VCE Specialist Mathematics
Units 3 and 4

Sample application task – reduction formulas for integration

Introduction

A context such as the following can be used to investigate the use of recursion to develop reduction formulas for integration. Recursion is a process that expresses a given stage of a procedure or construction in terms of earlier stages of the same procedure or construction. There are many examples of recursive processes in mathematics.

In calculus, a ***reduction formula*** for integration can be used to successively reduce the complexity of an integral involving a natural number parameter.

This investigation will involve differentiation and integration of combined functions, the development of recursive formulas, the graphing of functions, their derivative functions and anti-derivative functions, and the identification and analysis of key features and relations between these functions and their graphs.

Where parameters are involved in the specification of the rule of a family of functions, formulas, relations and key features of graphs should be determined with respect to these parameters and different types of graphs classified.

Component 1

Let, where  and .

1. Consider the special case where  and . Graph the function, its derivative function and an anti-derivative function.
2. Show how the graph of the derivative function can be obtained from the graph of the function, and how the graph of an antiderivative function can be obtained from the graph of the function.
3. Explain how differentiating the function leads to finding an antiderivative of the function.
4. Choose several suitable values of  and , differentiating the function and using the results of this to determine an antiderivative of the function.
5. Discuss the role of  and  in terms of finding an antiderivative function.

Component 2

Consider a function  that is the product of two functions  and , that is .

1. Use the approach developed above to show that:



1. Let , determine  for from 1 to 3 by finding the result for a given value
of  from previous results.
2. Determine a reduction formula for  in terms of , ,  and  and use this to find 
3. Describe the graphs of  for small values of *.*

Component 3

Carry out a similar investigation for integrals of the form: , where  and

to determine and illustrate the corresponding reduction formula.

Areas of study

The following content from the areas of study is addressed through this task.

|  |  |
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| **Area of study** | **Content dot point** |
| Discrete mathematics | - |
| Functions relations and graphs | 1, 2 |
| Algebra number and structure | - |
| Calculus | 1, 5, 6, 7 |
| Space and measurement | - |
| Probability and statistics | - |

Outcomes

The following outcomes, key knowledge and key skills are addressed through this task.

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| --- | --- | --- |
| **Outcome** | **Key knowledge dot point** | **Key skill dot point** |
| **1** | 3, 6, 7, 8 | 2, 7 |
| **2** | 1, 2, 3, 4, 6 | 1, 2, 4, 5, 7 |
| **3** | 1, 2, 3, 4, 5, 7 | 2, 3, 4, 5, 6, 7, 9, 10, 11 |