**Paper 2: SENIOR SECONDARY MATHEMATICS IN VICTORIA   
WORKING TOWARDS CHANGE**

**Introduction**

Central to the work of curriculum and assessment authorities are principled and coherent responses to the questions of: *What mathematics*? (Selection from culture, discipline and domain knowledge, theory and applications); *For whom*? (Subsets of the cohort); *How*? (Curriculum and assessment requirements and advice on possible pedagogies); and *Why*? (Rationale and purpose). Australian states and territories have the constitutional responsibility for school education, set the curriculum and related assessment such as final year examinations and requirements, and oversee system specified school-based assessment.

In Victoria, review or development of a senior secondary study or group of related studies for the Victorian Certificate of Education (VCE) takes place by [Review Panels](http://www.vcaa.vic.edu.au/Documents/vce/rolesresponsibilitiesofVCEreviewpanels09.pdf) convened and working in accordance with the [VCAA’s Principles and procedures for the development and review of VCE studies,](https://www.vcaa.vic.edu.au/Documents/vce/2019_Principles_Procedures_VCE_review.docx) and general and specific terms of reference for that review. The work of Review Panels, VCAA Managers and Executive, the VCAA Senior Secondary Curriculum and Assessment Committee (SSCAC) and the VCAA Board is informed by various research, enrolment data and trends, examination and coursework audit reports, background and discussion papers, benchmarking reports and consultations.

This paper aims to inform and support in-depth review of senior secondary mathematics in Victoria. By in-depth review it is meant that the outcome *could* involve substantive change to the structure, nature, purpose and scope of the curriculum. The [current Victorian senior secondary mathematics curriculum](http://www.vcaa.vic.edu.au/Documents/vce/mathematics/MathematicsSD-2016.pdf) is accredited 2016–2019/20 and follows on from several cycles of progressive review and development of a model first introduced in 1993/4.It also incorporates aspects of recent collaborative Australian curriculum work involving the states and territories and the Commonwealth.

**Defining the task**

Consider the question:

*What could a senior secondary mathematics curriculum for a liberal democratic society in a developed country for* 2020–2030 *look like*?

and possible responses that are not limited by current thinking or beliefs or constrained by prior work. The following can be assumed as broad background:

* most students in the relevant cohort from birth, say 2002–2013, would progress to be engaged in senior secondary certificate studies for the final two years of secondary education in 2020–2030 (historically around 80% of this cohort have proceeded to VCE)
* most of these students would undertake some mathematical studies during their VCE; however, the study of mathematics is not compulsory in the senior secondary curriculum (historically just over 90 per cent of VCE students study a Units 1 and 2, or Year 11 level mathematics, and just over 80 per cent of VCE students study a Units 3 and 4, or Year 12 level mathematics)
* students would have access to relevant functionality from enabling technology as applicable
* a suitable combination of assessments, externally set and school-based, with and without the use of enabling technology, would be put in place in conjunction with the proposed curriculum
* there are no undue restrictions on the number and combinations of mathematical studies students can undertake
* there is a clear articulation of assumed knowledge and skills students would need to have to enable them to effectively undertake various pathways of study, and natural relations between preparatory and following studies. While there are currently no pre-requisites, there is clear articulation of assumed knowledge and skills, and schools exercise responsibility for ensuring students have appropriate preparation for the units they select to study.
* the Primary to Year 10 (compulsory years) mathematics curriculum these student would have studied (depending on year of birth) include the [Curriculum and Standards Framework II: Mathematics](http://pandora.nla.gov.au/pan/99103/20090505-1614/csf.vcaa.vic.edu.au/home.html) (2000–2004), the [Victorian Essential Learning Standards: Mathematics](http://pandora.nla.gov.au/pan/129125/20121206-0015/vels.vcaa.vic.edu.au/maths/index.html) (2005–2011), the [Australian Curriculum: Mathematics](http://pandora.nla.gov.au/pan/129125/20121206-0015/ausvels.vcaa.vic.edu.au/Mathematics/Overview/Rationale-and-Aims.html) (2012–2016) and the [Victorian Curriculum: Mathematics](http://victoriancurriculum.vcaa.vic.edu.au/mathematics/introduction/rationale-and-aims) (from 2017).

Generally, following a review and consultation process, new or revised courses are made available for teacher familiarisation a year before implementation begins. A state-wide program of professional learning is undertaken in that year and related advice documents and resources are developed and published to support the implementation. These include advice for teachers, and sample assessment tasks comprising sample examinations or questions (depending on the degree of change). In the year *preceding* the familiarisation year, during the review process, a consultation draft of the proposed curriculum is available for general comment and feedback, typically for term 3 (in a four-term year). This draft is usually close to the likely version proposed for accreditation, and at this stage commercial publishers are briefed so that they can commence preparation of supporting resources (which will be reviewed by academics, practising teachers and others). There are no approved or prescribed texts; schools select texts for their classes as they find suitable, based on the supplementary resources and shared professional experience.

This paper provides historical background to the Victorian context from 1980 to the present.

**Changes over the years**

**1970–1985**

**Structure and assessment**

From 1970–1985 senior secondary mathematics was based on three Higher School Certificate (HSC) Year 12 subjects: General Mathematics, Pure Mathematics and Applied Mathematics. In broad terms these can be considered as describing the *style* of mathematics students would undertake. All three subjects involved calculus, although only at a basic level for General Mathematics.

General Mathematics and Pure Mathematics were alternative subjects. The former was regarded as providing for a distinct ‘stream’ of students from Pure Mathematics.

The latter included ‘expected background’ for Applied Mathematics, although this was not a formal requirement as indicated in the Higher School Certificate Course Description (VISE, 1980, page 3):

Since the Applied Mathematics syllabus includes a substantial number of applications of calculus, it would probably be unwise for students to undertake this subject unless they were concurrently studying Pure Mathematics or has passed Pure Mathematics (or its equivalent) in a previous year.

The Victorian Universities and Schools Examination Board (VUSEB) specified the syllabus (content) and assessment from 1970 to 1978, this function was taken over by the Victorian Institute of Secondary Education (VISE) from 1979 to 1988. Year 11 subjects were not regulated; however, text publishers and authors (typically Fitzpatrick and Galbraith and Lucas and James) produced pairs of texts (Mathematics 1 and 2, Mathematics A and B) that provided coverage of relevant content. Schools typically implemented two types of courses based on these texts, a single mathematics subject as preparation for General Mathematics and a double mathematics subject as preparation for Pure Mathematics and possibly also Applied Mathematics.

Indeed, the Higher School Certificate Course Description (VISE, 1980, page 3) clearly stated:

… no formal requirements are made for entries into these courses. However, students are strongly urged to have completed a satisfactory Year 11 mathematics program. Thus, they will need an appropriate mathematical background and will need to be familiar with essential underlying concepts.

From 1981 the Year 12 subjects had a core and option structure in a 20:10-week ratio, with an assumed class time of ‘approximately four hours a week’. The core was externally assessed by a single three-hour examination and the options were internally assessed by around 10 hours of school-devised assessment, and statistically moderated with respect to the examinations (even though the options content was by definition distinctive from that of the core content). The final score assessment weighting was 70 for the examination and 30 for the school-based assessment. For each subject, there was a single three-hour examination comprising a collection of short and extended answer questions. School-based assessment comprised a combination of teacher set tests, assignments and projects, as indicated in the (very) brief advice (half of a column on a two column B4 size page) on assessment for the various options.

**Content**

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| --- | --- |
| General Mathematics | |
| *Core* | ***Options* (*select one*)** |
| Arrangements and selections | Computer applications in General Mathematics |
| Functions and their graphs | Mathematics of earth and space |
| Matrices and linear equations | Mathematics of growth and decay |
| Calculus | Applicable mathematics |
| Probability | **-** |

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| --- | --- |
| Pure Mathematics | |
| *Core* | ***Options* (*select one*)** |
| Real numbers | Numerical mathematics |
| Analytic geometry | Probability |
| Relations and functions | Complex numbers and matrices |
| Calculus | Computer applications in mathematics |

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| --- | --- |
| Applied Mathematics | |
| *Core* | ***Options* (*select one*)** |
| Vectors | The same set of options as for Pure Mathematics.  The selected options for the two subjects must be different. |
| Applications of calculus |
| Mechanics |

**Use of technology**

Until 1978, computational technology was slide rules and four-figure logarithm tables. After that year scientific calculators were allowed. Some of the options required students to do programming (FORTRAN or BASIC, flowcharts). This was facilitated by collaborative arrangements with universities allowing schools access to their facilities to run student programs. Increasingly from the early 1980s, some schools had their own mini-computers and shared access. From the mid-1980s these were replaced by collections of early micro-computers.

**Prerequisite status for tertiary study**

Pure Mathematics was a prerequisite for university science, engineering and mathematics courses, the latter two also requiring Applied Mathematics. General Mathematics was a prerequisite for various courses that required Year 12 Mathematics, such as biological sciences, commerce and economics.

Some institutions allowed students with a General Mathematics scoring grade of D or above to undertake science courses, however this was not encouraged.

**1986–1989  
Structure and assessment**

From 1985 to 1989, following the [Ministerial Review of Postcompulsory Schooling Report Volume 1, the Blackburn Report (1985)](http://www.voced.edu.au/content/ngv%3A39068), a transitional structure was put in place under the auspices of VISE while the original VCE structure was being developed. From 1989, the Victorian Curriculum and Assessment Board (VCAB) would itself replace VISE. Three mathematics subjects were available, two Group 1 subjects – Mathematics A and Mathematics B (which could be used for tertiary entrance purposes), and a Group 2 subject – Business Mathematics (which could not be used for tertiary entrance purposes). Mathematics A was a co-requisite for Mathematics B. In broad terms Mathematics A was a combination of aspects of the former General Mathematics and Pure Mathematics, while Mathematics B was a combination of aspects of the former Pure Mathematics and Applied Mathematics. The previous assessment structure was retained for the Group 1 subjects, that is, a single three-hour examination comprising a collection of short and extended answer questions, and school-based assessment comprising a combination of teacher set tests, assignments and projects, as indicated in the advice on assessment for the various options.

The Group 2 Business Mathematics subject was completely school-based assessed, subject to state-wide moderation. The subject titles used were indicators of the *purpose* of the mathematics being studied. Teaching time expectations for these subjects were the same as for the previous set of subjects.

**Content (Group 1 subjects)**

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| Mathematics A | |
| *Core* | ***Options* (*one of the following 10 week optional units*)** |
| Mensuration | Mathematics of earth and space |
| Probability | Mathematics of growth and decay |
| Functions and calculus | Computer applications in mathematics |
| Mechanics |
| Combination (any two of the following optional units: Business applications of sequences and series, Statistical sampling, Continuous probability distributions, Logic and proof, Transformation geometry, Linear programming) |

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| --- | --- |
| Mathematics B | |
| *Core* | ***Options* (*select one*)** |
| Functions and calculus | As for Mathematics A.  The selected options for the two subjects must be different. |
| Linear algebra |
| Vectors |
| Complex numbers |
| Analytical geometry |

**Content (Group 2 subject)**

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| --- | --- |
| Business Mathematics | |
| A course is constructed from a selection of topics and projects comprising 120 hours in total. | |
| *Topics* (*12 – 20 hours a topic*) | ***Projects* (*16 hours each*)** |
| Investment | Home CPI |
| Financial management | Quality control |
| Taxation | Tattslotto |
| Data collection, presentation and interpretation | Sampling |
| Axiomatic probability | Management games |
| Probability distributions | Computer project – queuing theory – Monte Carlo methods |
| Economic decision models | Stock market investment |
| Growth analysis | Starting a business |
| Allocation |  |
| Queuing theory |  |
| Technology in business |  |
| Computer programming |  |
| Computer programming for business |  |

**Use of technology**

Computational technology used in examinations was scientific calculators, with tables for computations related to probability distributions.

For some school-based assessments tasks spreadsheets became popular as computational software, as did first generation graphing software such as *Efofex* (1989).

Recent versions of these are still available and in use ([www.efofex.com/](http://www.efofex.com/)).

For some options and projects computer programs and programming were used (structured BASIC or PASCAL, flowcharts, N-S diagrams, structured English or pseudo-code).

**Prerequisite status for tertiary study**

Mathematics A was the main prerequisite subject for tertiary courses where such a prerequisite was required.

Group 2 subjects such as Business Mathematics did not qualify for tertiary prerequisite purposes. The increasing popularity of this subject with the student cohort (roughly 1 in 3 students) was in part a significant factor in the decision to move away from the Group 1 and 2 structures.

**VCE Mathematics 1.0[[1]](#footnote-1)\*: 1990–1993**

**Structure and assessment**

The original VCE Mathematics study structure, developed under the auspices of the Victorian Curriculum and Assessment Board (VCAB), was piloted in trial schools in 1989 and fully implemented from 1990 to 1992 (Units 1 and 2) and from 1990 to 1993 (Units 3 and 4). This structure was distinctive in that it was based on areas of mathematics (each called a block) studied from within the discipline (Space and Number, Reasoning and Data, Change and Approximation) and specified content for Year 11 (Units 1 and 2) as well as for Year 12 (Units 3 and 4). It was the first structure that regulated the penultimate (Year 11) as well as final (Year 12) years senior secondary curriculum. Each of these blocks could be done at either Unit 1 and 2 or Unit 3 and 4 levels, with a 60:40 weighting of prescribed and optional topics. If a block were studied at the Unit 1 and 2 level, the *extension* of that block or one of the other blocks would be studied at the Unit 3 and 4 level. The study titles used were broad indicators of the *nature* of mathematics being studied. While there continued to be no formal prerequisites for entry into Units 1 to 3 of any study, Unit 3 was a prerequisite for Unit 4, Units 3 and 4 being designed as a paired sequence. A unit comprised approximately 100 hours of study, of which 50 to 60 hours was in class time.

There were three work requirements:

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| Work requirements | |
| 1 | Skills practice and standard applications |
| 2 | Problem solving and modelling |
| 3 | Projects |

Each work requirement was to be allocated a minimum of 20 hours and cover all areas of study between them, with computers or calculators used in at least one of the work requirements.

Assessment was based on two components: satisfactory completion of units (the basis for award of the VCE), and level of achievement, the latter by two one and a half hour examinations and two centrally set school-based extended assessment tasks. Collectively these were referred to as Common Assessment Tasks (CATs). CAT 1: Investigative Project was a four-week extended mathematical investigation where students selected a starting point from several for a centrally set theme. CAT 2: Challenging Problem was a two-week mathematical problem-solving task where students selected a starting point from several for a centrally set context. These tasks were completed in annually designated periods. CAT 3: Facts, skills and applications was a multiple-choice examination; and CAT 4: Analysis task was an extended response written examination. The extended CATs were subject to a comprehensive verification process based on expert review, random sampling and criterion-based assessment. The tasks were published by the VCAB as an A4 booklet of several pages. This provided an overview of the theme for the task, several starting points related to the theme, from which students had to select one, and specifications for the working log book and final task report, which formed the student’s response. The strengths, limitations and difficulties of this structure have been well documented in Australian mathematics education literature. For a variety of reasons and factors, significantly simplicity and ease of transition from secondary to tertiary study pathways involving mathematics (and related decisions about entrance scores), VCE Mathematics 1.0 was substantially revised in 1992/3 leading to the initial version of the current structure.

One of the key features of the VCE is that it separates the requirements for the award of the certificate (based on satisfactory completion of units) from the assessment of levels of achievement (originally based on CATs). Thus the ‘minimum mark to pass a subject’ problem is not a consideration for award of the VCE. In the first iteration of the VCE all four CAT scores were assigned a 25 per cent weighting and aggregated directly without any scaling.

**Content**

Each study was structured around broad areas and clusters of content, covering both required and additional material.

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| Space and number block | |
| *Units 3 and 4 areas of study* | ***Units 3 and 4: Extensions*** |
| Arithmetic | Extension of content in Arithmetic, Geometry, Algebra and content from any one of Probability, Statistics, Logic or Calculus from the other two blocks. |
| Geometry |
| Trigonometry |
| Algebra |

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| Change and approximation block | |
| *Units 3 and 4 areas of study* | ***Units 3 and 4: Extensions*** |
| Coordinate geometry | Extension of content in Coordinate geometry, Calculus, Algebra and content from any one of Arithmetic, Trigonometry, Statistics or Probability from the other two blocks. |
| Calculus |
| Algebra |

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| Reasoning and data block | |
| *Units 3 and 4 areas of study* | ***Units 3 and 4: Extensions*** |
| Probability | Extension of content in Probability, Statistics, Algebra and content from any one of calculus, Arithmetic or Geometry from the other two blocks. |
| Statistics |
| Logic |
| Algebra |

**Use of technology**

While the permitted technology for examinations continued to be a scientific calculator, the extended CATs led to students using a variety of technologies: computer programs, the then newly available graphics calculators (from 1989) and software such as spreadsheets, graphing software and early versions of computer algebra system (CAS) software (Derive and *Mathematica*) and CAS mini-computers/calculators such as the TI-92.

It was also a time when students increasingly began to use word processing software to complete the reports on which their assessment for these components would be based, and to integrate graphical and other output from various software.

From the beginning of the VCE, for mathematics examinations, students were permitted to bring in four pages of their own notes (two sides of two A4 sheets), handwritten and/or typed. Some very small font sizes were commonly seen, along with very ‘dense’ sections of text, diagrams and formulas. This practice continued until 2006 when it was replaced by the bound reference.

**Prerequisite status for tertiary study**

A key principle of the VCE is that all Units 3 and 4 studies can have a study score compiled, and that these all counted towards the ENTER (now [ATAR](http://www.vtac.edu.au/results-offers/atar-explained.html)) tertiary entrance score.

Due to the distinctive structure of the VCE Mathematics 1.0 study design, both Reasoning and Data Units 3 and 4 and Change and Approximation Units 3 and 4, or the Extension study for either of these studies, could be used to satisfy prerequisite requirements for a broad range of university courses. Extensions: Change and Approximation has the same status as the former Mathematics B. Space and Number. Units 3 and 4 were used to meet general requirements such as ‘a Unit 3 and 4 level mathematics’.

**1994 to present**

**VCE Mathematics 2.0[[2]](#footnote-2)\*: 1994–1996**

**Structure and assessment**

The current VCE Mathematics study structure was introduced over two years in 1993 (Units 1 and 2) and 1994 (Units 3 and 4) and has been in place with various modifications and refinements since then. It was developed under the auspices of the Victorian Board of Studies (BOS), and represented a significantly different paradigm from the original VCE Mathematics study structure 1989 (pilot) to 1992, one more aligned with the at the time ‘traditional’ expectations. Unlike its predecessors, the Board of Studies had carriage of P–12 curriculum and assessment, which would lead in the first instance to the introduction of the Curriculum and Standards Framework for the compulsory years for government schools in 1995 (CSF95).

In its original form, VCE Mathematics 2.0 comprised two Unit 1 and 2 studies, General Mathematics and Mathematical Methods, and three Unit 3 and 4 studies, Further Mathematics, Mathematical Methods and Specialist Mathematics. The study titles used were indicators of the *cohort* of students studying each mathematics study.

General Mathematics and Further Mathematics were constructed as non-calculus studies, developed from the Space and Number study, and the statistical and other discrete mathematics aspects of the Reasoning and Data study. Mathematical Methods was constructed from the Change and Approximation study and probability aspects of the Reasoning and Data study. Specialist Mathematics was only implemented at Units 3 and 4 and constructed with a core and options structure from Extensions: Change and Approximation, and selected aspects of Extensions: Space and Number and Extensions: Reasoning and Data.

General Mathematics Unit 1 was completely prescribed and comprised topics in univariate and bivariate statistics, practical/financial applications of arithmetic and graphs, and algebra of linear relations and functions.

General Mathematics Unit 2 comprised a prescribed topic *Geometry* (essentially practical mensuration) and a teacher selection of optional topics for additional material from a broad list of possible topics, from very practical to non-calculus advanced mathematics. These were designed as preparatory units for Further Mathematics Units 3 and 4. Further Mathematics Units 3 and 4 comprised a compulsory core of data analysis, followed by a teacher selection of three from six possible optional modules, with a 40:60 weighting.

Mathematical Methods Units 1 and 2 were completely prescribed, comprising topics from four areas of study: algebra, functions and graphs, calculus, and probability. They were designed as preparatory units for Mathematical Methods Units 3 and 4 which covered the same areas of study and extended on previous content as well as introducing new content. This prescription of content provided universities with a confidence that sufficient essential and common mathematics of this kind would be covered by all students who undertook the study.

Specialist Mathematics was only offered at Units 3 and 4, and assumed previous completion of Mathematical Methods Units 1 and 2, and concurrent study of Mathematical Methods Units 3 and 4. In addition, students were expected to be ‘familiar with the solution of triangles in two-dimensional situations’. It comprised a compulsory core of new (and extension) advanced topics in functions and graphs, algebra (simple rational functions and complex numbers), calculus (including differential equations) and vectors and a selection of one from four possible optional modules (Statistics and probability, Geometry, Mechanics, and Logic) in a 70:30 weighting. Suitable background for new topics could be obtained by studying a selection of optional topics from General Mathematics Unit 2.

With respect to assessment the centrally set common assessment tasks (CATs) were reduced from two tasks to a single task for each Unit 3 and 4 study. For Further Mathematics and Mathematical Methods, the four-week Investigative Project was retained. For Specialist Mathematics, the two-week Challenging Problem was retained, and a Board set test (based on a similar context to that of the challenging problems) was introduced and run as a min-exam, after the Challenging Problem was completed. The weighting of the problem task to the test was 60:40.

These tasks were undertaken in a period prescribed annually by the Board of Studies, and subject to a Board run independent review process of tasks from all schools.

The examination structure was changed to two one and a half hour examinations. For Mathematical Methods and Specialist Mathematics, Examination 1 was a collection of 28 multiple-choice questions and a set of short-answer questions worth a total of 60 marks. Examination 2 was a collection of four extended response questions also worth a total of 60 marks.

For Further Mathematics Examination 1 was a collection of multiple-choice questions covering the core and modules and a set of short-answer questions worth a total of 60 marks. Examination 2 was a collection of short-answer application questions covering the core and modules also worth a total of 60 marks. Students only answered module questions relating to the three selected modules for their school. A small number of students occasionally chose to do a different module than that studied at school.

The contributions to final score were adjusted accordingly to be one-third school-based assessment (now a single CAT) and two-thirds examination assessment, with each examination equally weighted.

**Content**

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| Further Mathematics Units 3 and 4 | |
| *Core* (*40% content weighting*) | ***Modules* (*60% content weighting – three selected from six options)*** |
| Probability and statistics | Arithmetic and applications |
| Probability and statistics (sampling and control) |
| Geometry and trigonometry |
| Graphs and relations |
| Business related mathematics |
| Networks and decision mathematics |

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| Mathematical Methods Units 3 and 4 | |
| A completely prescribed course comprising the following areas of study | |
| Area of Study 1 | Coordinate geometry |
| Area of Study 2 | Trigonometric functions |
| Area of Study 3 | Calculus |
| Area of Study 4 | Algebra |
| Area of Study 5 | Statistics and probability |

The statistics component included introductory sampling and estimation from the former Reasoning and data block.

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| Specialist Mathematics Units 3 and 4 | |
| *Core (70 % content weighting) – advanced and extension material in each of the following areas of study* | ***Module (30% content weighting – one selected from four options)*** |
| Coordinate geometry (including reciprocal quadratic functions) | Statistics and probability (including continuous random variables and sampling and estimation) |
| Trigonometric functions (including identities) | Geometry (including deductive two-dimensional geometry, analytic geometry and transformation geometry with matrices) |
| Calculus (including differential and integral calculus and applications, differential equations and kinematics) | Mechanics (including Newton’s laws of motion, simple harmonic motion and vector calculus) |
| Algebra (including partial fractions, complex numbers) | Logic (including elementary set theory, propositional logic, boolean algebra, logic circuits and boolean algebra, and methods of proof). |
| Vectors in two and three dimensions | Statistics and probability (including continuous random variables and sampling and estimation) |

**Use of technology**As for the previous accreditation period.

**Prerequisite status for tertiary study**

Mathematical Methods Units 3 and 4 was the study used to meet prerequisite requirements for university courses in areas such as medicine, science, engineering, some economics and commerce courses and aviation.

Specialist Mathematics was also a prerequisite for a small number of courses. Further Mathematics could be used to meet general prerequisite requirements based on students having undertaken a Unit 3 and 4 mathematics study.

**VCE Mathematics 2.1[[3]](#footnote-3)\*: 1997–1999 (originally accredited to 2000, then shortened to 1999)**

Student and teacher workload associated with satisfactory completion of work requirements and the extended CATs, led to a major review of school-based assessment in the VCE in 1997 with the report [*VCE Enhancing their Futures*](https://digitised-collections.unimelb.edu.au/bitstream/handle/11343/115550/scpp-01179-vic-1997.pdf?sequence=1) published in December 1997. This resulted in foreshortening of all VCE study accreditation periods and implementation from 2000 of a revised school-based assessment structure. This new structure replaced satisfactory completion of work requirements with satisfactory demonstration of a set of outcomes as the basis for satisfactory completion of a unit, and hence award of the VCE, as discussed in more detail in VCE Mathematics 2.2 below.

The previous accreditation cycle was one in which there were some significant developments in availability and access of enabling technologies (software and hand-held devices) with symbolic and graphical and numerical functionality. In particular, portable hand-held devices such as ‘graphing’ or ‘graphics’ calculators had become increasingly popular. In 1995 and 1997 respectively these (including in some cases early CAS calculators) were incorporated into senior secondary curriculum and assessment, including examinations, in various systems and jurisdictions including in the US, France, Denmark and Western Australia. The affordances of technology would increasingly become a significant driver for developments in mathematics curriculum and assessment.

**Structure and assessment**

Apart from minor changes and refinements to some of the studies, the structure of VCE Mathematics 2.1 was essentially unchanged from that of VCE Mathematics 2.0. For Specialist Mathematics, the number of possible modules was reduced from four to two, Mechanics and Geometry, in the (as it turned out false) hope that numbers for the selected modules would become more evenly distributed. For Mathematical Methods, a test component was introduced for the Investigative Project CAT, as previously done with the Specialist Mathematics Challenging Problem CAT. This counted for 25 per cent of the Investigative Project CAT score.

**Content**

As for the previous accreditation period, with the changes noted above.

**Use of technology**

In 1997, the use of graphics calculators was permitted for both examinations in Mathematical Methods and Specialist Mathematics and from 1998 student access to this technology was assumed. A preliminary list of approved calculators for these examinations was published in late 1997 and confirmed in March 1998. For Further Mathematics, assumed access to an approved graphics calculator for examinations was delayed until 2000, with scientific calculators with bivariate statistical functionality assumed for 1999.

From 1998 a formal ongoing process was established for monitoring and reviewing available technologies. Subsequently, approval of some technologies was confirmed, and an updated list of approved graphics calculators for the *following* year’s examinations was published in the October Bulletin of the preceding year. That is, in October 1998, the Bulletin included the list of approved graphics calculators for end-of-year examinations for applicable studies in November 1999, and so on.

Two other key developments occurred around this time:

* the emergence of the first series of hand-held Computer Algebra System (CAS) calculators (HP 48G, Casio Algebra, TI-89) which were effectively graphics calculators with symbolic manipulation functionality, and
* the increasing memory capacity of graphics calculators and range of third-party propriety programs/applications (developed in graphics calculator programming languages) that gave graphics calculator increased functionality, for example programs such as *Symbolic*, *Factor*9, and various antidifferentiation and integration applications.

To address these developments and related considerations, the Victorian Board of Studies, in partnership with the Department of Science and Mathematics Education at The University of Melbourne, applied for and secured a major three-year Australian Research Council SPIRT grant 2000–2002 to investigate the use and impact of Computer Algebra System (CAS) calculator technology in senior secondary mathematics curriculum and assessment, initially in relation to the Mathematical Methods study.

Following on from recommendation 15 of the *VCE Enhancing their Futures*report, a new Unit 1 and 2 sequence: Foundation Mathematics was developed in 1998 and piloted in 1999. This study was developed to create a Year 11 standard course that provided students with access to age appropriate further mathematical studies in a strongly context-application based learning environment. It also acknowledged that the General Mathematics topic-based study structure was *not* adequately meeting the needs of a particular cohort of students.

**Prerequisite status for tertiary study**

As for the previous accreditation period.

**VCE Mathematics 2.2[[4]](#footnote-4)\*: 2000–2005 (originally accredited to 2003, extended to 2005)**

The Victorian Board of Studies was replaced by two complementary statutory Authorities from 2000, the [Victorian Curriculum and Assessment Authority (VCAA)](http://www.legislation.vic.gov.au/Domino/Web_Notes/LDMS/PubStatbook.nsf/51dea49770555ea6ca256da4001b90cd/2226EE6AF4F7E5B6CA256E5B00213F5D/$FILE/00-096a.pdf) and the [Victorian Registration and Qualifications Authority (VRQA)](http://www.vrqa.vic.gov.au/Pages/default.aspx). The VCAA retained the former Board’s curriculum and assessment functions while the VRQA was assigned the Board’s accreditation functions and other functions. VCE certificates were co-signed by the Chairs of both Authorities. This arrangement has continued in place with various developments and refinements over the years.

The overall structure of the VCE Mathematics study design remained the same with respect to types of studies, with the exceptions of the introduction of Foundation Mathematics Units 1 and 2, and the pilot Mathematical Methods (CAS) Units 1–4. The recommendations of the *Enhancing their Futures*report led to a fundamental change to the basis for the award of the VCE, satisfactory completion of a unit, and for school-based assessment (with level of achievement judgment). A new model was introduced which has continued, with various developments and refinements. Key drivers were to reduce workload, address authentication issues and reduce costs associated with the former verification and independent review processes.

Satisfactory completion of a unit was changed from being based on satisfactory completion of three work requirements to being based on demonstration of a set of outcomes for each unit. For all VCE Mathematics studies, a set of three outcomes was derived from the following three aims:

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| Aim | Activity |
| *Apply knowledge and skills* | The study of aspects of the existing body of mathematical knowledge through learning and practising mathematical algorithms, routines and techniques, and using them to find solutions to standard problems. |
| *Model, investigate and solve problems* | The creative application of mathematical knowledge and skills in unfamiliar situations, including real-life situations, which require investigative, modelling or problem-solving approaches. |
| *Use technology* | The effective and appropriate use of technology to produce results which support learning mathematics and its application in different context. |

A corresponding outcome was developed for each aim. Thus, Outcome 1 for each study was essentially about knowledge, skills, standard techniques and routine applications, Outcome 2 was about the use of modelling, problem-solving and investigative aspects of working mathematically and mathematical inquiry in more open-ended and non-routine contexts, and Outcome 3 was about the use of technology as a means of obtaining results (computational tool) and mathematical inquiry in relation to Outcomes 1 and 2.

Each outcome was described by a brief defining statement, elaborated by a set of accompanying key knowledge and key skill statements (see [current study design](http://www.vcaa.vic.edu.au/Documents/vce/mathematics/MathematicsSD-2016.pdf) for details of the latest version of these).

A consequence of this new structure was that extended common assessment tasks were replaced in most studies, including VCE Mathematics, by a prescribed set of school developed tasks for school-assessed coursework (popularly called SACs) according to the requirements of the relevant study design, and assessed in relation to mark weightings for the outcomes.

The idea was that teachers would locally devise tasks for their students of a specified nature, purpose and scope. Overall, these would comprise a collection of several smaller tasks, and reduce the overall time taken for students to complete them.

Individual student work would no longer be sampled and reviewed, rather an audit of teacher developed school coursework tasks and implementation would be conducted to ensure conformance to the requirements of the study design with respect to school-based coursework assessment. The scores for school-assessed coursework would be statistically moderated with respect to the examinations.

**Structure and assessment**

The introduction of Foundation Mathematics Units 1 and 2 was very successful, with student enrolments increasing from around 100 in the 1999 pilot to around 5000 within a few years. As anticipated, while many of these students did come from what would have formerly been General Mathematics enrolments, there was also a slight increase in overall enrolments across the two studies. These enrolment numbers have continued in a range of 4000 – 5000 since the early 2000s.

**Content**

Foundation Mathematics Units 1 and 2 was based on an extensive national and international review of past approaches and subjects, courses and studies, and the development of the study as a VCE Year 11 standard study, to avoid being typecast, in the vernacular as a ‘Veggie – maths’ subject.

It was developed using the same design principles as any other VCE study, with a strong context and applications-based approach to content from four areas of study:

|  |  |
| --- | --- |
| Foundation Mathematics Units 1 and 2 | |
| A completely prescribed course comprising the following areas of study | |
| Area of Study 1 | Shape and space |
| Area of Study 2 | Patterns in number |
| Area of Study 3 | Handling data |
| Area of Study 4 | Measurement and design |

In conjunction with the VCAA and University of Melbourne CAS–CAT research project, international benchmarking and meta-analysis of the research literature, the VCAA convened a review panel in 2000, and the VRQA accredited the pilot Mathematical Methods (CAS) Units 1 to 4 for 2001–2003, with examinations in 2003 for 80 volunteer students from three schools, one school each using one of the three CAS calculators (HP 48G, Casio Algebra, TI-89). The pilot study had its own examination panels, with some members also on the Mathematical Methods panels. The two sets of examinations had around 75 per cent common questions, and the Mathematical Methods content was a substantive subset of the Mathematical Methods (CAS) content. The pilot was expanded in 2004–2005 to include schools using CAS software Derive and *Mathematica*. For the next accreditation period from 2006, Mathematical Methods (graphics calculator enabled) and Mathematical Methods (CAS) (CAS calculator or software enabled) were run as alternative but equivalent studies, with the former lapsing and [transitioning](http://www.vcaa.vic.edu.au/documents/vce/drdavidleigh-lancaster.pdf) to the latter by 2009 for Units 1 and 2 and 2010 for Units 3 and 4. Non-pilot schools wishing to introduce Mathematical Methods (CAS) Units 3 and 4 were permitted to implement the pilot Mathematical Methods (CAS) Units 1 and 2 in 2005. While clearly related, the CAS version of the study had a more general approach, not restricted to by-hand symbolic computation.

In the interim, Mathematical Methods Examination 1 and 2 questions were carefully constructed to not afford undue advantage to those students who had supplemented their graphics calculators with various [algebra](http://www.ticalc.org/pub/83plus/basic/math/algebra/), [calculus](http://www.ticalc.org/pub/83plus/basic/math/calculus/) and other applications and programs that effectively made them into quasi-CAS devices.

There was no requirement in mathematics examinations that the memories of graphics calculators be cleared (such a process was required in other studies, but compliance became increasing fraught as various programs were able to simulate, cheat and hide content and programs from clearing processes).

**General Mathematics Units 1 and 2**

Units 1 and 2 of General Mathematics were significantly re-structured, with a unit course of study constructed according to a set of three rules applied to a set of topics from six areas of study:

* for each unit, material should cover four or more topics selected from at least three different areas of study
* courses intended as preparation for study at the Units 3 and 4 level should include selection of material from areas of study which provide a suitable background for these studies
* selected material from an area of study should provide a clear progression in key knowledge and key skills from Unit 1 to Unit 2.

|  |  |
| --- | --- |
| Generals Mathematics Units 1 and 2 | |
| *Areas of study* | ***Topics*** |
| Statistics and probability | Univariate data |
| Bivariate data |
| Sampling for attributes |
| Combinatorics |
| Introductory probability |
| Arithmetic | Applications of arithmetic |
| Financial arithmetic |
| Variation |
| Sequences and series |
| Functions and graphs | Linear graphs and modelling |
| Linear programming |
| Sketching and interpreting graphs |
| Algebra | Linear relations and equations |
| Non-linear relations and equations |
| Matrices |
| Algebra and logic |
| Geometry | Shape and measurement |
| Coordinate geometry |
| Vectors |
| Geometry in two and three dimensions |
| Undirected graphs and networks |
| Trigonometry | Trigonometric ratios and their applications |

While Outcomes 2 and 3 and their key knowledge and key skill statements applied across all the areas of study, Outcome 1 presented a different set of key knowledge and key skill statements for each area of study. In corresponding advice for teachers several different sample courses were outlined, including one suitable as desirable (but not required) additional preparation for Specialist Mathematics Units 3 and 4 in conjunction with Mathematical Methods Units 1 and 2. Topics assumed as background for Further Mathematics Units 3 and 4 were specified. These included: Univariate data; Bivariate data; Linear graphs and modelling and Linear relations and equations.

While the content of Further Mathematics Units 3 and 4 was reviewed, there were no major changes to its structure. The Statistics and probability core area of study was reconceptualised as the Data analysis core area of study; however, the number of modules available was reduced from six to five with the Probability and statistics (sampling and control) module removed. The modules were used to form the Applications area of study. While Outcomes 1 and 2 and their key knowledge and key skill statements applied across both the Core (Data analysis) and Applications (modules) areas of study, separate key knowledge and key skill statements were developed for Outcome 1 Core Data analysis, and each of the Applications modules.

|  |  |
| --- | --- |
| Further Mathematics Units 3 and 4 | |
| *Core* (*40% content weighting*) | ***Modules* (*60% content weighting– three selected from six options)*** |
| Data analysis | Number patterns and applications (including financial applications as one of several modelling scenarios) |
| Geometry and trigonometry |
| Graphs and relations |
| Business-related mathematics |
| Networks and decision mathematics |

For Specialist Mathematics Units 3 and 4, the study description incorporated a more comprehensive list of ‘additional assumed background’, essentially corresponding to the topics geometry in two and three dimensions and trigonometric ratios and their applications.

Specialist Mathematics Units 3 and 4 became completely prescribed with six areas of study:

|  |  |
| --- | --- |
| Specialist Mathematics Units 3 and 4 | |
| A completely prescribed course comprising the following areas of study | |
| Area of Study 1 | Coordinate geometry |
| Area of Study 2 | Circular (trigonometric) functions |
| Area of Study 3 | Algebra |
| Area of Study 4 | Calculus |
| Area of Study 5 | Vectors in two and three dimensions |
| Area of Study 6 | Mechanics |

Selected material from the two modules of the previous study was incorporated. This material included Mechanics (Newton’s laws, with some minor reduction of content covered, and vector calculus, but not simple harmonic motion) and Geometry (vector proofs, analytic geometry and relations and regions in the complex plane, but not deductive two-dimensional geometry). Some new material was also incorporated – on numerical solution of differential equations using technology (including Euler’s method first order approximations).

The VCAA also developed a [discussion paper](http://www.vcaa.vic.edu.au/Documents/bulletin/03DECSU2.pdf) and in 2003 conducted a [major and extended consultation](http://www.vcaa.vic.edu.au/Documents/bulletin/03DECSU2.pdf) on VCE Mathematics examination structures with respect to several possible models, based on international experience (US College Board, Denmark, France, Austria, Switzerland and Germany).

The outcome of this consultation led to the introduction from 2006 of a one-hour technology free examination and a two-hour technology active examination for Mathematical Methods, Mathematical Methods (CAS) and Specialist Mathematics. The two one-and-a-half hour technology active examinations were retained for Further Mathematics.

This was clearly the most strongly supported model, with retention of two technology active examinations for all studies also attracting good support. There was only limited support for a no technology at all or scientific calculator only technology model.

School-based assessment counted for 34 per cent of the final student score. It comprised specified school-assessed coursework tasks (application task, analysis task, test) or SACs for each study and each of Units 3 and 4 as indicated in the following table. As indicated by the asterisk, the bracketed figures show the proportion of the coursework component allocated to these tasks as a mark weighting out of 100.

In this model, Mathematics was distinctive from other VCE studies, with mark weightings assigned to *tasks*, and the three outcomes understood to be applicable across areas of study and the set of tasks. That is, there was a many-to-many relationship between the tasks and the outcomes, naturally applied across and drawing on several areas of study as applicable.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Task  Type | Further Mathematics | | Mathematical Methods | | Specialist Mathematics | |
| **Unit 3** | **Unit 4** | **Unit 3** | **Unit 4** | **Unit 3** | **Unit 4** |
| Application (40)\*  (6–8 hours over 1–2 weeks) | Data analysis core |  | Function and calculus |  |  | Problem-solving or modelling |
| Analysis (20)\*  (2–4 hours over 1 week) | Module 1 | Module 2 Module 3 |  | Task 1 Task 2 (Statistics and probability) | Task 1 Task 2 |  |
| Test (10)\*  (1 hour) |  |  | Test 1 Test 2 |  |  | Test 1 Test 2 |

The bracketed figures indicate the contribution (out of 100) of the school-assessed coursework to the study score.

To provide teachers with some choice and flexibility, there were four types of analysis task; in each study two of the analysis tasks had to be of a different type:

|  |
| --- |
| An assignment where students have the opportunity to work on a broader range of problems, or |
| A short and focused investigation, challenging problem or modelling task, or |
| A set of application questions requiring extended analysis in relation to a particular topic or topics, or |
| An item response analysis for a collection of multiple-choice questions |

This approach was successful in reducing the load associated with coursework as intended, especially given the many-to-many relationship between the tasks and outcomes. Further details about this research are available in Brew et al (MERGA, 2001).

The main issue for the transition was the change from centrally set and prescribed tasks (which teachers had to ‘run’ and assess according to VCAA criteria sheets and marking schemes, with supporting notes) to a small set of tasks that teachers had to create, run and assess within parameters specified by the VCAA. These tasks were to be assessed according to the mark weightings for the outcomes.

Several mechanisms were put in place to support teachers in the transition from work requirements and extended CATs to outcomes and SACs. These included:

* The publication of a larger study design to incorporate both mandatory and advisory content. The latter took the form of an *Advice for teachers* section for each set of units for a study. The advice included teaching sequences, sample courses, learning activities and sample assessment tasks related to the outcomes as well as the areas of study. These advisory materials also included diagrams, graphs, and sample outputs from technology.
* State-wide implementation preparation programs with experienced teachers briefed and trained by the VCAA presenting sample approaches, the ‘this is how I would do it’ model.
* The publication of an advisory *Assessment Guide*, with examples, providing teachers with two models (one based on global descriptors, the other based on criteria) on how they could approach assessment of student work or develop alternative approaches of their own, for example rubrics or marking schemes based on the outcomes.
* Support for the Mathematical Association of Victoria (MAV) to annually run ongoing professional learning workshops based on developing sample assessment tasks.
* Annual publication (1999–2004) in the *VCAA Bulletin* (Supplements) in December of the preceding year for implementation in the following year of suggested themes and possible starting points for the development of application tasks, see, for example, [2002](http://www.vcaa.vic.edu.au/Documents/bulletin/02decsu2.pdf), [2003](http://www.vcaa.vic.edu.au/Documents/bulletin/03DECSU1.pdf) and [2004](http://www.vcaa.vic.edu.au/Documents/bulletin/04DECSU2.pdf) editions. For the Further Mathematics data analysis application task, [data sets](http://www.vcaa.vic.edu.au/Pages/vce/studies/mathematics/further/furthermathindex.aspx) were also provided.
* Partnership with Cambridge publications (following an RFQ process) to have all VCAB/VBOS Mathematics CATs 1989–1999 moved onto editable digital (Word document) format on CD-ROM, and to develop an accompanying set of sample application and analysis tasks to illustrate how teachers could draw on these materials as a basis for writing their own school-based assessment tasks. These can be accessed [here.](http://www.cambridge.edu.au/go/titles/Mathematical-Methods-VCE-Units-34:Cambridge-Senior-Mathematics-Australian-Curriculum---VCE/)

A new model for quality assurance of school-based assessment was adopted for school- assessed coursework. Student results were statistically moderated with respect to examinations, both components of which now addressed the same content in each study. In addition, an audit of tasks was put in place to ensure that schools and teachers were implementing school-assessed coursework in accordance with the requirements of the study design. Student work was no longer inspected; however, the tasks used and the related assessment processes were now under scrutiny. Approximately 20 per cent of the schools teaching each study were audited on a random basis, with all schools effectively audited across a five-year cycle. Some schools were re-audited on a follow-up basis, and schools could be audited and/or re-audited on a discretionary basis as required.

**Use of technology**

An approved graphics calculator was the assumed technology for all VCE Mathematics examinations, except for the Mathematical Methods (CAS) pilot and expanded pilot study, where an approved CAS calculator or CAS software could be used. To use CAS software as a computational tool only, an additional approval process was required to ensure that a suitable computer set up was in place. Students could also use a scientific calculator if desired.

Calculator memories were *not* required to be cleared for mathematics examinations, and in this accreditation period students included supplementary programs or applications developed by third parties and written in the language of the propriety software for each device. The preamble for each unit included a paragraph specifying the expected use of enabling technology, not only for the teaching and learning of mathematics but also for working mathematically.

The *Advice for teachers* sections of the study design (comprising half of the 200 pages) included detailed examples explicitly linking outcomes and areas of study with learning activities and assessment tasks. These materials incorporated various sample inputs and outputs for a range of approved technologies.

**Prerequisite status for tertiary study**

Mathematical Methods and the Mathematical Methods (CAS) pilot were considered equivalent studies for prerequisite purposes. These were the studies used to meet prerequisite requirements for university courses in areas such as medicine, science, engineering, some economics and commerce courses and aviation.

For a small number of courses, Specialist Mathematics was one of a collection of several studies such as Physics or Chemistry, which, in conjunction with Mathematical Methods and Mathematical Methods (CAS), was required to meet prerequisite purposes.

Further Mathematics could be used to meet prerequisite requirements for a range of general courses, but not those requiring a function, algebra and calculus background.

**VCE Mathematics 2.3[[5]](#footnote-5)\*: 2006–2009**

**Structure and assessment**

The general structure of the VCE Mathematics study design, and the particular studies, continued as in the previous accreditation period. There was one main change: Mathematical Methods Units 1–4 and Mathematical Methods (CAS) Units 1–4 were run as parallel and equivalent studies, the former with an approved graphics calculator as the assumed enabling technology, and the latter with an approved CAS calculator or CAS software as the assumed enabling technology. Each study had its own accredited study design developed by review panels with some common members, the CAS study encompassing the non-CAS version with respect to content and generality of approach. Implementation was designed so that at the end of the accreditation period, Mathematical Methods Units 1–4 would lapse and Mathematical Methods (CAS) Units 1–4 would continue. This gave schools that had not been involved in the pilot four years to make the transition, with a large body of resources developed and related professional learning provided by the VCAA, the Mathematical Association of Victoria, universities, technology companies and publishers. During this period publishers produced both Mathematical Methods and Mathematical Methods (CAS) texts. An approved graphics calculator or an approved CAS (calculator or software) was the assumed enabling technology for Further Mathematics and Specialist Mathematics.

The transition period was extended by a year so that all providers implemented Mathematical Methods (CAS) Units 1 and 2 in 2009 and Mathematical Methods (CAS) Units 3 and 4 (from 2010). By this time there was also a set of examination resources ([past papers and reports](http://www.vcaa.vic.edu.au/Pages/vce/studies/mathematics/cas/casexams.aspx#H2N10034)) for Mathematical Methods (CAS) going back to 2002. The growth in schools taking up Mathematical Methods (CAS) Units 1–4 was exponential, from about 1000 students in 2006, essentially doubling each year until 2009 when around 8000 students undertook each version of the study.

Based on the extensive consultation conducted in the previous review, the examination structure for both Mathematical Methods and Mathematical Methods (CAS) and Specialist Mathematics was changed from two one and a half hour examinations, to a one-hour technology free Examination 1, and a two-hour technology assumed Examination 2.

The weightings of question types were also adjusted. Examination 1 had previously comprised an equal weighting of multiple-choice and short-answer questions with an emphasis on Outcome 1 and related aspects of Outcome 1, while Examination 2 comprised extended response questions with some unfamiliar and non-routine components, and an emphasis on Outcome 2 and related aspects of Outcome 3. The consultation indicated a strong preference for a technology free–technology active structure with a greater weighting allocated to the technology active component. The next most supported view was for both examinations to continue to be technology active.

In the revised examination structure the multiple-choice component was reduced and re-allocated to Examination 2. The weighting was adjusted so that Examination 2 had a larger extended response component, with a focus on Outcome 2 and related aspects of   
Outcome 3, while Examination 1 comprised short-answer and some extended response questions, with a focus on Outcome 1.

Mathematical Methods and Mathematical Methods (CAS) had a common Examination 1 based on the content for Mathematical Methods. Mathematical Methods and Mathematical Methods (CAS) Examinations 2 had 70–80 per cent common questions. Students undertook the same Specialist Mathematics examinations with either Mathematical Methods or Mathematical Methods (CAS) as the concurrent study; Examination 2 questions were set to be technology active but graphics calculator/CAS neutral.

For Further Mathematics, with its numerical and graphical approach to practical problems in context, either enabling technology could be used in both of its examinations, the first of which continued to comprise multiple-choice questions and the second short-answer questions.

For School-assessed coursework, Units 3 and 4 Mathematics studies were required to have mark weightings mapped to the *outcomes* rather than to the tasks. This change was to align Mathematics studies with other studies, which were often less content-based than mathematics and had a one-to-one relationship between areas of study, outcomes and tasks for School-assessed Coursework. Thus, the previous table for school-based assessment was reconstructed with the outcomes as the primary construct; the following shows this for Mathematical Methods and Mathematical Methods (CAS) Units 3 and 4:

|  |  |  |  |
| --- | --- | --- | --- |
| Unit 3 | | | |
| Outcome | **Marks** | **Assessment task** | |
| Outcome 1  Define and explain key concepts as specified in the areas of study, and apply a range of related mathematical routines and procedures. | 30 | 15 | Application task |
| 15 | Two tests |
| Outcome 2  Apply mathematical processes in non-routine contexts, and analyse and discuss these applications of mathematics. | 20 | 20 | Application task |
| Outcome 3  Select and appropriately use technology to develop mathematical ideas, produce results and carry out analysis in situations requiring problem-solving, modelling or investigative approaches. | 10 | 5 | Application task |
| 5 | Two tests |
| Total marks 60 (out of a coursework total of 100 contributing 34% to the final score) | | | |

|  |  |  |  |
| --- | --- | --- | --- |
| Unit 4 | | | |
| Outcome | **Marks** | **Assessment task** | |
| Outcome 1  Define and explain key concepts as specified in the areas of study, and apply a range of related mathematical routines and procedures. | 15 | 7 | Analysis task 1 |
| 8 | Analysis task 2 |
| Outcome 2  Apply mathematical processes in non-routine contexts, and analyse and discuss these applications of mathematics. | 15 | 8 | Analysis task 1 |
| 7 | Analysis task 2 |
| Outcome 3  Select and appropriately use technology to develop mathematical ideas, produce results and carry out analysis in situations requiring problem-solving, modelling or investigative approaches. | 10 | 5 | Analysis task 1 |
| 5 | Analysis task 2 |
| Total marks 40 (out of a coursework total of 100 contributing 34% to the final score) | | | |

Prior to 2007, students were unable to enrol in all three Unit 3 and 4 studies (Further Mathematics, Mathematical Methods or Mathematical Methods (CAS), and Specialist Mathematics) in the same year. This was prevented by scheduling examinations for Further Mathematics and Specialist Mathematics concurrently. However, a few hundred students had for several years included all three in their VCE by means of advanced sequencing of General Mathematics Units 1 and 2 in Year 10 and Further Mathematics Units 3 and 4 in Year 11, in conjunction with Mathematical Methods or Mathematical Methods (CAS) Units   
1–4 studied across Years 11 and 12 respectively and Specialist Mathematics Units 3 and 4 in Year 12.

At this time there was a focus at the VCAA in removing undue restrictions in study combinations across all studies. From 2007 students could undertake all three Unit 3 and 4 sequences in the same year, and the Further Mathematics and Specialist Mathematics examinations were unblocked. The result of this is that each year 300 to 400 students include all three sequences in their VCE through the advanced staggered approach, and a similar number include all three sequences in their VCE in the same year.

The VCAA also decided to take a light approach to monitoring and audit of school-assessed coursework. It reduced the scope of the audit and removed specifications around the nature of tasks, with teachers expected to develop tasks broadly in line with the kinds indicated as sufficient in scope and fit for purpose with respect to the outcomes.

For VCE Mathematics studies only the analysis task retained a time specification of 2–4 hours. An effect of this approach across all studies was for teachers to increasingly use school- assessed coursework for examination preparation purposes rather than its original intention to enhance validity. Consequently, it became increasingly difficult for state reviewers to both assess non-compliance and to argue for change in approach to enhance compliance. In VCE Mathematics studies this led to an increasing use of mix-and-match combinations of analysis tasks types, and increasingly closed and directive application tasks. In particular, the ‘non-routine’ and ‘… requiring problem-solving, modelling or investigative approaches’ aspects of Outcomes 2 and 3 respectively were addressed with less diligence and fidelity.

**Content**

The content for Foundation Mathematics Units 1 and 2 was revised, refined and further detailed, within the same areas of study and with the same set of outcomes. The *Advice for teachers* was revised to illustrate how implementation could be based on a series of mathematical investigations in a variety of contexts.

General Mathematics Units 1 and 2 had been thoroughly reviewed, taking into account the extent to which schools and teachers were effectively availing themselves of its flexibility. The structure of the study was retained, with areas of study, topics and content all being revised and refined.

In particular, topics on Combinatorics and Probability that overlapped with Mathematical Methods Units 1 and 2 content were removed. Some topics were re-conceptualised and some new topics supporting preparatory implementation for Specialist Mathematics Units 3 and 4 were introduced. These included:

|  |  |
| --- | --- |
| Generals Mathematics Units 1 and 2 | |
| *Areas of study* | ***Topics*** |
| Arithmetic | Matrices |
| Integer and rational number systems |
| Real and complex number systems |
| Sequences and series |
| Data analysis and simulation | Univariate data |
| Bivariate data |
| Simulation |
| Algebra | Linear relations and equations |
| Non-linear relations and equations |
| Algebra and logic |
| Graphs of linear and non-linear relations | Linear graphs and modelling |
| Sketching and interpreting linear and non-linear graphs |
| Variation |
| Kinematics |
| Decision and business mathematics | Linear programming |
| Financial arithmetic |
| Geometry and trigonometry | Shape and measurement |
| Geometry in two and three dimensions |
| Coordinate geometry |
| Vectors |
| Trigonometric ratios and their applications |

The *Advice for teachers* included several sample courses illustrating implementations for several possible pathways: students wishing to do General Mathematics in its own right as a Year 11 course; as a preparation for subsequent study of Further Mathematics Units 3   
and 4; as a complement to study of Mathematical Methods or Mathematical Methods (CAS); and as a strong preparation for Specialist Mathematics Units 3 and 4 (in conjunction with the Mathematical Methods or Mathematical Methods (CAS), Units 1–4 sequence). Schools would adopt their own informal nomenclature for these courses in their school-based documentation.

Units 3 and 4 in Further Mathematics were again progressively refined and revised; for example, residual analysis as a check of fit was introduced into the Data analysis core, Fibonacci and related sequences were introduced into the Number patterns module, and a new Matrices module was included. The latter was based on broad consultation and involved application such as population modelling, with computation scaffolded by technology. The then Post-compulsory Curriculum and Assessment Committee (PCCAC) approved this inclusion, with a request for monitoring uptake. By the end of the 2010 - 2015 accreditation period it was the most popular module.

|  |  |
| --- | --- |
| Further Mathematics Units 3 and 4 | |
| *Core* (*40% content weighting*) | ***Modules* (*60% content weighting – three selected from six options)*** |
| Probability and statistics | Number patterns |
| Geometry and trigonometry |
| Graphs and relations |
| Business-related mathematics |
| Networks and decision mathematics |
|  | Matrices |

There were also changes to module ‘selection’ practices. Generally, the selection of modules was made by the school or teacher, informed to varying degrees by consultation with students; that is, it was a school/teacher driven process rather than a student driven process. However, increasingly, some students were choosing to independently study an additional module ‘off their own bat’ and in their own time (especially if they didn’t like one in particular chosen by the school or teacher) and would answer the questions related to this module on their examinations. The following tables show the percentage of students for each selected module (totals should round to 300 per cent as there were three selected modules).

|  |  |  |
| --- | --- | --- |
| Module | % 2008 | % 2009 |
| 1. Number patterns | 37 | 37 |
| 1. Geometry and trigonometry | 86 | 85 |
| 1. Graphs and relations | 49 | 49 |
| 1. Business related mathematics | 48 | 46 |
| 1. Networks and decision mathematics | 43 | 46 |
| 1. Matrices | 36 | 48 |

|  |  |
| --- | --- |
| Module | % 2015 |
| 1. Number patterns | 27 |
| 1. Geometry and trigonometry | 65 |
| 1. Graphs and relations | 46 |
| 1. Business related mathematics | 31 |
| 1. Networks and decision mathematics | 49 |
| 1. Matrices | 83 |

The areas of study for Mathematical Methods or Mathematical Methods (CAS) Units 3 and 4 and Specialist Mathematics Units 3 and 4 were rationalised to a broader set of common groupings:

|  |  |
| --- | --- |
| Mathematical Methods/Mathematical Methods (CAS) Units 3 and 4 | |
| A completely prescribed course comprising the following areas of study | |
| Area of Study 1 | Functions and graphs |
| Area of Study 2 | Algebra |
| Area of Study 3 | Calculus |
| Area of Study 4 | Probability |

Mathematical Methods (CAS) subsumed and generalised the content from Mathematical Methods, and included material related to simple functional equations, and the use of matrices for transformations of the plane, systems of simultaneous linear equations and Markov chains (where the functionality of technology enabled these to be computed via matrices). In Probability, the study of the hypergeometric distribution was omitted, with the binomial and normal distributions studied as distinct distributions. However, other discrete and continuous distributions could be defined and used in context for examination questions, as applicable. Probability mass functions for distributions of discrete random variables could be defined through tables or rules, and a range of probability density functions for distributions of continuous random variables could be defined through hybrid functions, and related graphs and integrals obtained using graphic, numeric or symbolic functionality of CAS.

|  |  |
| --- | --- |
| Specialist Mathematics Units 3 and 4 | |
| A completely prescribed course comprising the following areas of study | |
| Area of Study 1 | Functions, relations and graphs |
| Area of Study 2 | Algebra |
| Area of Study 3 | Calculus |
| Area of Study 4 | Vectors |
| Area of Study 5 | Mechanics |

This was the last VCE Mathematics study design published and distributed in print format.

**Use of technology**

An approved graphics calculator was the assumed enabling technology for Further Mathematics, Mathematical Methods and Specialist Mathematics and applicable examinations. An approved CAS (calculator or software) calculator was the assumed enabling technology for Mathematical Methods (CAS). To use CAS software as a computational tool only, an additional approval process was required to ensure that a suitable computer set up was in place.

**Prerequisite status for tertiary study**

Mathematical Methods and Mathematical Methods (CAS) pilot were equivalent studies for prerequisite purposes. These were the studies used to meet prerequisite requirements for university courses in areas such as medicine, science, engineering, some economics and commerce courses and aviation. For a small number of courses, Specialist Mathematics was one of a collection of several studies such as Physics or Chemistry, which, in conjunction with Mathematical Methods or Mathematical Methods (CAS), was required to meet prerequisite purposes.Further Mathematics could be used to meet prerequisite requirements for a range of general courses, but not those requiring a function, algebra and calculus background.

**VCE Mathematics 2.4[[6]](#footnote-6)\*: 2010–2015**

**Structure and assessment**

The start of thisaccreditation period corresponded to the final stage of the transition from Mathematical Methods to Mathematical Methods (CAS), with no change to the structure of the study, areas of study, content, outcomes or related assessment of any of the other VCE Mathematics studies. Mathematical Methods was removed and Mathematical Methods (CAS) Units 1–4 retained, with the *Advice for Teachers* having some minor revisions. This corresponded with the Mathematics study design being the first VCE study design to be available (from 2010) only in digital format. A hard copy from the pdf file could be down loaded and printed if desired. The *Assessment Handbook* was also updated to reflect the transition from Mathematical Methods or Mathematical Methods (CAS), to only Mathematical Methods (CAS). The (CAS) post nominal was retained for this accreditation period and all past study materials referring to Mathematical Methods removed from the VCAA website.

The VCE Mathematics study was due for its VCAA review in 2010 (preliminary work had been undertaken in the form of several invited discussion papers from 2008). However, this process was deferred with the advent of the [Australian Curriculum](http://www.australiancurriculum.edu.au/seniorsecondary/overview) (AC) collaborative development process between the states and territories and the Commonwealth. During this time Unit 1–4 sequences across Years 11 and 12 were developed for four subjects: Essential Mathematics, General Mathematics, Mathematical Methods and Specialist Mathematics. The discussion papers and other materials were made available to ACARA to assist in the AC development process. As work was undertaken, the accreditation period for the then current suite of VCE Mathematics studies was progressively extended on an annual basis to 2015. As the AC work concluded, the states and territories re-accredited their senior secondary curriculums accordingly.

In Victoria this work took place through 2013 and 2014, starting with endorsement of a [proposed directions paper](https://www.google.com.au/url?sa=t&rct=j&q=&esrc=s&source=web&cd=1&cad=rja&uact=8&ved=0ahUKEwiao--q_YLUAhXEKJQKHQTkDgQQFgglMAA&url=http%3A%2F%2Fwww.vcaa.vic.edu.au%2Fdocuments%2Fvce%2Fvceconsult%2Fproposed-directions-review-of-english-history-maths-and-science-discussion-paper.pdf&usg=AFQjCNH8cc5umnmomEGyUbRzaCNNpfx0vA) (June 2013) and subsequent consultations, leading to the current suite of VCE Mathematics studies.

From 2011 to 2015 the VCAA undertook a [Computer Based Examination trial](https://www.researchgate.net/profile/David_Leigh-Lancaster/publications) (CBE) for Mathematical Methods (CAS). This involved using the CAS software *Mathematica* and specially constructed applications (notebooks programmed in *Mathematica*) to develop a model that produced the examination paper (the Production Palette) and then ran the examination (the Student Palette) as a notebook with *Mathematica* as the computational tool naturally accessed in the exam. The trial was completed successfully and the VCAA proceeded to a phase of [expanded implementation](http://www.vcaa.vic.edu.au/Pages/correspondence/notices/2016/96.aspx). The Computer Based Examination (CBE) has the same questions as the paper-based examination; however, it uses a digital mode of delivery and response. It is *not* an online examination.

While the approach to school-assessed coursework continued, the model for audit was changed to incorporate a two-stage process for each of Unit 3 and Unit 4. Initially, audited schools responded to an online questionnaire (effectively a self-audit) followed up by requests for further evidence and a full audit based on state reviewer evaluation of the self-audit responses. A small random sample of schools with satisfactory questionnaire responses continued to be subject to full audit. The issues identified previously continued to be relevant, with some schools included in follow-up audits due to non-compliance.

**Content**

The content of VCE Mathematics studies was unchanged.

**Use of technology**

The assumed enabling technology for Mathematical Methods (CAS) and Specialist Mathematics was an [approved](http://www.vcaa.vic.edu.au/Documents/bulletin/2010OCTBULL.pdf) CAS (calculator or software) and for Further Mathematics an approved graphics calculator or an approved CAS (calculator or software). To use CAS software as a computational tool only, an additional approval process was required to ensure that a suitable computer set up was in place.

**Prerequisite status for tertiary study**

Mathematical Methods (CAS) was the study used to meet prerequisite requirements for university courses in areas such as medicine, science, engineering, some economics and commerce courses and aviation.

For a small number of courses, Specialist Mathematics was one of a collection of several studies such as Physics or Chemistry, which, in conjunction with Mathematical Methods or Mathematical Methods (CAS), was required to meet prerequisite purposes.

Further Mathematics could be used to meet prerequisite requirements for a range of general courses, but not those requiring a function, algebra and calculus background.

**VCE Mathematics 2.5[[7]](#footnote-7)\* 2016–2019/20**

**Structure and assessment**

The VCE Mathematics study essentially skipped a review cycle from 2010–15, with an extended accreditation period running from 1 January 2006 to 31 December 2015.

Thus, the review undertaken in 2013–2014 initially led to a shorter accreditation period 2015–2018, while strategic in-depth review work continued, with the intention of maintaining momentum for a more in-depth review informed by developments in society, mathematics and mathematics education.

Nonetheless, the 2013–2014 review provided the opportunity and stimulus for several key structural and content developments, based on extensive consultation with respect to the AC work with key stakeholders, but also more in-depth strategic consultation with leaders in education, business, [academia](https://theconversation.com/optimising-the-future-with-mathematics-22122), industry and research. Four key focuses emerged:

* the importance of stochastic processes, [developments in statistics,](https://theconversation.com/new-statistical-methods-would-let-researchers-deal-with-data-in-better-more-robust-ways-67981) and the familiarity and capability of students with these and related reasoning
* the importance of algorithmic thinking and coding (broadly interpreted)
* the importance of discrete mathematics
* the effective use of technology for computation and working mathematically, and a good sense of reasonableness that is essential for such use.

These align with the views of various experts and commentators in these fields from around the world. For each VCE Mathematics study, a major curriculum and assessment benchmarking report was produced with respect to several jurisdictions internationally (an Australian state and territory benchmarking was conducted as part of the AC work). The range of jurisdictions varied in relation to whether similar courses existed for each study. The inter-relationship between pathways was also considered in meta-analysis of the benchmarking work.

Developments in technology have meant that so called CAS devices are now multi-functionality devices that handle numeric, graphic, statistical, geometric and symbolic computations, and can deploy the relevant functionality as applicable to the problem at hand. Students also have ready access to software such as [Wolfram Alpha](https://www.wolframalpha.com/) on personal devices such as iPads, tablets and mobile phones. Through a Department of Education and Training (DET) initiative, all Victorian secondary teachers and students (irrespective of sector, government, catholic or independent) currently have free at school and at home access to [Mathematica and Wolfram Alpha Pro.](http://wolfram.global2.vic.edu.au/) An approved CAS is the assumed technology for VCE Unit 3 and 4 Mathematics studies.

The revised [VCE Mathematics study design](http://www.vcaa.vic.edu.au/Documents/vce/mathematics/MathematicsSD-2016.pdf) is a smaller 90-page digital pdf document with heading links, containing only mandated material, with the corresponding *Advice for teachers* published digitally on the web, for example [Further Mathematics](http://www.vcaa.vic.edu.au/Pages/vce/adviceforteachers/furthermaths/introduction.aspx), [Mathematical Methods](http://www.vcaa.vic.edu.au/Pages/vce/adviceforteachers/mathsmethods/introduction.aspx) and [Specialist Mathematics](http://www.vcaa.vic.edu.au/Pages/vce/studies/mathematics/specialist/specialmathindex.aspx). The digital publication has enabled multiple links within the sections of the advice web pages to various VCAA documents (such as pas[t exams and reports](http://www.vcaa.vic.edu.au/Pages/vce/studies/mathematics/cas/casexams.aspx), and the [Administrative Handbook](http://www.vcaa.vic.edu.au/Pages/schooladmin/handbook/handbook.aspx)) and to other resources. Teachers can access a [sample course implementation](http://www.vcaa.vic.edu.au/Pages/vce/adviceforteachers/mathsmethods/samplecourse.aspx) with links to such [resources](http://www.amsi.org.au/ESA_Senior_Years/SeniorTopic4/4_md/SeniorTopic4g.html), sample assessment tasks for each of [Further Mathematics](http://www.vcaa.vic.edu.au/Pages/vce/adviceforteachers/furthermaths/units_assessment_tasks.aspx), [Mathematical Methods](http://www.vcaa.vic.edu.au/Pages/vce/adviceforteachers/mathsmethods/units_assessment_tasks.aspx) and [Specialist Mathematics](http://www.vcaa.vic.edu.au/Pages/vce/adviceforteachers/specialistmaths/units_assessment_tasks.aspx), which can be downloaded as use as editable templates (click-copy-paste-modify) for teacher devised tasks, and [sample assessment criteria and record sheets](http://www.vcaa.vic.edu.au/Pages/vce/adviceforteachers/mathsmethods/units_performance_criteria.aspx). These materials can and have been updated from time to time, for example, the inclusion of additional sample assessment tasks in 2017 and 2018.

These sample tasks include a summary table indicating content covered from the areas of study and key knowledge and key skills addressed with respect to each of the outcomes.

The issues identified with school-assessed coursework during 2006–2015 across VCE studies, including mathematics, were further addressed in part by:

* re-introduction of more detailed specification of task nature, purpose and scope
* additional sample task materials
* increased audit of schools.

In mathematics these issues were also addressed specifically by reducing the range of possible tasks to only two task types: an *application task* and a *modelling or problem-solving task*, across all Units 3 and 4 mathematics studies, as indicated in the following table.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Task  type | Further Mathematics | | Mathematical Methods | | Specialist Mathematics | |
| **Unit 3** | **Unit 4** | **Unit 3** | **Unit 4** | **Unit 3** | **Unit 4** |
| Application task (4–6 hours over  1–2 weeks) | Data analysis core (40)\* |  | Function and calculus (50)\* |  | Mathematical investigation (50)\* |  |
| Modelling problem-solving task (2–3 hours over (1 week) | Financial modelling and recursion core (20)\* | Module 1 (20)\*  Module 2 (20)\* |  | Task 1 (25)\*  Task 2 Probability and statistics area of study (25)\* |  | Task 1 (25)\*  Task 2 Mechanics or Probability and statistics area of study (25)\* |

\*The bracketed figures indicate the contribution (out of 100) of the School-assessed Coursework to the study score.

Additionally, following the first year of implementation of a revised study, a coursework audit report was published for each study: [Further Mathematics](http://www.vcaa.vic.edu.au/Documents/vce/mathematics/Further_mathematics_SAC_report.doc), [Mathematical Methods](http://www.vcaa.vic.edu.au/Documents/vce/mathematics/Maths_Methods_SAC_report.docx) and [Specialist Mathematics](http://www.vcaa.vic.edu.au/Documents/vce/mathematics/Specialist_Mathematics_SAC_report.docx) along with a [companion document](http://www.vcaa.vic.edu.au/Documents/vce/mathematics/Maths_Methods-modelling_problem_solving.docx) on modelling and problem-solving. The coursework audit surveys were also refined to direct the teachers completing them to the relevant documentation (and to indicate that they had done so) at the beginning of the survey.

**Content**

Foundation Mathematics underwent a major review in 2013–2014, following a minor review in 2003–2004. A key question following on from the AC process was whether a Unit 3 and 4 sequence should be developed for Foundation Mathematics, as in the Australian Curriculum Essential Mathematics Units 1–4.

The review panel noted that the original rationale for Foundation Mathematics had been very successful in developing a course that attracted and retained a cohort of around 5000 students, most of whom successfully completed this practical context-based Year 11 standard mathematics. Part of the rationale for the decision *not* to extend this to a Unit 3   
and 4 sequence was that at this level the VCE structure of examinations and coursework would apply, and students of this cohort might not perform as well under examination conditions.

There was also concern that the AC Essential Mathematics Units 1–4 was a bit like an AC General Mathematics Units 1–4 ‘lite’ subject with less breadth and depth of content and without the contextual focus that made Foundation Mathematics Units 1 and 2 successful for its cohort. While the Foundation Mathematics Units 1 and 2 study design made explicit that the study was not intended, nor suitable as preparation, for Further Mathematics Units 3 and 4 (General Mathematics Units 1 and 2 being the suitable pathway), informally students who had done well in their study of Foundation Mathematics and gained confidence in the process could, with the support of their teacher, undertake additional study of the relevant content (typically in Unit 2) to prepare to enrol in Further Mathematics Units 3 and 4 in the following year. However it was noted that this would be subject to monitoring and further consideration in subsequent review.

Based on the work of the review panel this pathway was formalised. The additional study was specified in terms of General Mathematics Units 1 and 2 topics and their key knowledge and skills required as background for the Further Mathematics Units 3 and 4 Core area of study. Some Australian jurisdictions such as Queensland have, as part of their senior secondary review cycles, developed an Essential Mathematics Units 3 and 4 have made it a ‘Group 2’ type subject, which cannot count towards a tertiary entrance score as a primary subject, while South Australia has developed an Essential Mathematics course that does count as a primary subject.

The content and outcome key knowledge and key skills for Foundation Mathematics Units 1 and 2 were substantially refined, revised and updated with respect to contemporary workplace practice and numeracy in everyday life, within the following areas of study:

|  |  |
| --- | --- |
| Foundation Mathematics Units 1 and 2 | |
| A completely prescribed course comprising the following areas of study | |
| Area of Study 1 | Shape, space and design |
| Area of Study 2 | Patterns and number |
| Area of Study 3 | Data |
| Area of Study 4 | Measurement |

General Mathematics Units 1 and 2 retained the same nature, purpose and scope; however, topics within the areas of study were significantly revised, and a complementary separate study for Specialist Mathematics Units 1 and 2 developed and accredited. The structure of Specialist Mathematics Units 1 and 2 is the same as for General Mathematics Units 1 and 2 with some additional specification of required topics. Around one in three of all VCE providers were only able to offer one class of the former General Mathematics Units 1 and 2, this usually being a preparatory implementation for Further Mathematics Units 3 and 4. However, the flexible nature of the study meant that a small number of students within such a class could undertake some alternative topics as helpful additional preparation for Specialist Mathematics Units 3 and 4. During the 2013–2014 review cycle, it was decided to:

* retain the existing structure for General Mathematics Units 1 and 2
* introduce a new study Specialist Mathematics Units 1 and 2 with the same structure and additional design requirements (prescribed topics)
* substantially re-develop the range of topics from the previous General Mathematics
* have the same areas of study and allow cross-selection of topics between the two studies
* formalise requirements for preparatory topics or subsequent study of Further Mathematics Units 3 and 4 or/and Specialist Mathematics Units 3 and 4.

For General Mathematics Units 1 and 2, to suit the range of students entering the study, content must be selected from the six areas of study using the following rules:

* for each unit, content covers four or more topics in their entirety, selected from at least three different areas of study
* courses intended as preparation for study at the Units 3 and 4 level should include a selection of topics from areas of study that provide a suitable background for these studies
* topics can also be selected from those available for Specialist Mathematics Units 1   
  and 2
* content covered from an area of study provides a clear progression in knowledge and skills from Unit 1 to Unit 2.

For Specialist Mathematics Units 1 and 2, to suit the range of students entering the study, and cover the four prescribed topics, content must be selected from the six areas of study using the following rules:

* for each unit, content covers four or more topics in their entirety, selected from at least three different areas of study
* each unit must include two of the prescribed topics: Number systems and recursion; Vectors in the plane; Geometry in the plane and proof; and Graphs of non-linear relations
* other topics can be selected from those included in the areas of study for Specialist Mathematics Units 1 and 2 and/or General Mathematics Units 1 and 2
* courses intended as preparation for study at the Units 3 and 4 level should include selection of content from areas of study that provide a suitable background for these studies
* content from an area of study provides a clear progression in knowledge and skills from Unit 1 to Unit 2.

The content is summarised in the following table:

| *Areas of study* | *Topics* (all available for both studies) | |
| --- | --- | --- |
|  | General Mathematics | Specialist Mathematics |
| 1: Algebra and structure | Linear relations and equations (assumed for Further Mathematics Units 3 and 4) | Logic and algebra  Transformations, trigonometry and matrices |
| 2: Arithmetic and number | Computation and practical arithmetic (assumed for Further Mathematics Units 3 and 4)  Financial arithmetic | Number systems and recursion (prescribed topic, assumed for Specialist Mathematics Units 3 and 4)  Principles of counting |
| 3: Discrete mathematics | Matrices  Graphs and networks  Number patterns and recursion (assumed for Further Mathematics Units 3 and 4) | Graph theory |
| 4: Geometry, measurement and trigonometry | Shape and measurement  Applications of trigonometry | (prescribed topic, assumed for Specialist Mathematics Units 3 and 4)  Vectors in the plane  (prescribed topic) |
| 5: Graphs of linear and non-linear relations | Linear graphs and models  (assumed for Further Mathematics Units 3 and 4)  Inequalities and linear programming  Variation | Graphs of non-linear relations  (prescribed topic) |
| 6: Statistics | Investigating and comparing data distributions  (assumed for Further Mathematics Units 3 and 4)  Investigating relationships between two numerical variables (assumed for Further Mathematics Units 3 and 4) | Simulation, sampling and sampling distributions |

Various [sample courses](http://www.vcaa.vic.edu.au/Pages/vce/adviceforteachers/generalmaths/samplecourse.aspx) for General Mathematics Units 1 and 2 and [sample courses](http://www.vcaa.vic.edu.au/Pages/vce/adviceforteachers/specialistmaths/samplecourse_units1and2.aspx) for Specialist Mathematics Units 1 and 2 are given in the corresponding *Advice for teachers*. For each module in Further Mathematics Units 3 and 4 there are related preparatory topics in General Mathematics Units 1 and 2.

Further Mathematics Units 3 and 4 was re-structured so that the core area of study now includes financial modelling and recursion, in response to broader expectations of financial literacy for all students in such a course. This had an equivalent weighting to one module, and replaced aspects of the former Business-related mathematics and Number patterns modules, although it was developed from scratch rather than as a ‘blend’ of the two former modules. The data core content was updated and revised, as were the remaining four modules.

|  |  |
| --- | --- |
| Further Mathematics Units 3 and 4 | |
| *Core area of study* (60% *content weighting*) | *Applications area of study – modules* (40% *content weighting – two selected from four options)* |
| Data analysis  (40% content weighting)  Financial modelling and recursion  (20% content weighting) | Matrices |
| Networks and decision mathematics |
| Geometry and measurement |
| Graphs and relations |

As Mathematical Methods Units 1–4 assume CAS enabling technology, content was revised, simplified and generalised. Numerical methods is now covered in Units 1 and 2, with a greater focus on functional relations, while elementary statistical inference based on sample proportions is covered in Units 3 and 4 as a practical application of binomial and normal distributions. Various minor topics were also ‘pruned’ to accommodate these changes. Expectations for by-hand computation were articulated through the key knowledge and key skill statements for Outcome 1. The Probability area of study once again became the Probability and statistics area of study; however, now technology could be used to effectively carry out related simulations simply and effectively, and construct sampling distributions and undertake similar tasks.

The Australian Mathematical Sciences Institute (AMSI) developed a comprehensive range of [supporting resources](http://amsi.org.au/ESA_Senior_Years/SeniorTopic4/4_md/SeniorTopic4.html) for teacher content knowledge and pedagogical knowledge with respect to implementation of this revised content, which was last incorporated in the 1996–1999 accreditation of Mathematical Methods. The inclusion of this content followed on from the AC work, the input of key stakeholders, and benchmarking, which indicated similar developments in other jurisdictions and systems.

|  |  |
| --- | --- |
| Mathematical Methods Units 3 and 4 | |
| A completely prescribed course comprising the following areas of study | |
| Area of Study 1 | Functions and graphs |
| Area of Study 2 | Algebra |
| Area of Study 3 | Calculus |
| Area of Study 4 | Probability and statistics |

Specialist Mathematics Units 3 and 4 assumes previous or concurrent study of Mathematical Methods Units 3 and 4 (and prior to this Units 1 and 2), as well as knowledge and skills from the two prescribed and assumed topics of Specialist Mathematics Units 1 and 2. It comprises the following six areas of study:

|  |  |
| --- | --- |
| Specialist Mathematics Units 3 and 4 | |
| A completely prescribed course comprising the following areas of study | |
| Area of Study 1 | Functions and graphs |
| Area of Study 2 | Algebra |
| Area of Study 3 | Calculus |
| Area of Study 4 | Vectors |
| Area of Study 5 | Mechanics |
| Area of Study 6 | Probability and statistics |

As with Mathematical Methods, Specialist Mathematics Units 3 and 4 assume CAS enabling technology. Content was revised, simplified and generalised, and elementary statistical inference based on sample means was introduced in Units 3 and 4, with several topics ‘pruned’ to accommodate these changes. Expectations for by-hand computation were articulated through the key knowledge and key skill statements for Outcome 1.

The Probability and statistics area of study (including topics: linear combinations of random variables, sample means, confidence intervals for means, hypothesis testing) was introduced, with technology used to effectively carry out simulations, and construct sampling distributions and undertake similar tasks.

**Use of technology**

The assumed enabling technology for Further Mathematics, Mathematical Methods and Specialist Mathematics is an approved CAS (calculator or software). The list of approved technology has been simplified to refer to a [range of models or software within a given type](http://www.vcaa.vic.edu.au/Pages/correspondence/bulletins/2016/October/vce_assessment.aspx#5) that have suitable functionality, rather than individual models.

CAS software can be used as a computational tool only for applicable examinations, subject to an approval process to ensure that a suitable computer set up is in place.

While not formally required as assessable content, programming in various languages (for example [Python](https://www.python.org/), [Wolfram Language](https://www.wolfram.com/language/), [Lua](https://education.ti.com/en/resources/lua-scripting), [RPL/HP PPL](http://www.hp-prime.de/en/category/10-programming)) has been incorporated in course implementation to illustrate algorithms that are related to content in mathematics studies, such as recurrence relations in Financial modelling and recursion, [Kruskal’s](https://visualgo.net/en/mst) or [Prim’s](https://visualgo.net/en/mst?slide=1) in networks, [numerical equation solving methods](https://www.lua.org/cgi-bin/demo?bisect) (bisection and Newton’s) and [Euler’s method](http://www.me.rochester.edu/~clark/ME163Web/webexamp/euler.pdf) for differential equations.

**Prerequisite status for tertiary study**

Mathematical Methods continues to be the study used to meet prerequisite requirements for university courses in areas such as medicine, science, engineering, some economics and commerce courses and aviation.

For a small number of courses, Specialist Mathematics is one of a collection of several studies such as Physics or Chemistry, which, in conjunction with Mathematical Methods, is required to meet prerequisite purposes.

Further Mathematics can be used to meet prerequisite requirements for a range of general courses, but not those requiring a function, algebra and calculus background.

1. \* The use of this labelling is a notational convenience, not part of a formal course title. [↑](#footnote-ref-1)
2. \* The use of this labelling is a notational convenience, not part of a formal course title. [↑](#footnote-ref-2)
3. \* The use of this labelling is a notational convenience, not part of a formal course title. [↑](#footnote-ref-3)
4. \* The use of this labelling is a notational convenience, not part of a formal course title. [↑](#footnote-ref-4)
5. \* The use of this labelling is a notational convenience, not part of a formal course title. [↑](#footnote-ref-5)
6. \* The use of this labelling is a notational convenience, not part of a formal course title. [↑](#footnote-ref-6)
7. \* The use of this labelling is a notational convenience, not part of a formal course title. [↑](#footnote-ref-7)