe properties, symmetry and equivalence
- the concept of an inverse function, connection between domain and range of the original function and its inverse relation and the conditions for existence of an inverse function, including the form of the graph of the inverse function for specified functions
- the concept of combined functions, and the connection between domain and range of the functions involved and the domain and range of the combined functions
- the features which enable the recognition of general forms of possible models for data presented in graphical or tabular form
- index (exponent) laws, and logarithm laws
- analytical, graphical and numerical approaches to solving equations and the nature of corresponding solutions (real, exact or approximate) and the effect of domain restrictions
- features which link the graph of a function to the graph of the corresponding gradient function or its numerical values, the tangent to a curve at a given point and how the sign and magnitude of the derivative of a function can be used to describe key features of the function and its derivative function
- the sum, difference, chain, product and quotient rules for differentiation
- the properties of anti-derivatives and definite integrals
- the concept of approximation to the area under a curve using rectangles, the ideas underlying the fundamental theorem of calculus and the relationship between the definite integral and area
- the concepts of a random variable (discrete and continuous), bernoulli trials and probability distributions, the parameters used to define a distribution and properties of probability distributions and their graphs
- the conditions under which a bernoulli trial or a probability distribution may be selected to suitably model various situations
- the definition of sample proportion as a random variable and key features of the distribution of sample proportions
- the concept of confidence intervals for proportions, variation in confidence intervals between samples and confidence intervals for estimates.

### Key skills

- identify key features and properties of the graph of a function or relation and draw the graphs of specified functions and relations, clearly identifying their key features and properties
- describe the effect of transformations on the graphs of a function or relation
- apply matrices to transformations of functions and their graphs
- find the rule of an inverse function and give its domain and range
- find the rule of a composite function and give its domain and range
- sketch by hand graphs of polynomial functions up to degree 4; simple power functions,  $y = a^x$  (using key points  $\left(-1, \frac{1}{a}\right)$ ,  $(0, 1)$  and  $(1, a)$ );  $\log_e(x)$ ;  $\log_{10}(x)$ ; and simple transformations of these
- apply a range of analytical, graphical and numerical processes, as appropriate, to obtain general and specific solutions (exact or approximate) to equations (including literal equations) over a given domain and be able to verify solutions to a particular equation or equations over a given domain
- solve by hand equations of the form  $\sin(ax + b) = c$ ,  $\cos(ax + b) = c$  and  $\tan(ax + b) = c$  with exact value solutions over a given interval
- apply algebraic, logarithmic and circular function properties to the simplification of expressions and the solution of equations
- evaluate derivatives of basic, transformed and combined functions and apply differentiation to curve sketching and related optimisation problems

- find derivatives of polynomial functions and power functions, functions of the form  $f(ax + b)$  where  $f$  is  $x^n$ , for  $n \in \mathcal{Q}$ , sine, cosine; tangent,  $e^x$ , or  $\log_e(x)$  and simple linear combinations of these, using pattern recognition, or by hand
- apply the product, chain and quotient rules for differentiation to simple combinations of functions by hand
- find derivatives of basic and more complicated functions and apply differentiation to curve sketching and optimisation problems
- find anti-derivatives of polynomial functions and power functions, functions of the form  $f(ax + b)$  where  $f$  is  $x^n$ , for  $n \in \mathcal{Q}$ ,  $e^x$ , sine or cosine, and simple linear combinations of these, using pattern recognition, or by hand
- evaluate rectangular area approximations to the area under a curve, find and verify anti-derivatives of specified functions and evaluate definite integrals
- apply definite integrals to the evaluation of the area under a curve and between curves over a specified interval
- analyse a probability mass function or probability density function and the shape of its graph in terms of the defining parameters for the probability distribution and the mean and variance of the probability distribution
- calculate and interpret the probabilities of various events associated with a given probability distribution, by hand in cases where simple arithmetic computations can be carried out
- apply probability distributions to modelling and solving related problems
- simulate repeated random sampling and interpret the results, for a variety of population proportions and a range of sample sizes, to illustrate the distribution of sample proportions and variations in confidence intervals
- calculate sample proportions and confidence intervals for population proportions.

## Outcome 2

On completion of each unit the student should be able to apply mathematical processes in non-routine contexts, including situations requiring problem-solving, modelling or investigative techniques or approaches, and analyse and discuss these applications of mathematics.

To achieve this outcome the student will draw on knowledge and skills outlined in one or more areas of study.

### Key knowledge

- the key mathematical content from one or more areas of study related to a given context
- specific and general formulations of concepts used to derive results for analysis within a given context
- the role of examples, counter-examples and general cases in working mathematically
- inferences from analysis and their use to draw valid conclusions related to a given context.

### Key skills

- specify the relevance of key mathematical content from one or more areas of study to the investigation of various questions in a given context
- develop mathematical formulations of specific and general cases used to derive results for analysis within a given context
- use a variety of techniques to verify results
- make inferences from analysis and use these to draw valid conclusions related to a given context
- communicate conclusions using both mathematical expression and everyday language, in particular, the interpretation of mathematics with respect to the context.

## Outcome 3

On completion of each unit the student should be able to select and appropriately use numerical, graphical, symbolic and statistical functionalities of technology to develop mathematical ideas, produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches.

To achieve this outcome the student will draw on knowledge and related skills outlined in all the areas of study.

### Key knowledge

- the exact and approximate specification of mathematical information such as numerical data, graphical forms and general or specific forms of solutions of equations produced by use of technology
- domain and range requirements for specification of graphs of functions and relations, when using technology
- the role of parameters in specifying general forms of functions and equations
- the relation between numerical, graphical and symbolic forms of information about functions and equations and the corresponding features of those functions and equations
- similarities and differences between formal mathematical expressions and their representation by technology
- the selection of an appropriate functionality of technology in a variety of mathematical contexts.

### Key skills

- distinguish between exact and approximate presentations of mathematical results produced by technology, and interpret these results to a specified degree of accuracy
- use technology to carry out numerical, graphical and symbolic computation as applicable
- produce results using a technology which identify examples or counter-examples for propositions
- produce tables of values, families of graphs and collections of other results using technology, which support general analysis in problem-solving, investigative and modelling contexts
- use appropriate domain and range specifications to illustrate key features of graphs of functions and relations
- identify the relation between numerical, graphical and symbolic forms of information about functions and equations and the corresponding features of those functions and equations
- specify the similarities and differences between formal mathematical expressions and their representation by technology, in particular, equivalent forms of symbolic expressions
- select an appropriate functionality of technology in a variety of mathematical contexts, and provide a rationale for these selections
- apply suitable constraints and conditions, as applicable, to carry out required computations
- relate the results from a particular technology application to the nature of a particular mathematical task (investigative, problem solving or modelling) and verify these results
- specify the process used to develop a solution to a problem using technology, and communicate the key stages of mathematical reasoning (formulation, solution, interpretation) used in this process.

## School-based assessment

### Satisfactory completion

The award of satisfactory completion for a unit is based on whether the student has demonstrated the set of outcomes specified for the unit. Teachers should use a variety of learning activities and assessment tasks to provide a range of opportunities for students to demonstrate the key knowledge and key skills in the outcomes.

The areas of study and key knowledge and key skills listed for the outcomes should be used for course design and the development of learning activities and assessment tasks.

## Assessment of levels of achievement

The student's level of achievement for Units 3 and 4 will be determined by School-assessed Coursework. School-assessed Coursework tasks must be a part of the regular teaching and learning program and must not unduly add to the workload associated with that program. They must be completed mainly in class and within a limited timeframe.

Where teachers provide a range of options for the same School-assessed Coursework task, they should ensure that the options are of comparable scope and demand.

The types and range of forms of School-assessed Coursework for the outcomes are prescribed within the study design. The VCAA publishes *Advice for teachers* for this study, which includes advice on the design of assessment tasks and the assessment of student work for a level of achievement.

Teachers will provide to the VCAA a numerical score representing an assessment of the student's level of achievement. The score must be based on the teacher's assessment of the performance of each student on the tasks set out in the following tables.

### Contribution to final assessment

School-assessed Coursework will contribute 17 per cent to the study score for each of Units 3 and 4.

### Unit 3

Outcomes	Marks allocated*	Assessment tasks
<p><b>Outcome 1</b> Define and explain key concepts as specified in the content from the areas of study, and apply a range of related mathematical routines and procedures.</p>	15	<p><b>Application task</b></p> <p>A function and calculus-based mathematical investigation of a practical or theoretical context involving content from two or more areas of study, with the following three components of increasing complexity:</p> <ul style="list-style-type: none"> <li>introduction of the context through specific cases or examples</li> <li>consideration of general features of the context</li> <li>variation or further specification of assumption or conditions involved in the context to focus on a particular feature or aspect related to the context.</li> </ul> <p>The application task is to be of 4–6 hours duration over a period of 1–2 weeks.</p>
<p><b>Outcome 2</b> Apply mathematical processes in non-routine contexts, including situations requiring problem-solving, modelling or investigative techniques or approaches, and analyse and discuss these applications of mathematics.</p>	20	
<p><b>Outcome 3</b> Select and appropriately use numerical, graphical, symbolic and statistical functionalities of technology to develop mathematical ideas, produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches.</p>	15	
<b>Total marks</b>	<b>50</b>	

\*School-assessed Coursework for Unit 3 contributes 17 per cent.

## Unit 4

Outcomes	Marks allocated*	Assessment tasks
<b>Outcome 1</b> Define and explain key concepts as specified in the content from the areas of study, and apply a range of related mathematical routines and procedures.	15	8 Modelling or problem-solving task 1
		7 Modelling or problem-solving task 2
<b>Outcome 2</b> Apply mathematical processes in non-routine contexts, including situations requiring problem-solving, modelling or investigative techniques or approaches, and analyse and discuss these applications of mathematics.	20	10 Modelling or problem-solving task 1
		10 Modelling or problem-solving task 2
<b>Outcome 3</b> Select and appropriately use numerical, graphical, symbolic and statistical functionalities of technology to develop mathematical ideas, produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches.	15	7 Modelling or problem-solving task 1
		8 Modelling or problem-solving task 2
<b>Total marks</b>	<b>50</b>	One of the modelling or problem-solving tasks is to be related to the <b>Probability and statistics</b> area of study. The modelling or problem-solving tasks are to be of 2–3 hours duration over a period of 1 week.

\*School-assessed coursework for Unit 4 contributes 17 per cent.

## External assessment

The level of achievement for Units 3 and 4 is also assessed by two end-of-year examinations.

### Contribution to final assessment

The examinations will contribute 22 and 44 per cent respectively.

## End-of-year examinations

### Description

All of the content from the areas of study and the key knowledge and key skills that underpin the outcomes in Units 3 and 4 are examinable.

### Examination 1

#### Description

This examination comprises short-answer and some extended-answer questions covering all areas of study in relation to Outcome 1. It is designed to assess students' knowledge of mathematical concepts, their skills in carrying out mathematical algorithms without the use of technology and their ability to apply concepts and skills.

**Conditions**

The examination will be of one hour duration and no technology (calculators or software) or notes of any kind are permitted. A sheet of formulas will be provided with the examination. VCAA examination rules will apply.

**Contribution to final assessment**

The examination will contribute 22 per cent.

**Examination 2****Description**

This examination comprises multiple-choice questions and extended-answer questions covering all areas of the study in relation to all three outcomes, with an emphasis on Outcome 2. The examination is designed to assess students' ability to understand and communicate mathematical ideas, and to interpret, analyse and solve both routine and non-routine problems.

**Conditions**

The examination will be of two hours duration and student access to an approved technology with numerical, graphical, symbolic and statistical functionality will be assumed. One bound reference, text (which may be annotated) or lecture pad, may be brought into the examination. VCAA examination rules will apply.

**Contribution to final assessment**

The examination will contribute 44 per cent.

**Further advice**

The VCAA publishes specifications for all VCE examinations on the VCAA website. Examination specifications include details about the sections of the examination, their weighting, the question format/s and any other essential information. The specifications are published in the first year of implementation of the revised Units 3 and 4 sequence together with any sample material.



# Specialist Mathematics Units 3 and 4

Specialist Mathematics Units 3 and 4 consist of the areas of study: 'Functions and graphs', 'Algebra', 'Calculus', 'Vectors', 'Mechanics' and 'Probability and statistics'. The development of course content should highlight mathematical structure, reasoning and applications across a range of modelling contexts with an appropriate selection of content for each of Unit 3 and Unit 4. The selection of content for Unit 3 and Unit 4 should be constructed so that there is a balanced and progressive development of knowledge and skills with connections among the areas of study being developed as appropriate across Unit 3 and Unit 4.

Specialist Mathematics Units 3 and 4 assumes familiarity with the key knowledge and skills from Mathematical Methods Units 1 and 2, the key knowledge and skills from Specialist Mathematics Units 1 and 2 topics 'Number systems and recursion' and 'Geometry in the plane and proof', and concurrent or previous study of Mathematical Methods Units 3 and 4. Together these cover the assumed knowledge and skills for Specialist Mathematics, which are drawn on as applicable in the development of content from the areas of study and key knowledge and skills for the outcomes.

In Unit 3 a study of Specialist Mathematics would typically include content from 'Functions and graphs' and a selection of material from the 'Algebra', 'Calculus' and 'Vectors' areas of study. In Unit 4 this selection would typically consist of the remaining content from the 'Algebra', 'Calculus', and 'Vectors' areas of study and the content from the 'Mechanics' and 'Probability and statistics' areas of study.

In undertaking these units, students are expected to be able to apply techniques, routines and processes involving rational, real and complex arithmetic, sets, lists and tables, diagrams and geometric constructions, algebraic manipulation, equations, graphs, differentiation, anti-differentiation and integration and inference with and without the use of technology. They should have facility with relevant mental and by-hand approaches to estimation and computation. The use of numerical, graphical, geometric, symbolic and statistical functionality of technology for teaching and learning mathematics, for working mathematically, and in related assessment, is to be incorporated throughout each unit as applicable.

## Area of Study 1

### Functions and graphs

In this area of study students cover inverse circular functions, reciprocal functions, rational functions and other simple quotient functions, the absolute value function, graphical representation of these functions, and the analysis of key features of their graphs including intercepts, asymptotic behaviour and the nature and location of stationary points, points of inflection, periodicity, and symmetry.

This area of study includes:

- graphs of rational functions of low degree, their asymptotic behaviour and nature and location of stationary points
- absolute value function, its graph and simple transformations of the graph
- graphs of the reciprocal circular functions cosecant, secant and cotangent, and simple transformations of these
- compound and double angle formulas for sine, cosine and tangent and the identities:  $\sec^2(x) = 1 + \tan^2(x)$  and  $\operatorname{cosec}^2(x) = 1 + \cot^2(x)$
- graphs of the restricted circular functions of sine, cosine and tangent over principal domains and their respective inverse functions  $\sin^{-1}$ ,  $\cos^{-1}$  and  $\tan^{-1}$  (students should be familiar with alternative notations) and simple transformations of these graphs
- graphs of simple quotient functions.

## Area of Study 2

### Algebra

In this area of study students cover the expression of simple rational functions as a sum of partial fractions; the arithmetic and algebra of complex numbers, including polar form; points and curves in the complex plane; introduction to factorisation of polynomial functions over the complex field; and an informal treatment of the fundamental theorem of algebra.

This area of study includes:

*Rational functions of a real variable, including:*

- definition of a rational function and expression of rational functions of low degree as sums of partial fractions.

*Complex numbers, including:*

- $C$ , the set of numbers  $z$  of the form  $z = x + yi$  where  $x, y$  are real numbers and  $i^2 = -1$ , real and imaginary parts, complex conjugates, modulus
- use of an argand diagram to represent points, lines, rays and circles in the complex plane
- equality, addition, subtraction, multiplication and division of complex numbers
- polar form (modulus and argument); multiplication and division in polar form, including their geometric representation and interpretation, proof of basic identities involving modulus and argument
- De Moivre's theorem, proof for integral powers, powers and roots of complex numbers in polar form, and their geometric representation and interpretation
- $n^{\text{th}}$  roots of unity and other complex numbers and their location in the complex plane
- factors over  $C$  of polynomials with integer coefficients; and informal introduction to the fundamental theorem of algebra
- factorisation of polynomial functions of a single variable over  $C$ , for example,  $z^8 + 1$ ,  $z^2 - i$ ,  $z^3 - (2 - i)z^2 + z - 2 + i$
- solution over  $C$  of corresponding polynomial equations by completing the square, factorisation and the conjugate root theorem.

## Area of Study 3

### Calculus

In this area of study students cover advanced calculus techniques for analytic and numeric differentiation and integration of a range of functions, and combinations of functions; and their application in a variety of theoretical and practical situations, including curve sketching, evaluation of arc length, area and volume, differential equations and kinematics.

This area of study includes:

*Differential and integral calculus, including:*

- derivatives of inverse circular functions
- second derivatives, use of notations  $f''(x)$  and  $\frac{d^2y}{dx^2}$  and their application to the analysis of graphs of functions, including points of inflection and concavity
- applications of chain rule to related rates of change and implicit differentiation; for example, implicit differentiation of the relations  $x^2 + y^2 = 9$  and  $3xy^2 = x + y$

- techniques of anti-differentiation and for the evaluation of definite integrals:
  - anti-differentiation of  $\frac{1}{x}$  to obtain  $\log_e |x|$
  - anti-differentiation of  $\frac{1}{\sqrt{a^2 - x^2}}$  and  $\frac{a}{a^2 + x^2}$  by recognition that they are derivatives of corresponding inverse circular functions
  - use of the substitution  $u = g(x)$  to anti-differentiate expressions
  - use of the trigonometric identities  $\sin^2(ax) = \frac{1}{2}(1 - \cos(2ax))$ ,  $\cos^2(ax) = \frac{1}{2}(1 + \cos(2ax))$ , in anti-differentiation techniques
  - anti-differentiation using partial fractions of rational functions
- relationship between the graph of a function and the graphs of its anti-derivative functions
- numeric and symbolic integration using technology
- application of integration, arc lengths of curves, areas of regions bounded by curves and volumes of solids of revolution of a region about either coordinate axis.

*Differential equations, including:*

- formulation of differential equations from contexts in, for example, physics, chemistry, biology and economics, in situations where rates are involved (including some differential equations whose analytic solutions are not required, but can be solved numerically using technology)
- verification of solutions of differential equations and their representation using direction (slope) fields
- solution of simple differential equations of the form  $\frac{dy}{dx} = f(x)$ ,  $\frac{dy}{dx} = g(y)$ , and in general differential equations of the form  $\frac{dy}{dx} = f(x)g(y)$  using separation of variables and differential equations of the form  $\frac{d^2y}{dx^2} = f(x)$
- numerical solution by Euler's method (first order approximation).

*Kinematics: rectilinear motion, including:*

- application of differentiation, anti-differentiation and solution of differential equations to rectilinear motion of a single particle, including the different derivative forms for acceleration  $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx} \left( \frac{1}{2} v^2 \right)$
- use of velocity–time graphs to describe and analyse rectilinear motion.

## Area of Study 4

### Vectors

In this area of study students cover the arithmetic and algebra of vectors, linear dependence and independence of a set of vectors, proof of geometric results using vectors, vector representation of curves in the plane and vector kinematics in one and two dimensions.

This area of study includes:

*Vectors, including:*

- addition and subtraction of vectors and their multiplication by a scalar, and position vectors
- linear dependence and independence of a set of vectors and geometric interpretation
- magnitude of a vector, unit vector, and the orthogonal unit vectors  $\underline{i}$ ,  $\underline{j}$  and  $\underline{k}$
- resolution of a vector into rectangular components
- scalar (dot) product of two vectors, deduction of dot product for  $\underline{i}$ ,  $\underline{j}$ ,  $\underline{k}$  system; its use to find scalar and vector resolutes
- parallel and perpendicular vectors
- vector proofs of simple geometric results, for example the diagonals of a rhombus are perpendicular, the medians of a triangle are concurrent, the angle subtended by a diameter in a circle is a right angle.

*Vector calculus, including:*

- position vector as a function of time  $\mathbf{r}(t)$ ; and sketching the corresponding path given  $\mathbf{r}(t)$ , including circles, ellipses and hyperbolas in cartesian or parametric forms
- differentiation and anti-differentiation of a vector function with respect to time and applying vector calculus to motion in a plane including projectile and circular motion.

## Area of Study 5

### Mechanics

In this area of study students cover an introduction to Newtonian mechanics, for both constant and variable acceleration.

This area of study includes:

- inertial mass, momentum, including change of momentum (conservation of momentum and impulse are not required), force, resultant force, weight, action and reaction
- equations of motion using absolute units (Equations of motion should be described from a diagram, showing all the forces acting on the body, and then writing down the equation of motion. Extensions could include cases involving a system of two or more connected particles. Examples are to be restricted to rectilinear motion, including motion on an inclined plane.)
- motion of a body, regarded as a particle under the action of concurrent coplanar forces (the case of equilibrium should be regarded as an application, where net force is zero).

## Area of Study 6

### Probability and statistics

In this area of study students cover statistical inference related to the definition and distribution of sample means, simulations and confidence interval.

*Linear combinations of random variables, including:*

- for random variables  $X$  and  $Y$ ,  $E(aX + b) = aE(X) + b$  and  $E(aX + bY) = aE(X) + bE(Y)$
- for random variables  $X$  and  $Y$ ,  $\text{Var}(aX + b) = a^2\text{Var}(X)$  and for independent random variables  $X$  and  $Y$ ,  $\text{Var}(aX + bY) = a^2\text{Var}(X) + b^2\text{Var}(Y)$
- for independent random variables  $X$  and  $Y$  with normal distributions then  $aX + bY$  also has a normal distribution.

*Sample means, including:*

- concept of the sample mean  $\bar{X}$  as a random variable whose value varies between samples where  $X$  is a random variable with mean  $\mu$  and standard deviation  $\sigma$
- simulation of repeated random sampling, from a variety of distributions and a range of sample sizes, to illustrate properties of the distribution of  $\bar{X}$  across samples of a fixed size  $n$  including its mean  $\mu$  and its standard deviation  $\frac{\sigma}{\sqrt{n}}$  (where  $\mu$  and  $\sigma$  are the mean and standard deviation of  $X$ ) and its approximate normality if  $n$  is large.

*Confidence intervals for means, including:*

- determination of confidence intervals for means and the use of simulation to illustrate variations in confidence intervals between samples and to show that most but not all confidence intervals contain  $\mu$
- construction of an approximate confidence interval  $\left( \bar{x} - z \frac{s}{\sqrt{n}}, \bar{x} + z \frac{s}{\sqrt{n}} \right)$ , where  $s$  is the sample standard deviation and  $z$  is the appropriate quantile for the standard normal distribution, in particular the 95% confidence interval as an example of such an interval where  $z \approx 1.96$  (the term standard error may be used but is not required).

*Hypothesis testing for a population mean with a sample drawn from a normal distribution of known variance or for a large sample, including:*

- $p$  values for hypothesis testing related to the mean
- formulation of a null hypothesis and an alternative hypothesis
- errors in hypothesis testing.

## Outcomes

For each unit the student is required to demonstrate achievement of three outcomes. As a set these outcomes encompass all of the areas of study for each unit. For each of Unit 3 and Unit 4 the outcomes apply to the content from the areas of study selected for that unit.

### Outcome 1

On the completion of each unit the student should be able to define and explain key concepts as specified in the content from the areas of study, and apply a range of related mathematical routines and procedures.

To achieve this outcome the student will draw on knowledge and skills outlined in all the areas of study.

#### Key knowledge

- functions and relations, the form of their sketch graphs and their key features, including asymptotic behaviour
- complex numbers, cartesian and polar forms, operations and properties and representation in the complex plane
- geometric interpretation of vectors in the plane and of complex numbers in the complex plane
- specification of curves in the complex plane using complex relations
- techniques for finding derivatives of explicit and implicit functions, and the meaning of first and second derivatives of a function
- techniques for finding anti-derivatives of functions, the relationship between the graph of a function and the graph of its anti-derivative functions, and graphical interpretation of definite integrals
- analytical, graphical and numerical techniques for setting up and solving equations involving functions and relations
- simple modelling contexts for setting up differential equations and associated solution techniques, including numerical approaches and representation of direction(slope) fields
- the definition and properties of vectors, vector operations, the geometric representation of vectors and the geometric interpretation of linear dependence and independence
- standard contexts for the application of vectors to the motion of a particle and to geometric problems
- techniques for solving kinematics problems in one and two dimensions
- Newton's laws of motion and related concepts
- the distribution of sample means
- linear combinations of independent random variables
- hypothesis testing for a sample mean.

**Key skills**

- sketch graphs and describe behaviour of specified functions and relations with and without the assistance of technology, clearly identifying their key features and using the concepts of first and second derivatives
- perform operations on complex numbers expressed in cartesian form or polar form and interpret them geometrically
- represent curves on an argand diagram using complex relations
- apply implicit differentiation, by hand in simple cases
- use analytic techniques to find derivatives and anti-derivatives by pattern recognition, and apply anti-derivatives to evaluate definite integrals
- set up and evaluate definite integrals to calculate arc lengths, areas and volumes
- set up and solve differential equations of specified forms
- represent and interpret differential equations by direction(slope) fields
- perform operations on vectors and interpret them geometrically
- apply vectors to motion of a particle and to geometric problems
- solve kinematics problems using a variety of techniques
- set up and solve problems involving Newton's laws of motion
- apply a range of analytical, graphical and numerical processes to obtain solutions (exact or approximate) to equations
- set up and solve problems involving the distribution of sample means
- construct approximate confidence intervals for sample means
- undertake a hypothesis test for a mean of a sample from a normal distribution or a large sample.

**Outcome 2**

On the completion of each unit the student should be able to apply mathematical processes, with an emphasis on general cases, in non-routine contexts, and analyse and discuss these applications of mathematics.

To achieve this outcome the student will draw on knowledge and skills outlined in one or more areas of study.

**Key knowledge**

- the key mathematical content from one or more areas of study relating to a given application context
- specific and general formulations of concepts used to derive results for analysis within a given application context
- the role of examples, counter-examples and general cases in developing mathematical analysis
- the role of proof in establishing a general result
- the use of inferences from analysis to draw valid conclusions related to a given application context.

**Key skills**

- specify the relevance of key mathematical content from one or more areas of study to the investigation of various questions related to a given context
- give mathematical formulations of specific and general cases used to derive results for analysis within a given application context
- develop functions as possible models for data presented in graphical form and apply a variety of techniques to decide which function provides an appropriate model
- use a variety of techniques to verify results
- establish proofs for general case results
- make inferences from analysis and use these to draw valid conclusions related to a given application context
- communicate conclusions using both mathematical expression and everyday language, in particular in relation to a given application context.

## Outcome 3

On completion of each unit the student should be able to select and appropriately use numerical, graphical, symbolic and statistical functionalities of technology to develop mathematical ideas, produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches.

To achieve this outcome the student will draw on knowledge and related skills outlined in all the areas of study.

### Key knowledge

- the exact and approximate specification of mathematical information such as numerical data, graphical forms and general or specific forms of solutions of equations produced by technology
- domain and range requirements for specification of graphs of functions and relations, when using technology
- the role of parameters in specifying general forms of functions and equations
- the relation between numerical, graphical and symbolic forms of information about functions and equations and the corresponding features of those functions and equations
- similarities and differences between formal mathematical expressions and their representation by technology
- the selection of an appropriate functionality of technology in a variety of mathematical contexts.

### Key skills

- distinguish between exact and approximate presentations of mathematical results produced by technology, and interpret these results to a specified degree of accuracy
- use technology to carry out numerical, graphical and symbolic computation as applicable
- produce results using a technology which identify examples or counter-examples for propositions
- produce tables of values, symbolic expressions, families of graphs and collections of other results using technology, which support general analysis in problem-solving, investigative and modelling contexts
- use appropriate domain and range specifications to illustrate key features of graphs of functions and relations
- identify the relation between numerical, graphical and symbolic forms of information about functions and equations and the corresponding features of those functions and equations
- specify the similarities and differences between formal mathematical expressions and their representation by technology, in particular, equivalent forms of symbolic expressions
- select an appropriate functionality of technology in a variety of mathematical contexts, and provide a rationale for these selections
- apply suitable constraints and conditions, as applicable, to carry out required computations
- relate the results from a particular technology application to the nature of a particular mathematical task (investigative, problem solving or modelling) and verify these results
- specify the process used to develop a solution to a problem using technology, and communicate the key stages of mathematical reasoning (formulation, solution, interpretation) used in this process.

## School-based assessment

### Satisfactory completion

The award of satisfactory completion for a unit is based on whether the student has demonstrated the set of outcomes specified for the unit. Teachers should use a variety of learning activities and assessment tasks to provide a range of opportunities for students to demonstrate the key knowledge and key skills in the outcomes.

The areas of study and key knowledge and key skills listed for the outcomes should be used for course design and the development of learning activities and assessment tasks.

## Assessment of levels of achievement

The student's level of achievement for Units 3 and 4 will be determined by School-assessed Coursework. School-assessed Coursework tasks must be a part of the regular teaching and learning program and must not unduly add to the workload associated with that program. They must be completed mainly in class and within a limited timeframe.

Where teachers provide a range of options for the same School-assessed Coursework task, they should ensure that the options are of comparable scope and demand.

The types and range of forms of School-assessed Coursework for the outcomes are prescribed within the study design. The VCAA publishes *Advice for teachers* for this study, which includes advice on the design of assessment tasks and the assessment of student work for a level of achievement.

Teachers will provide to the VCAA a numerical score representing an assessment of the student's level of achievement. The score must be based on the teacher's assessment of the performance of each student on the tasks set out in the following tables.

### Contribution to final assessment

School-assessed Coursework will contribute 17 per cent to the study score for each of Units 3 and 4.

### Unit 3

Outcomes	Marks allocated*	Assessment tasks
<p><b>Outcome 1</b> Define and explain key concepts as specified in the content from the areas of study, and apply a range of related mathematical routines and procedures.</p>	15	<p><b>Application task</b></p> <p>A mathematical investigation of a practical or theoretical context involving content from two or more areas of study, with the following three components of increasing complexity:</p> <ul style="list-style-type: none"> <li>introduction of the context through specific cases or examples</li> <li>consideration of general features of the context</li> <li>variation or further specification of assumption or conditions involved in the context to focus on a particular feature or aspect related to the context.</li> </ul> <p>The application task is to be of 4–6 hours duration over a period of 1–2 weeks.</p>
<p><b>Outcome 2</b> Apply mathematical processes, with an emphasis on general cases, in non-routine contexts, and analyse and discuss these applications of mathematics.</p>	20	
<p><b>Outcome 3</b> Select and appropriately use numerical, graphical, symbolic and statistical functionalities of technology to develop mathematical ideas, produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches.</p>	15	
<b>Total marks</b>	<b>50</b>	

\*School-assessed Coursework for Unit 3 contributes 17 per cent.



## Unit 4

Outcomes	Marks allocated*	Assessment tasks
<b>Outcome 1</b> Define and explain key concepts as specified in the content from the areas of study, and apply a range of related mathematical routines and procedures.	15	8 Modelling or problem-solving task 1
		7 Modelling or problem-solving task 2
<b>Outcome 2</b> Apply mathematical processes, with an emphasis on general cases, in non-routine contexts, and analyse and discuss these applications of mathematics.	20	10 Modelling or problem-solving task 1
		10 Modelling or problem-solving task 2
<b>Outcome 3</b> Select and appropriately use numerical, graphical, symbolic and statistical functionalities of technology to develop mathematical ideas, produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches.	15	7 Modelling or problem-solving task 1
		8 Modelling or problem-solving task 2
<b>Total marks</b>	<b>50</b>	One of the modelling or problem-solving tasks is to be related to the <b>Mechanics</b> or <b>Probability and statistics</b> area of study. The modelling or problem-solving tasks are to be of 2–3 hours duration over a period of 1 week.

\*School-assessed coursework for Unit 4 contributes 17 per cent.

## External assessment

The level of achievement for Units 3 and 4 will also be assessed by two end-of-year examinations. The examinations will contribute 22 and 44 per cent respectively.

## End-of-year examinations

### Description

All of the content from the areas of study and the key knowledge and key skills that underpin the outcomes in Units 3 and 4 are examinable.

### Examination 1

#### Description

This examination comprises short-answer and some extended-answer questions covering all areas of study in relation to Outcome 1. It is designed to assess students' knowledge of mathematical concepts, their skills in carrying out mathematical algorithms without the use of technology and their ability to apply concepts and skills.

**Conditions**

The examination will be of one hour duration and no technology (calculators or software) or notes of any kind are permitted. A sheet of formulas will be provided with the examination. VCAA examination rules will apply.

**Contribution to final assessment**

The examination will contribute 22 per cent.

**Examination 2****Description**

This examination comprises multiple-choice questions and extended-answer questions covering all areas of the study in relation to all three outcomes, with an emphasis on Outcome 2. The examination is designed to assess students' ability to understand and communicate mathematical ideas, and to interpret, analyse and solve both routine and non-routine problems.

**Conditions**

The examination will be of two hours duration and student access to an approved technology with numerical, graphical, symbolic and statistical functionality will be assumed. One bound reference, text (which may be annotated) or lecture pad, may be brought into the examination. VCAA examination rules will apply.

**Contribution to final assessment**

The examination will contribute 44 per cent.

**Further advice**

The VCAA publishes specifications for all VCE examinations on the VCAA website. Examination specifications include details about the sections of the examination, their weighting, the question format/s and any other essential information. The specifications are published in the first year of implementation of the revised Units 3 and 4 sequence together with any sample material.