

What could a suitable senior secondary mathematics curriculum for a liberal democratic society in a developed country for 2020 – 2030 look like?



Wolfram – ComputerBasedMath.Org

Discussion Paper

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### Annex A:

Example module titles showing linkages to STEM subjects, the concepts, tools and outcomes acquired

### Annex B:

CBM Outcomes - 11 dimensions.

# 1 The current educational landscape and why it needs to change

## 1.1 Mainstream school mathematics

At no time in history has new machinery threatened to take over from humans as it does now. Previous eras of mechanisation have been largely confined to replacing then scaling up physical activities. Instead computers are continuing to take over intelligence and knowledge-based activities—areas previously considered quintessentially human.

How should education react? Do we still need to learn skills that computers now perform? If not, what should we learn instead?

It is strongly our view that standing on new powers of automation and enabling humans to go further and take on new challenges is the urgent priority, not trying to continue to do tasks that compete with them. This means learning to handle harder, more complex problems earlier (to mimic growing complexity in the real world) as well as gaining experience of managing and interfacing with our new machinery (computers and artificial intelligence (AI)). It also means jettisoning most of skills that computers take over.

In mainstream school education, mathematics is starkly at the centre of this issue: it's the core technical subject - and curricula everywhere still retain hand-calculating as their focus. Yet in the real world—where maths skills are so coveted—almost all calculating is by computer adding much more conceptual complexity and very different approaches for which students are today ill-prepared. School maths today is perhaps 80% content that will not be used outside education and however well that subject is taught with whatever IT provision in its pedagogy, it will still fail to match what's now needed.

For all education, in a rapidly changing world, particularly one with increasing AI, we regularly need to be answering both what are “Today's human survival skills?” and what are “Top human value-adds” or we rapidly find a mismatch between what's learnt at school and what's in fact needed both by individuals and more generally for society.

One such core human skill that attaches to both is “Computational Thinking”. A recently rejuvenated term, it is being heralded as a new imperative of education and we believe it is a useful banner for the core technical school subject that combines many of the needed aspects of today's maths, computer-based maths and coding.

We see ability at computational thinking (with a modern toolset) as an important strand while our most affluent societies transition from Knowledge Economies (in which direct knowledge is the key driver to success) to what we term Computational Knowledge Economies (in which knowledge of applying computational thinking is key).

Should “Computational Thinking” replace “Maths” as the core subject in a modern secondary curriculum? The terminology is largely a political decision driven by whether reform or restart is the best route, but in essence a change to this subject is the key element needed to achieve a successful “mathematics curriculum for a liberal democratic society in a developed country for 2020 – 2030”.

We detail the case, the practicalities and the achievements and indeed the necessity of such a position in the remainder of this submission. There's a good reason why we may take a more radical and different view from many in maths or education communities. For nearly 30 years, Wolfram has been at the centre of mathematics worldwide in more ways than any other organisation: as employers, suppliers of technology, users of mathematics for creating technology, and including the world's companies, governments, universities, schools as customers for doing mathematics. Wolfram is sometimes credited with having strongly contributed to the increased use of

mathematics in the real-world during the last few decades. It is out of this uniquely broad and real-world basis for understanding the world's mathematics that our views have been formed.

It is interesting to note that in 1988 Steve Jobs expressed what should be achieved by our technology such as ours but while this shift has been more evident than anyone predicted in the real-world, it is still to materialise in education.

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*"Mathematica will revolutionize the teaching and learning of math by focusing on the prose of mathematics without getting lost in the grammar."*

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## 1.2 How is today's real-world mathematics different?

The very reason mathematics is seen as so important in education today is because mathematical or computational thinking skills appear so widespread and deeply embedded in all walks of life. But this is a relatively new phenomenon. Before mechanised computing, mathematics use above very basic arithmetic was much narrower: only applicable to fields such as some of areas of physics and accountancy. It did not work well on anything requiring larger amounts of data, messier or less immediately quantitative problems. Computers have made this fundamental change and without them many other fields now and rapidly emerging would not exist, for example:

- Computational biology
- Experimental maths
- Data-driven physics
- Medical imaging
- Virtual prototyping
- Archaeological surveying
- Signal processing
- Encryption and compression
- Machine learning
- Data science

It is therefore crucial to understand these real-world changes to the use of mathematics if we are to consider what the educational subject should be. And it's no less crucial to ensure that computers are available and assumed for that educational subject. Without them, real, modern, complex contexts cannot be achieved, just like they could not be outside education.

It should be noted that while data science is a broad field that underlies many of these areas and contexts, its current prominence should not eclipse other toolsets for a computational thinking approach.

## 1.3 The broader context of computers in education

In 2012, 96% of 15-year-old students in OECD countries reported that they have a computer at home, but only 72% reported that they use a desktop, laptop or tablet computer at school. Australia was top for students browsing the internet for schoolwork at least once a week at 80.8% (OECD, 2015). The report summarises "PISA results show no appreciable improvements in student achievement in reading, mathematics or science in the countries that had invested heavily in ICT for education."

It is easy to be misled in analysing this observation. Most educational use has been focused on automating pedagogy not changing the subject taught. Whether they have been used effectively in

pedagogy is not directly connected with whether they are critical to teaching the right subject. Almost all the research relating to the use of computers in education fails to separate these fundamentally different aspects or explicitly only considers how computers have affected pedagogy with implicit assumption that the subject to be taught is not transformed too. This research therefore must not inform fundamental changes needed in the subject of mathematics to make it computer-based.

Actually, in our view, there are many possible pedagogical benefits of introducing computers to the classroom and many reasons teachers use them: to provide context, dynamic simulation and ease of communication. However, where the students themselves use the computers in maths today, the tasks are generally around *supporting the learning of hand-manipulation procedures* of the current curriculum.

This is role-reversal of the real-life where the computer is the tool for computation and the human should be instructing what computation it does. When students have access to computers, their time can be spent learning how to manage the computer, often involving programming, and not primarily using the time watching the computer present material that tells the student what to do.

In poorly managed environments, worse problems can accrue. The computer opens up a world of distraction from the intended educational objective. Which may be a factor behind the finding "... levels of computer use above the current OECD average are associated with significantly poorer results." Cause or effect? Is the reason there are poorer results due to the use of computers? Or does increased computer use stem from teaching students with poorer results, already disengaged with the traditional subjects, which in the case of maths may be made contextualised and engaging by being computer-based?

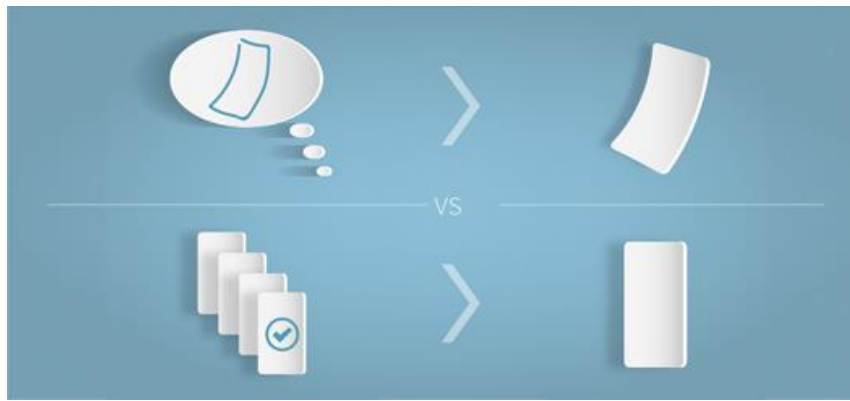
We concur with what the OECD adds so far as it goes: "One interpretation of these findings is that it takes educators time and effort to learn how to use technology in education while staying firmly focused on student learning." However the key issues remain - separation of subject from pedagogy and understanding of the extent of its divergence from the real-world.

#### 1.4 Innovation and evidence

It is crucial to understand how evidence can be employed to improve standards and promote not stifle innovation which we see as a major issue in a required change of educational subjects. Broadly, we'd categorise this as two different approaches: "innovation-led evidence" and "evidence-led innovation". Education has long insisted on the latter while fundamental innovation in all sectors requires the former, exposing a key ecosystem problem in achieving particularly subject change.

The difference in approaches is whether you build your "product" (e.g. phone, drug, curriculum) first, then test it (using those tests for iterative refinement or rejection) or whether formal evidence that exists from previous products becomes the arbiter of any new products you build.

The former—"innovation-led evidence"—is highly productive in achieving outcomes, though of course care must be taken that those outcomes represent your objectives effectively. The latter—"evidence-led innovation" almost by definition excludes fundamental innovation because it means only building stuff that past evidence said would work.



When you build something significantly new it isn't just a matter of formally assembling evidence from the past in a predictable way. A leap is needed, or several. Different insights. A new viewpoint. Often in practice these will occur from a mixture of observation, experience and what still appears to be very human-style intelligence. But wherever it comes from, it isn't straightforwardly "evidence-led".

"Evidence-led innovation" stifles major innovation - it locks out the guess -yet that's what most of "evidence-led education" while often not applying much "innovation-led evidence".

In an age of massive real-world change, correctly and rapidly reflecting this change in education is crucial to future curricula, their effective deployment, and achieving optimisation for the right educational outcomes.

Considering how to make systemic change to promote innovation is crucial to a 2020-30 curriculum for Victoria, not only for the initial changes but to continually upgrade through that period and beyond.

## 1.5 Assessment

There are really two underlying reasons why assessment needs to change.

First, assessments are so high-stakes that agreement of points scored, and ease of delivery and efficiency of marking, often become the dominant driver.

In mathematics, the adherence to calculation-based assessment (closed-ended, right-or-wrong) - exams for maths does not match open-ended messier problems in real life and often stops computers being available for calculating. This is particularly driven in most countries by 'point system' approaches to university entrance, based on assessments which lend themselves to drill-and-practice learning. These assessment lynchpins drive behaviours in both children and parents to such an extent that the introduction of any fundamentally new content, particularly an open-ended, problem-solving curriculum, is exceptionally challenging.

Secondly, across the assessment system, there is major inertia preventing content change, from perceived risks for policy-makers, through to a wish for continuity for comparison between years.

Sometimes, as in Estonia and Ireland, there is a government policy-level vision for curriculum change, but even then, shifting assessments, particularly for university admissions, is often the key.

## 1.6 Enfranchisement

It is clear that today's mathematics curricula work well only for a relatively small fraction of the population often showing worse performance for girls, ethnic minorities and those from disadvantaged backgrounds even when compared to their performance on other subjects. A key problem is how abstract the subject appears at the outset. Those from disadvantaged background often lack the confidence initially to push through abstract ideas ahead of seeing their context and yet this is exactly how almost all today's maths curricula work.

Instead, by starting with a problem, and using abstraction so that mathematical computation can be applied to that problem, many new groups become engaged. Even today's "problem-led" curricula often appear abstract because the problems cannot reflect real-world complexity as they have to assume hand-calculating for their computation. The mathematics curriculum being computer-based is therefore vital for improving enfranchisement so abstraction can be seen as a crucial life skill for problem-solving not a way to put off students from trying to solve problems.

Not only does abstraction lead today's curriculum, but so does the "mechanics-inside" of the mathematics. Whilst this is attractive to a small group of students, often boys, it is highly off-putting to those not specifically interested upfront in the mechanics, particularly girls.

## 2 Effecting change

### 2.1 The nature of a 2020-2030 Curriculum and delivery of a specification

Although change to a computer-based maths, problem-centric subject is the key focus needed for a 2020-2030 curriculum, delivering it well entails a change to how curricula are constructed, manifested and deployed to teachers and students

Traditionally curricula have been written as specifications for what to cover ranging in scope from general topic areas (e.g. algebra) to specifics (e.g. recalling and applying the quadratic formula). Different jurisdictions opt for more or less prescriptiveness in *how* to cover what is specified. From this specification teaching materials are then deduced, for example books, assessments and example examinations - sometimes by the same organisation or closely connected others. They typically take the material and produce content that covers the scope of the course and will identify discrete problems that enable students to drill, practice and learn techniques. The content will usually mix mathematical objects (e.g. equations) with techniques and procedures (e.g. differentiation) but often without explicit delineation.

There are notable problems with this approach when thinking of a modern, computer-based maths curriculum. Firstly this new subject is highly complex to map out as those capable of so doing are all trained traditionally and therefore find it hard to ask whether a topic to be covered is genuinely needed or simply the basis of each contributor's background. Even at Wolfram, with the wealth of experience of real-world maths already highlighted, we have been unable to prescribe what's areas are needed without walking through actual problems.

Secondly, the delivery of a curriculum needs to involve scaffolding with interactivity, coding and ideally summative assessment. The exact learning outcomes are heavily influenced by how this richer interaction with the machinery occurs and is very inefficient and ineffective to write down in a specification upfront rather than by making the materials that represent this.

Thirdly, production of good materials requires a mixture of software and pedagogical skills and is typically much more expensive in isolation than are traditional materials such as books. Separating these tasks from curriculum specification is unlikely to be cost-effective, produce high quality or deliver results in a reasonable timeframe. To the contrary, a materials-led computer-based curriculum can be less expensive to deliver overall.

We believe the best approach is for a 2020-2030 curriculum definition to address the challenge completely the other way round. It will begin with the question "What sort of problems do we need students to solve and what skills and conceptual understanding will they need?" The curriculum start point will then be the problem sets themselves and the mathematical content is then shaped to underpin the problems, introducing material by increasing conceptual not procedural or calculation complexity. Specifications are then deduced from the materials with today's genuine gaps (as opposed to legacy inclusions) highlighted and filled in the materials. This is an iterative process. The basis for inclusion or rejection of content will be based on its usefulness in solving problems and the problem set extended or modified if needed.

It should be noted here (but will be detailed later) that outcomes for a modern maths curriculum need to be much broader in type than today's and this makes it more essential that the curriculum building process is reformed along the lines described above.

This is a very different framework for creating a curriculum and not only moves from a vision for the right subject to an effective build-out, through alignment of the assessment, but also helps to



market and sell the new curriculum and approach to all stakeholders much more effectively: they can see upfront examples what students would be tackling, not just through examination papers.

## 2.2 The areas of mathematics for a 2020-2030 curriculum

Traditional areas of mathematics like algebra, calculus or trigonometry—which represent the machinery of maths—seem less helpful in subdividing the subject than by application areas that support broad uses across STEM and beyond. But why subdivide at all? In a sense, you should not. The expert mathematician utilises whichever maths areas help solve the problem at hand. Breadth and ingenuity of application is often the key.

But mathematics represents a massive body of knowledge and expertise. Subdividing helps us to think about different areas, so curricula can focus their energies enough that there's sufficient depth of experience gained by students at a given time to get a foothold.

Subdivisions by modern uses of maths, not ancient divisions of tools are not mutually exclusive groups. We find these 5 categories helpful.



- Data Science (everything data, incorporating but expanding statistics and probability)
- Geometry (an ancient subject, but highly relevant today)
- Information Theory (everything to do with communication, whether datasets, images, sound or objects)
- Modelling (techniques for good application of maths for real-world problems)
- Architecture of Maths (understanding the coherence of maths that builds its power, closely related to coding)

Based upon experience of Wolfram technology users and how they utilise mathematics, the modern areas of mathematics will include many new concepts not currently included at the secondary level.

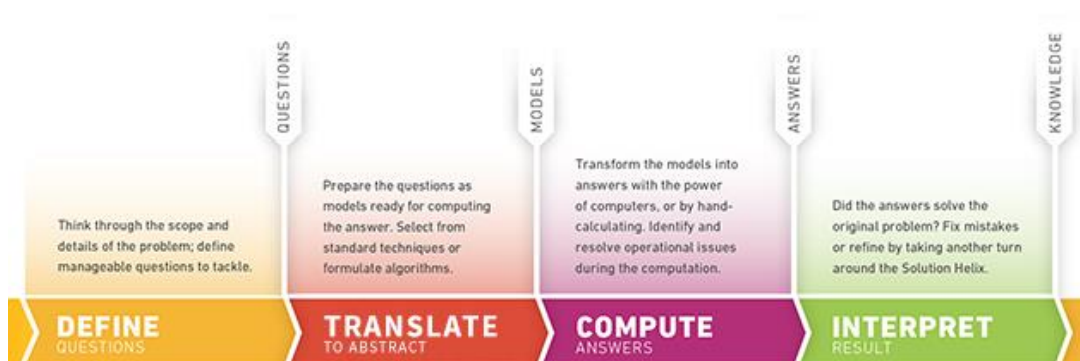
New mathematics or maths-related topics	
<ul style="list-style-type: none"> <li>• Monte Carlo simulation.</li> <li>• Significance and risk.</li> <li>• Hypothesis creation, testing and interpretation.</li> <li>• Model creation and validation techniques.</li> <li>• Pattern matching.</li> <li>• Graphs and networks</li> </ul>	<ul style="list-style-type: none"> <li>• Encryption.</li> <li>• Compression and loss.</li> <li>• Optimisation.</li> <li>• Specification of location and orientation in 3D.</li> <li>• Digital description of 3D objects.</li> <li>• Machine learning</li> </ul>

## 2.3 The practicalities of delivering a computer-based curriculum

The vision and approach we are proposing for a 2020-2030 curriculum is built on our own vision and a unique, tested, robust and completely re-imagined interactive content built from the ground up. It is worth explaining the approach in some detail as it underpins the proposed approach we would recommend.

The delivery uses the interactive Mathematica notebook format – as already available across Victoria schools - with synchronised student and teacher editions including interactive communication between teacher and student, and point-of-need professional development material for teachers. This curriculum has been developed through our Computerbasedmath.org (CBM) initiative and with our own proven approach to developing content in line with the principles outlined above. The CBM vision is to tackle all of the challenges outlined in section 1 to reflect the real world with computer-based computation at its heart. It connects mathematics and computational thinking in a fundamental way.

Thinking computationally is a mode of thinking about life in which you creatively apply a four-step problem-solving process to ideas, challenges and opportunities you encounter to make progress with them.



The start point is **defining the question** that is under consideration—a step shared with most definitions of "critical thinking" and problem solving.

But computational thinking follows this with a crucial transitional step 2 in which the questions are **translated into abstract** computational language—be that code, diagrams or algorithms. This has several purposes. It means that hundreds of years' worth of concepts and tools can be brought to bear on the question (usually by computer), because the question has been turned into a form ready for this high-fidelity machinery to do its work. Another purpose of step 2 is to force a more precise definition of the question. In many cases this abstraction step is the most demanding of conceptual understanding, creativity, experience and insight.

After abstraction comes the **computation** itself—step 3—where the question is transformed into an abstract answer—usually by a computer.

In step 4 we take this abstract answer and **interpret the results**, re-contextualising them in the scope of our original questions and sceptically verifying them.

The process rarely stops at that point because it can be applied over and over again, with output informing the next input until the answers are deemed sufficiently good. This might take just a minute for a simple estimation or a whole lifetime for a scientific discovery.

It is helpful to represent this iteration as ascending a helix made up of a roadway of the four steps, repeating in sequence until 'success' is declared.

Figure 1: Computational Thinking Helix



This computational thinking approach should be at the heart of a computer-based mathematics curriculum which assumes that students and teachers will have access to the computational power that is available to ordinary, everyday people and industry. It teaches the use of these resources to best advantage. Through this process, deeper conceptual understanding is built and experience of abstraction and stronger problem-solving skills are possible than through traditional methods.

Our unique experience in secondary education in other countries such as Estonia and Sweden and with students in Africa shows that if the student is engaged in solving a problem they can grasp and are interested in, the abstraction to mathematics becomes a useful tool for helping them, not a hindrance that fails to engage. Similarly, even though the mathematical content of our curriculum is in many ways more challenging than that of a traditional curriculum, teachers who understand the approach are more engaged because they too see relevant purpose in the subject.

There is another key difference too between a traditional maths way of thinking about a problem and a modern computational thinking approach, and it has to do with the cost-benefit analysis between the four steps of the helix.

Figure 2: Computation is the fast and cheap step



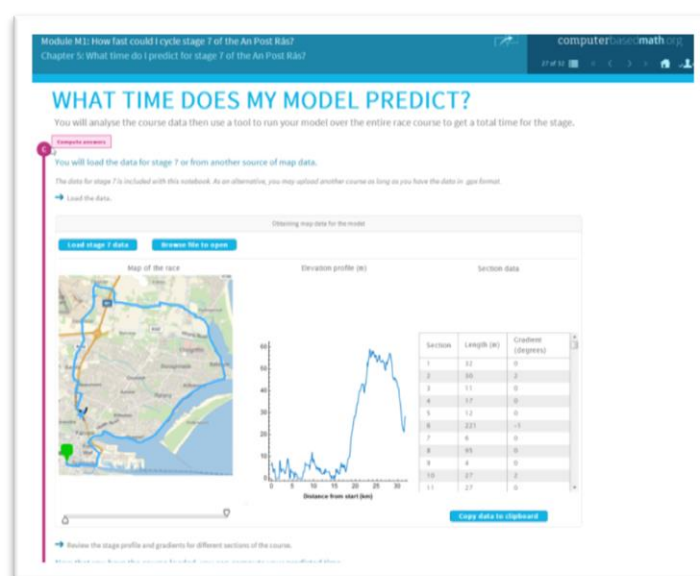
Before modern computers, step 3—computation—was very expensive because it had to be done by hand. Therefore in real life you'd try very hard to minimise the amount of computation at the expense of much more upfront deliberation in steps 1 (defining the question) and 2 (abstracting). It was a very deliberate process. Now, more often than not, you might have a much more scientific or experimental approach with a looser initial question for step 1 (like "can I find something interesting in this data"), an abstraction in step 2 to a multiplicity of computations (like "let me try plotting correlation of all the pairs of data") because computation of step 3 is so cheap and effective you can

try it repeatedly and not worry if there's wastage at that step. Modern technology has dramatically shifted the effective process because you don't get stuck on your helix roadway at step 3, so you may as well zoom up more turns of the track faster.

As we have developed CBM content, the problems we have selected have been drawn from across the STEM curriculum and this is the approach we would recommend for a modern 2020-2030 but extending the problem sets beyond STEM too. We have built our CBM materials around topics as diverse as how to win a bicycle race, marketing the 'best' mobile phone, controlling a quadcopter or deciding whether boys are better than girls at mathematics. The aim should always be to choose problems which will:

- Be as realistic as possible to real problems they'll actually face
- Motivate students to enjoy mathematics and want to learn more
- Build mathematical skills by introducing increasingly complex concepts rather than increasingly complex processes and procedures
- Build an understanding of and competence in using an iterative four-step problem-solving methodology that has broad applicability
- Give students as broad an experience as possible of today's mathematical tools (e.g. machine learning)
- Develop complementary coding skills
- Address a rather different set of mathematics outcomes than has been seen in traditional mathematics education

Figure 3: A sample screen from a CBM module



The content is not restricted by the need to learn hours of hand-calculation techniques before being able to progress. Thus we can order the curriculum by conceptual complexity and concentrate on interesting problems that require the students to think about the problem and not the mechanical procedure of computation. For example, the concept of a solution to an equation is the same for solving linear, quadratic, cubic, or any polynomial - it's the clutter of the hand-calculation procedure that separates them into extremely difficult different ideas and alters the stage at which the concept is learnt.

For illustration purposes some of the features of a typical module are itemised in Figure 7.

Figure 7: An example module entry page

**Module summary and outline**  
The entry page shows a summary of the narrative and content covered.

**Lesson planning**  
Creates a printable summary for planning the lessons.

**Student view**  
Opens a new window on an extended display of the student materials.

**Welcome screen**  
Opens the welcome screen in front of the module. Enables primers or other modules to be opened.

**User account**  
Students can edit account information, change their displayed name, their email, or their class.

**Activities**  
Every activity title is a button to enable you to jump directly to the activity.

**Magnification**  
Can be used to zoom in or out of the module display

**Chapters**  
A list of chapter titles for the module. Click the title to open the activity titles within.

**Coverage**  
Lists of objectives, skills, and concepts covered by the module.

Module G4: How do I design controls for my game?  
Summary

How do I design controls for my game?

MODULE SUMMARY  
How many controls do you need for a game and which ones should you pick?  
In this module you will learn how to design a controller system for a game. You will first need to understand how to describe an object's of rules known as coordinate systems and learn how to choose a suitable coordinate system and how to communicate positions and movement. Once you are familiar with coordinate systems you will learn how controllers can be used to control the movement of an object within the movement performed by the object.

CONTENTS

- How many numbers are needed to describe where it is and what it's doing?
- Which controls give which actions on my computer screen?
- How do you optimise the mapping for the controls?
- Project

After completing the module you will:

- Concepts
- Primers
- Tools
- Outcomes
- Image credits

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Summary

1: How many numbers are needed to describe where it is and what it's doing?

- How many ways can it move?
- How are movement and location related?
- What are the standard ways to specify location?
- What are the standard ways to specify location - 3D?
- Is position all we need?
- What if the object is not stationary?
- 3D movement challenge
- Chapter summary

2: Which controls give which actions on my computer screen?

- How do you describe the position of something on the screen?
- How does your pointing device map to the cursor on screen?
- How do analog sticks control the cursor?
- Optional independent task - How does my games console know where I am?
- Chapter summary - Can you describe the controllers and coordinate systems?

3: How do you optimise the mapping for the controls?

- From here or from there? What's the best viewpoint?
- What happens when the controls are rewired?
- Chapter summary

4: Project

- Design your own controller simulation



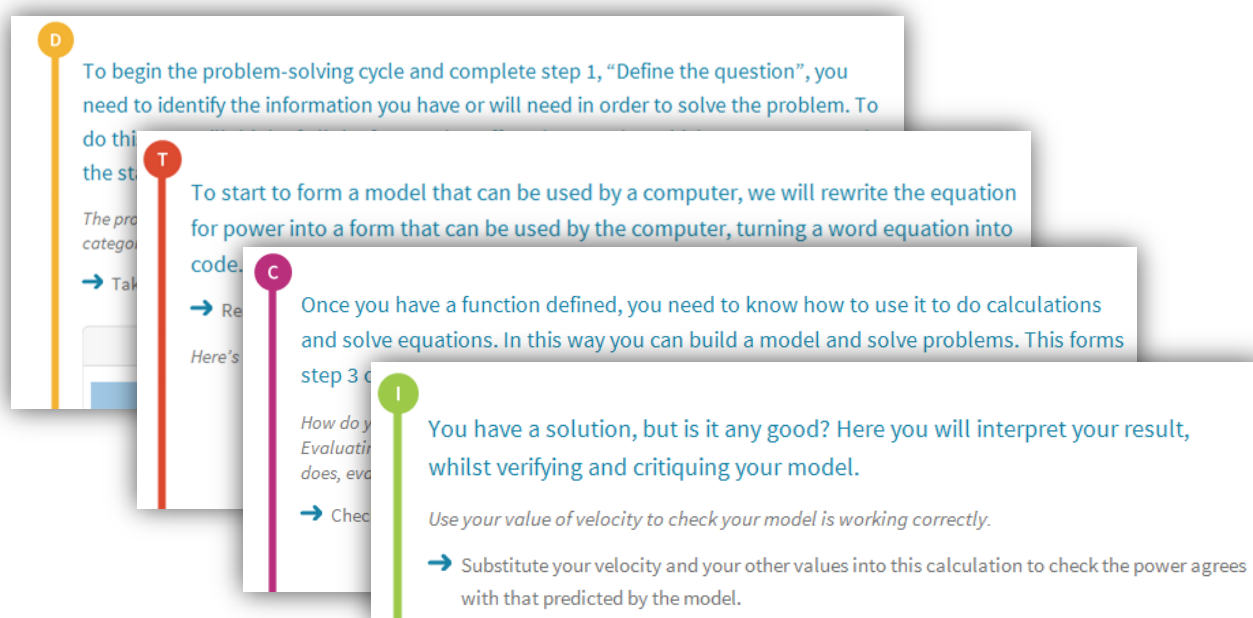
We include an exceptionally wide range of learning modes and pedagogies by varying the modality used to give a variety of challenges and stimulate learners. All of the modalities below have been used extensively in materials:

Abstract a diagram	Manipulate to discover
Assess validity	Modify code
Brainstorm	Obtain pre-existing data
Calculate answer	Pose a question
Sorting /Classify/ Ordering	Presentation
Compose/collect raw data	Primer
Comprehension	Quiz
Cross examine	Role-play
Data structuring	Run a simulation
Distil opinions	Summarise
Essay or report	Reinforce learning
Estimate a value	Synthesise code
Experiment	Video essay
Find the information	Visualise data
Find the mistake	Watch video
Guided discussion	Write instructions
Interpret a chart	

Many of these modalities are underpinned by interactive activities. For example, 'Sorting /Classify/ Ordering' is generally tackled through a drag and drop template that enables students to categorise objects; building and reinforcing understanding of key concepts.

To reinforce the manner of the problem-solving cycle, each relevant modality is colour coded and labelled with the name of the step that the student will be undertaking. Introductory statements to each modality also make it clear what is expected. Figure shows examples of such modalities.

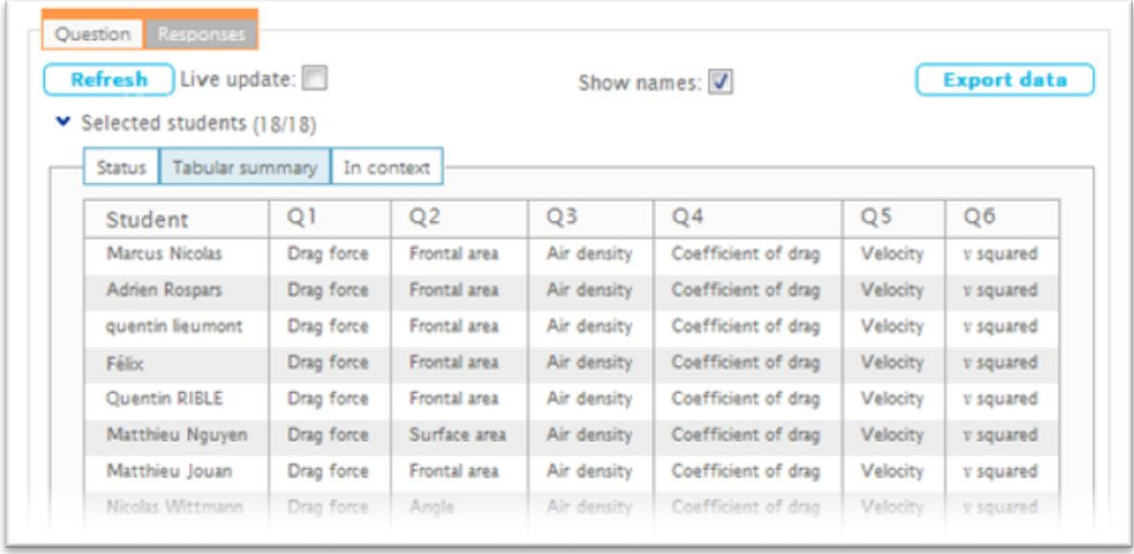
Figure 8: Modalities with problem-solving steps indicated



Assessment opportunities can occur every few minutes with the teacher being able to read and digest their students' thoughts (discretely on their own computer) before summarising the learning and picking up on misconceptions using the classroom large display, see Figure 9. Slightly longer assessments happen at the end of each chapter after new learning has taken place and allow the teacher to ensure progression of all students together. Summative assessments are planned at the

end of each module when a transfer of skills and knowledge into another related context is attempted by students either individually or in groups. Larger summative assessments happen at key points in the course design to include skills and knowledge from a collection of modules simultaneously.

Figure 9: Student responses for the teacher to review



Student	Q1	Q2	Q3	Q4	Q5	Q6
Marcus Nicolas	Drag force	Frontal area	Air density	Coefficient of drag	Velocity	$v$ squared
Adrien Rospars	Drag force	Frontal area	Air density	Coefficient of drag	Velocity	$v$ squared
quentin lieumont	Drag force	Frontal area	Air density	Coefficient of drag	Velocity	$v$ squared
Félix	Drag force	Frontal area	Air density	Coefficient of drag	Velocity	$v$ squared
Quentin RIBLE	Drag force	Frontal area	Air density	Coefficient of drag	Velocity	$v$ squared
Matthieu Nguyen	Drag force	Surface area	Air density	Coefficient of drag	Velocity	$v$ squared
Matthieu Jouan	Drag force	Frontal area	Air density	Coefficient of drag	Velocity	$v$ squared
Nicolas Wittmann	Drag force	Angle	Air density	Coefficient of drag	Velocity	$v$ squared

Reinforcement and the opportunity to correct misunderstandings happens through carefully chosen switches of context, applying very similar concepts and tools in a new context, usually with less granular direction in the materials but still hung on the four step problem-solving cycle. This more hands-off approach is used in the final project of each module.

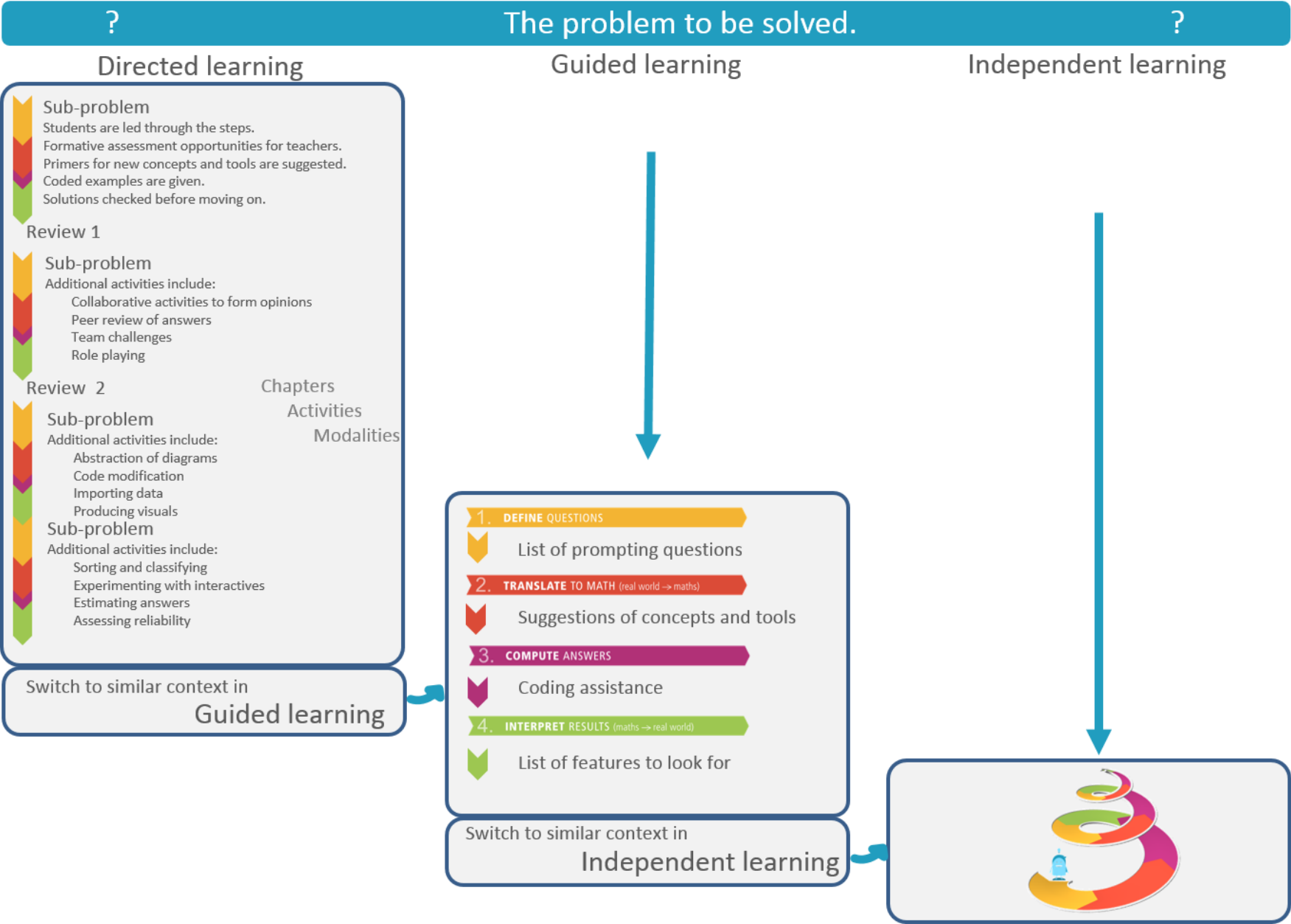
## 2.4 Progression paths, abstraction and mixed abilities

As mentioned earlier, in a computer-based approach skills, concepts and tools used, should be reinforced through the use of projects at the end of each module. Projects are a less built out version of a full 'directed learning' style module, they share a similar structure based upon the problem-solving helix. This cut-down version is referred to as the 'guided learning' style. The relationship between directed, guided, and a final independent style is shown in Figure 10.

This is one of the approaches that can underpin the ability of CBM to support mixed ability level classes – increased emphasis on directed learning is always possible and more able students can be released earlier to independent learning.

In coping with mixed ability, questions around when and how to tackle abstraction are also addressed with CBM. Coding tasks introduce a level of abstraction and the more able student can again be left to step outside the problem scenario and explore further abstraction.

Figure 10: CBM resource deployment options.





## 2.5 The outcomes of F-12 education in the core technical subject

CBM has rethought educational outcomes to target real-world needs in line with a forward thinking 2020-2030 curriculum. This has involved a much broader view of mathematical understanding and skills, as well as a systematic approach to fitting multiple dimensions of need together coherently.

These are the guiding principles for the construction of an outcomes list that we believe fits a 2020-2030 curriculum:

### 1. Outcomes should be more than just the mechanistic tools.

Do not limit the outcomes to specific mathematics concepts and tools. The computational thinking process is far more than being able to show competence at working out an answer. For example, being able to solve a quadratic by factorisation (when  $a=1$  of course), or deciding whether to use the mean or median for the average of a dataset. As described earlier, the four-step process is an iterative, multi-layered, complex task, and linking outcomes to it cannot be limited to mathematics concepts and tools being used.

### 2. Outcomes are not dependent upon assessment.

Include desirable outcomes even if there is no current assessment methodology. State the outcomes we desire and strive for; that will enable you to design a relevant curriculum and provide learning experiences to students that are useful, meaningful and interesting. If the summative assessment cannot measure the outcome, that is not a reason to remove it from the curriculum design principle.

### 3. Relate the process of doing maths (cf. computational thinking Helix) to required outcomes.

This is tricky because there are outcomes about the whole process, one step of it and super-processes enabling its effective application. What's definitely wrong is to try to tether every outcome to each step, or to claim that the process is not connected to outcomes (if it's the central maths process, it must be related to outcomes of learning maths), or to claim it's simply one outcome.

Annex B summarises the outcomes we have defined and full details are at:

[computerbasedmath.org/outcomes](http://computerbasedmath.org/outcomes)

## 2.6 Assessing the outcomes

The key change that is needed is the content of the instruments being used—utilising more open-ended questions that target the new outcomes of the computational thinking style curriculum. CBM defines mathematics topics as the concepts and tools that are applied to solve problems. CBM then defines mathematics skills as those required to solve problems using concepts and tools, assuming a computer is available as a default.

Current methods of summative assessment that are relevant to future mathematics:

1. Examination (<3 hours in controlled conditions)
2. Controlled project work (extended hours under restricted conditions)
3. Portfolio creation (evidence gathered throughout the duration of a course)
4. Peer reviewed and peer reviewing activities.
5. Presentation (multimedia prepared under unrestricted conditions)
6. Interview (oral questioning)



### 1. Examination

For measuring the understanding of the scope, limitations, pitfalls and use of concepts and tools, examinations provide a well-documented route to grading a student's abilities. As a means to measure skills, they are limited by their duration and the very restrictions that enable them to be fair and accurate.

We have identified four methods for developmental purposes:

- A. Paper-based (deploy and submission), no computer.
- B. Paper-based (deploy and submission), using a computer during the examination.
- C. Part paper (computer deployed and during, paper submitted)
- D. Computer-based assessment (computer deployed, during and submitted)

Method D would be the ideal for an assessment of a CBM curriculum, but the other options should be considered, as many problems arise with computer-based submission of formal exam entries in many jurisdictions.

### 2. Controlled project work

Adding this method allows the student more time to complete an extended task and use the skills associated with problem solving in the real world. Creating a rubric from the outcomes applicable to each project is a necessity, with exemplar items that represent each outcome.

### 3. Portfolio

As every problem-solving module consists of one directed-learning problem solution followed by a more open guided-learning problem, the students get regular opportunities to complete independent problem solutions. Each module adds to their experience and breadth of context awareness. Teacher ratification of student portfolios (against a rubric as in 2) would be a valuable method of assessing a student's performance.

### 4. Peer review

Smaller problem-solving activities lend themselves to structured peer review by students. This gives a three strand approach to the learning. First, the skills to be able to complete a problem solution to a sufficient degree to be reviewed by peers. Second, the reflection on their product when being reviewed by a peer and learning from another student's experience. Third, the skills of reviewing another student's product.

### 5. Presentation

Being able to present problem solutions to a variety of audiences in a variety of forms is an important outcome that can only be assessed through a presentation. This is a skill widely used in business and higher education but little used as an assessment tool at the secondary level.

### 6. Interview

A speaking and listening exam is often the method used to assess language understanding and skills but is rarely used in the STEM subjects at the secondary level. A mathematics oral is found at the university level in a number of jurisdictions but rarely in the UK (Iannone & Simpson, 2012). Hairer (Oral Examinations / Presentations, 2005) from the Mathematics Research Centre, University of Warwick, states "My experience ... is that it is easier to test the knowledge of a subject with a written examination and to test the understanding of a subject with an oral examination."

## 2.7 Supporting the teacher and learner

For the curriculum planner, the modules would be fully referenced, and tagged with learning dependencies that would restrict the reordering or deletion of certain problem modules. A



curriculum planning tool would allow teachers to choose a suite of problem modules that suited their students' needs and provided a sensible progression through the adoption of new key outcomes.

All resource delivery is done using the Wolfram Language and so is customisable by any teacher in possession of a suitable licence. Templates are available to add in new activities, new questions, new projects etc.

Aside from the three styles of the resource construction and use ('Directed', 'Guided', and 'Independent' learning), there are also a number of ways in which the delivery of the learning takes place. The manner in which the learner interacts with the curriculum learning content can be any one of the following:

Delivery type	Description
<b>Teacher-led, classroom-based, real time.</b>	Teacher is in the same room as the learners and the learning of the group moves forward together.
<b>Teacher-led, classroom-based, asynchronous.</b>	Teacher is present with the students but the group is not kept together. Individuals work at their own pace.
<b>Teacher-led, remote, real-time.</b>	Students are in dispersed locations and communicate via online meeting software. Teacher moves the group forward together.
<b>Teacher-led, remote, asynchronous.</b>	Students are in dispersed locations and communicate via online meeting software as and when necessary. Teacher gives a published list of office hours when they can be reached for assistance.
<b>Self-study.</b>	The computer leads the learning and gives feedback as far as possible. Supplementary input from video recordings is available on demand.

For the types where a teacher is the main facilitator for the learning, the content of the directed learning style modules comes in two linked versions. A student version that contains all of the material necessary for the student to interact with the class and take part in the learning, and a teacher version which adds support material within the student version.

The teacher version contains comprehensive support and guidance. Activities comprise a set of modalities and every modality provides the teacher with:

- Overview – duration and nature of activity (e.g. guided, whole class discussion)
- Purpose – what is this activity aiming to achieve?
- Steps – breaking the overarching activity into more manageable chunks
- What to say – for those teachers that want or need it, complete suggestions of what they can say at each step of the activity.

Most activities also contain two extra pieces of teacher support:

- Technical Manual – to support the use of any specific learning modalities or external data capture in the activity
- Answers- to specific questions within the activity

The teacher may also have extra support material directly relevant to the activity that may not be suitable for the students to see at the outset. Teachers can introduce this material as students' progress to a suitable point in their understanding. Examples include: Images to display as a discussion prompt to pose an alternative idea; snippets of code that can be displayed once students have attempted their own creation; further reading or primers that are appropriate once a suitable level has been achieved.

The teacher is able at any point to switch between this fully supported version and the student version which contains none of the above guidance. Giving the teacher both versions means that the

teacher can choose to display, on a screen or whiteboard for example, exactly the material the student sees.

Many of the activities require the student to submit responses which are amalgamated and then displayed on the teacher version. The teacher can then choose the point at which group answers are made available to students (see Figure 9). Having students send their answers to the teacher via a web server, means that teachers can remotely track their students' progress without disturbing their learning and only intervene when a misconception arises or progress is unsatisfactory.

Student versions of the resource contain not just the narrative of the problem solution, but also key concepts and tools. These are covered by separate, out-of-context primers that provide information needed for the student to progress whilst using such a concept or tool. Primers contain the key facts for the concept, useful interactive demonstrations, use cases, further reading and examples. All primers have a final self-marking 'Check your understanding' section to give feedback to the student.

## 2.8 Technology underpinnings required for delivery

Key technologies are important for implementation as described above:

- Electronic notebooks that enable live computation, coding and interactivity
- Linked student and teacher materials so tracking and information swapping are as easy as possible
- Student-teacher live exchange through the materials to enable classroom collaboration, teachers to pull together class-wide response and track progress
- Computation supporting algebra, graphics, numerics, sound and modern algorithms with as much automation as possible

In addition, to optimise development efficiency, the technology should:

- Provide automatic tagging of content so it can easily be seen which outcomes have been covered and materials can be upgraded iteratively.
- Enable automatic layout of interactive applets
- Have a direct source of computable data that makes it easy to pull in real datasets to problems.

### 3 Early suggestions for a specific VCE mathematics curriculum for 2020-2030

#### 3.1 Areas to deliver - problems sets

Everything we have outlined above is applicable to any liberal democratic society in a developed country for 2020 – 2030. How would Victoria progress a context-specific implementation of the ideas and approaches we have defined?

Victoria is in a pivotal position – strength in integrating computers in mathematics assessment, and state-wide adoption of industrial-strength computation technology combined with the political will for change.

We have been in a privileged position in recent years able to see and comment on a wide range of international mathematics curricula. We know that that amongst modern, traditional curricula the Victorian 2016-19 VCE curriculum is a leading example and reflects the effort that has been invested as part of becoming “The Education State”. Including algorithms and coding in the curriculum has been an important addition. Victoria can now make the next step forward from this strong base to embrace our proposed outcomes which are broader (e.g. covering computational thinking and coding) and assume computer-human interaction at their core.

The skill is in creating an appropriate set of problems and requires an iterative approach which considers:

- The breadth of the target audience
- Problem themes that will engage, inspire and motivate in a Victorian context
- Problem themes that have applicability to real-world, life-impacting issues
- Coverage of a robust and real world set of mathematical content

These are context specific and need to reflect the Victoria aspiration. For example, in our work in Ireland there has been a keen desire to attract students in humanities to tackle mathematics and computational thinking and the problem-sets have been selected accordingly. A different problem set would be appropriate if the emphasis is solely on STEM.

#### *Example of how Victoria could map-out a new curriculum*

We outline here *one possible* model for VCE. This model is categorised into a set of FOUR blocks which seek to address, at a high level, the diversity of interests students should be bringing to the subject as well as the core content of mathematics.

The entire approach is problem centric and each block introduces new mathematical concepts, tools and approaches. Block one is the ‘foundation block’ which introduces the core four step modelling approach, builds computational thinking and introduces key mathematical concepts. The other three blocks then focus on *application domains* into which the selected problems fall.

Each block is 50 hours of guided learning and the 50 hours then breaks into 25 hours of “scaffolded”, computer-based maths and 25 hours of more focused skill development outside the boundaries of the core problems. The level of scaffolding decreases as students progress through the course reflecting increasing confidence and skills and a growing computational thinking competence.

Possible titles for the four blocks are listed below but it is important to emphasise that these are *cross-discipline* in a variety of ways. Computational thinking in biology for example fits most obviously in a natural world problem-set but the problems straddle traditional discipline boundaries (even outside STEM, in history say). So a module built around a cycling race problem can embrace physics (friction, air resistance, forces), biology (energy conversion, nutrition) and health and social science (obesity, diabetes). With that in mind, the proposed blocks are:

- 1. Mathematical Modelling and the Tools of Mathematics (built around two major problems)**
- 2. Mathematics for problems in the Physical world (three problems)**
- 3. Mathematics for problems in the natural world (three problems)**
- 4. Mathematics for business and life (three problems)**

Again, as mentioned above, we have deliberately avoided calling the blocks by terms such as “mechanics” (which falls in the physical world block) or “Operational Research” (which falls in the business and life block). Likewise, although “data and decision” topics will be covered extensively there is no sign of a “probability and statistics” label. The topics traditionally covered under these familiar domains will be fully covered but they are introduced as and when they are needed across the problem sets. As described previously this allows the introduction of topics in a sequence of increasing conceptual rather than procedural complexity.

So data and decision topics will appear and be gradually developed across all four blocks – there is no point in the curriculum where the student says “this week I’m studying data and decision”. We will outline in full below *the content coverage* but for now we turn to explore each block.

In terms of the five areas of mathematics we defined in Section 2.3 these can be mapped to the blocks. All five areas – data science, information theory, modelling, geometry and the architecture of mathematics – will be represented to varying degree in each block depending on the final problem sets.

Two major problems would form the core of the mathematical modelling foundation block. These will be selected to be of wide interest but as examples here are two that we believe would work:

- Can I win a stage of the Official Tour Down Under cycle race?
- Can I locate a missing aircraft?

These two problems build on existing modules already created by the Wolfram CBM team although they would need review and re-work to make them more appealing to a Victorian audience. For example we would want to persuade Cadel Evans to appear in an opening video and to provide an inspiring and motivational opening for the cycling module – successful professional cycling is as much about maths and science as it is about training and conditioning. Note that these are simply suggested modules that help reduce the initial investment for Victoria by utilising pre-built materials but we would of course be happy to build bespoke problem areas instead.

Problem sets for the remaining three blocks can readily be created. We have formulated and continue to develop a wide list of potential topics some of which are listed in Annex A and these could be readily adopted to contribute to the VCE programme. However, there may be specific local contexts which mean that alternative problems should be considered and co-defined with the VCE team.



### 3.2 Pathway options to handle varying abilities

An important consideration remains the applicability of materials to both the high achieving as well as the low achieving students. Research suggests that the high achieving students can survive with almost any level of scaffolding within a teacher led class.

Our experience has shown that the highly scaffolded materials in CBM allow the lower achieving students to advance at a more rapid pace than traditional abstract, procedurally based content. The scaffolding approach in CBM supports handling of differing ability levels while alternative cohorts (by interest) can be tackled by having optional problem sets. This can give the option to put emphasis on particular STEM domains for example.

There remains the option to adopt a 'tiered' approach giving more able students an extra challenge. This can be achieved at the core block level with ascending conceptual difficulty meaning that some students could miss the most demanding problem in each block. But this would mean missing valuable content so a better route is extensions beyond the core.

The more able student can be given the opportunity to define problems of their own and to work independently on a valuable project.

### 3.3 Assessment options

As discussed in Section two the assessment content needs to change in line with the learning delivery content. This means utilising more open-ended questions that target the new outcomes of the computational thinking style curriculum. CBM defines mathematics topics as the concepts and tools that are applied to solve problems. CBM then defines mathematics skills as those required to solve problems using concepts and tools, assuming a computer is available as a default.

We argued that the examination (<3 hours in controlled conditions) remained a key instrument but the availability of Mathematica across the state enables this to be computer-based : carried out in the same environment and with same access to tools as for the learning. The remaining five key instruments are all still applicable for VCE.

1. Controlled project work (extended hours under restricted conditions)
2. Portfolio creation (evidence gathered throughout the duration of a course)
3. Peer reviewed and peer reviewing activities.
4. Presentation (multimedia prepared under unrestricted conditions)
5. Interview (oral questioning)



## 4 Conclusion

Victoria is in a strong position to leapfrog other liberal democratic societies in fundamentally reforming the mathematics curriculum to optimally support the move to a Computational Knowledge Economy. Key factors in Victoria's favour include being world-leaders in integrating computers in mathematics assessment, state-wide adoption of industrial-strength computation technology, a network of specialist schools promoting STEM, an ideal size and a political will for leading change inside Australia and beyond.

No jurisdiction will find the true magnitude of this change easy but those who are first to carry it out will benefit the most. Victoria can also benefit from long-standing collaboration with Wolfram both in terms of technology needed for the change but also in terms of Wolfram's spin-off [computerbasedmath.org](http://computerbasedmath.org) which is unique worldwide in working out a new core technical curriculum and starting to deliver it—saving years on an implementation in Victoria.



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**Annex A: Example module titles showing linkages to STEM subjects, the concepts, tools and outcomes acquired**

Example module titles and brief description of the purpose.	Possible stage	STEM subject links						Concepts used Areas of Mathematics covered by the problem.	Tools used Mathematical tools to solve the problem	CBM Outcomes (see section 2.5)
		Physics	Chemistry	Biology	Business	Engineering	Technology			
Do I know what I don't know? Understanding how making assumptions and clearly stating them is a necessary part of any model.	3-6	✓	✓	✓	✓	✓	✓	Probabilistic model, Assumptions, Data, Model, Simulation, Statistics,	Histogram, Distributions, Mean, Median, PDF, CDF,	AM1, AM2, AM4, CCD, CCV, CV1, GM2, IFE, IFP, IN1
What are the economics of behaviour? Analysing effective strategies for marketing, and population modelling.	7-12				✓	✓	✓	Data analysis, Modelling, Profiling, Probability distribution, Expectation	Mean, Median, Distributions, Extrapolation, Fitting,	AM1,AM2,AM3,AM4,AM5,AM6, CCA, CCD, CCG, CCP, CCQ, CCR, CCV, CV1, CV2, GM3, GM4, IN2, IN3, IN5, IN6, PM4, PMB, PMP, PMR
How do you get to the moon? Determining a model for a rocket launch from Earth.	7-12	✓				✓	✓	Modelling motion, Vectors, Gravity, Coordinate systems, Rates of change, Momentum, Forces, Differential equations	Derivative, Plot3D, Spherical coordinates, Lists	AM1,AM2,AM3,AM4,AM5,AM6, CCA, CCD, CCG, CCP, CCQ, CCR, CCV, CTA, CTI, CTK, CV1, CV2, CV3, CV4, CV5, CV6, GM1, GM2, GM3, GM4, IFE, IFP, IN1, IN2, PM4, PMB, PMP, PMR
How do computers detect and correct errors? Using real verification methods and understanding their limitations.	3-6						✓	Hamming codes, Verification, Error correcting codes, Moving averages, Fourier smoothing	Check sum	AM5,AM6,CCD,CCR,CTA,CTI,CTK, CV2, CV3, CV5, CV6, GM1, GM2, GM3, GM5, IFE, IFF, IFP, IN2, IN3, IN4, IN5, PM4, PMB, PMP, PMR
Can you find the best deal? Modelling a varied set of financial plans over time and forecasting the most effective.	3-6		✓	✓	✓	✓	✓	Optimization, Modelling, Multivariate problems, Extrapolation, Confidence	Region, Plot, Maximize, Minimize, Nearest, Furthest.	AM2,AM3,AM4,AM5,AM6,CCP, CCR, IFE, IFP, IN1, IN2, IN3, IN4, IN5, PM4, PMR
Can I crack your password? Comparing brute force techniques to more intelligent methods.	3-6			✓	✓	✓	✓	Exponential, Modelling, Behavioural analysis, Combinations.	Function, Loop, Count	AM5,AM6,CCD,CCR,CTA,CTI,CTK, CV2, CV3, CV5, CV6, GM1, GM2, GM3, GM5, IFE, IFF, IFP, IN2, IN3, IN4, IN5, PM4, PMB, PMP, PMR
Cause or just correlation? Realising that a correlation can be random, causation is not implied by correlation.	7-10	✓	✓	✓	✓	✓	✓	Multivariate data, Correlation, Dependence, Independence, Causation, Fitting.	Correlation coefficient, Plot,	AM1, AM4, AM6, CCA, CCD, CCP, CCQ, CCV, CTI, GM2, GM3, IN1, IN2, IN3, IN4, IN5, IN6, PM4, PMB, PMP
What resolution do you need? Is the retinal screen really needed? How many pixels do you need at home/cinema/phone?	3-6 / 7-12	✓		✓		✓	✓	Trigonometry, Density, Similarity, Image processing	Trigonometrical functions, Graphics,	AM3,AM4,CCD,CCG,CCP,CCV,CTI, CV5, GM3, GM4, IN1, IN2, IN3, IN4, IN5, IN6, PMB, PMP
How do populations vary over time? Using models of predator-prey relationships to gauge the impact of contributory factors.	7-12		✓	✓	✓	✓	✓	Modelling, Feedback loops, Control systems, Differential equations, Cellular automata	DSolve,	AM1, AM2, AM4, AM5, AM6, CCA, CCD, CCG, CCP, CCQ, CM3, CM4, CM5, CTA, CTI, CTT, CV1, CV2, GM1, GM2, GM3, GM4, GM5, IFE, IN1, IN2, IN3, IN4, IN6, PM4, PMB, PMP, TM2, TM3
Where should I build the distribution centre? Analysing networks for efficient layouts for distribution of services or communications	7-12				✓	✓	✓	Graph theory, Networks, Optimization, Modelling	Graph, FindShortestPath, FindMaximumFlow, ClosenessCentrality	AM1,AM3,AM4,AM5,AM6,CCA, CCD, CCG, CCP, CCV, CV1, CV6,IFE, IFF,IFP,IN1,IN2,IN3,IN4,IN5,IN6, PM4, PMB, PMP, PMR

Example module titles and brief description of the purpose.	Possible stage	STEM subject links							Concepts used Areas of Mathematics covered by the problem.	Tools used Mathematical tools to solve the problem	CBM Outcomes (see section 2.5)
		Physics	Chemistry	Biology	Business	Engineering	Technology	Humanities			
When will the next peak happen? Analysing trends from data, fitting a model to the data.	7-12	✓	✓	✓	✓	✓	✓	✓	Data analysis, Rates of change, Fitting a function to data, Minima, Maxima, Summation	Derivative, Fit, Plot, Integrate, Solve	AM1, AM4, AM5, AM6, CV1, CV2, CV3, CV4, CV5, IN3, IN6
How many words do I know? Extrapolating from a sample to estimate a population parameter.	7-12			✓	✓			✓	Data analysis, Sampling, Statistics, Parameter estimation, Variation, Distributions, Confidence, Error, Sampling bias, Probabilistic model	Mean, Median, Quantile, Histogram, Min, Max.	AM4, CCG, CCP, CV1, CV6, GM1, GM2, GM3, GM4, IFF, IFP, IN1, IN2, IN3, IN4, PM4, PMB
Fooling your eyes with geometry. Using geometry to reproduce 3D effects from 2D screens	7-10	✓				✓	✓		Trigonometry, Dimensions, Perspective, Projection, Transformation, Cartesian coordinates	Graphics, ArcSin, ArcTan, ArcCos, Line, HalfLine, InfiniteLine, Polygon, PolyhedronData, ImagePerspectiveTransformation, Texture, Graphics3D	AM1,AM2,AM3,AM4,AM5,AM6, CCA, CCD, CCG, CCP, CCV, CTI, CTK, CV2, CV3, GM1, GM3, GM4, GM5, IFF, IN2, IN3, IN4, PM4, PMB, PMP
How do you map the world? Producing 2D images of 3D surfaces and vice versa.	3-6 / 7-10	✓				✓	✓	✓	Projections-linear, Projections-functional, Dimensions, Transformation, Distortion, Great Circles, Nets, Polar coordinates, Limits,	Graphics, GeoPlot, Polygon, ImageTransformation, Projection,	AM3,AM4,AM5,AM6,CCD,CCG, CCP, CCV, CTI, CTK, CV1, CV2, CV4, GM1, GM2, GM3, GM4, IFE, IFF, IN1, IN2, IN3, IN4, IN5, IN6, PM4, PMB, PMP, PMR
How many cell towers do I need? Covering an area with a suitable strength signal.	3-6	✓	✓	✓	✓	✓	✓	✓	Networks, Cluster analysis, Density, Loci, Regions,	Graph, Mean, Area, GeoMap,	AM1,AM2,AM3,AM4,AM5,AM6, CCA, CCD, CCG, CCP, CCQ, CCR, CCV, CTA, CTK, CV1, CV2, CV3, CV4, CV5, CV6, GM1, GM2, GM3, GM4, GM5, IFE, IFF, IFP, IN1, IN2, IN3, IN4, IN5, IN6, PM4, PMB, PMP, PMR
How big could the biggest specimen be? Using a model distribution to estimate the maximum value.	7-12		✓	✓	✓			✓	Probability distributions, Distribution fitting, Sampling Bias,	DistributionFit, RandomChoice, Mean, Median.	AM1, AM4, AM5, AM6, CV1, CV2, CV3, CV4, CV5, IN3, IN6
Can I spot a cheat? Testing patterns against known distributions to determine confidence in randomness	7-10		✓	✓	✓	✓	✓	✓	Hypothesis Testing, Confidence Interval, Significance, Probability distribution, Expectation	Tally, Count, Tuple, Riffle, Quantile, Histogram, HypothesisTest	AM1, AM2, AM4, AM5, AM6, CCA, CCD, CCG, CCP, CCQ, CM3, CM4, CM5, CTA, CTI, CTT, CV1, CV2, GM1, GM2, GM3, GM4, GM5, IFE, IN1, IN2, IN3, IN4, IN6, PM4, PMB, PMP, PMR, TM2, TM3
Am I Normal? An introduction to measures of central tendency and how to combine characteristics.	3-6	✓	✓	✓	✓	✓	✓	✓	Data visualisation, Data analysis, Minima, Maxima, Mean, Median, Deviation, Spread/Range	Bar charts, Histograms, Pie charts, Scatter plot 2D and 3D, Mean, Median, Range	AM5,AM6,CCD,CCR,CTA,CTI,CTK, CV2, CV3, CV5, CV6, GM1, GM2, GM3, GM5, IFE, IFF, IFP, IN2, IN3, IN4, IN5, PM4, PMB, PMP, PMR



Annex B: We have defined 11 dimensions. The detail of each outcome can be seen at [computerbasedmath.org/outcomes](https://computerbasedmath.org/outcomes)

#### CT - CONFIDENCE TO TACKLE NEW PROBLEMS

Students show confidence to attempt solutions to new problems by application of the four-step process. They use the problem-solving process as a mechanism to overcome hard-to-handle or unknown scenarios and can adapt previously learnt methods, concepts and tools to new contexts. They are able to overcome sticking points in the process and teach themselves new tools as the need arises.

#### IF - INSTINCTIVE FEEL FOR MATHS

Students are able to use their experience to know when something just “smells” wrong. They are aware of common errors made and have a working mental knowledge of the use of maths concepts.

#### DQ - DEFINING THE QUESTION

Students begin the problem-solving process by organising the information needed to solve the problem and identifying suitable smaller tasks that can be solved. They understand assumptions and use them effectively to aid progress on the solution.

#### AM - ABSTRACTING TO MATHEMATICAL CONCEPTS

Students begin the translation to maths phase by taking their precise questions and working out strategies or mathematical concepts to explore. They organise their information and identify the relevant concepts and their suitability for the purpose.

#### CM - CONCEPTS OF MATHS

Concepts are what you want to get done (hang a picture, solve an equation, describe an event’s probability...). Tools are what you want to use to do it (glue, nail, screw, graph, formula, normal distribution...). Most concepts begin life with one tool; you invent the concept for a given problem and a tool to fix that. Though retrospectively, people might collect a number of tools and create an umbrella concept to cover them.

#### TM - TOOLS OF MATHS

Tools take the form of functions, methods or processes that enable a conversion from the abstracted form of the defined question into a form that is useful in answering the question. The tool may not necessarily be computer based. The most efficient manifestation of the tool for the purpose should be chosen.

#### MC - MANAGING COMPUTATIONS

The computation phase begins with students choosing the manifestation of the mathematical tool(s) to produce a result. This may be a trivial step for one tool with a simple input but could also be organisationally complex for combinations of a number of tools. Once the computation reaches a certain size, the process of performing the computation becomes a significant consideration.

#### IN - INTERPRETING

Students take the output of the computation stage and translate this back to the original real-world problem by relating the output to their precise question. They consider further areas of investigation as a result.

#### CV - CRITIQUING AND VERIFYING

Critiquing is a consideration of what could possibly be wrong with your process or solution. Asking the questions: Where? When? Why? What? Who? It is a constant process of scepticism towards results, from unexpected results to expected results. Verifying is comparing against a hypothesis to confirm an answer and being able to justify the result.

#### CC - COMMUNICATING AND COLLABORATING

Communicating and collaborating is a continual process that happens throughout all stages. Students use media fit for the purpose and combine multiple representations effectively for the intended audience to be able to follow the ideas presented.

#### GM - GENERALISING A MODEL/THEORY/APPROACH

Once a model has been built for a specific purpose, looking further afield for instances where the model may apply or providing sufficient documentation for others to adapt the model for their purpose