**[Trang Pham]** - Welcome to the 10 minute information bites of the topics which are considered to be new in the 2023 to 2027 study design. The 10 minute information bites will cover the concepts, and the examples of one, integration by parts, two, surface area of revolution, and three, logistic equation. We start off with an integration by parts. The method of integration by parts comes from the product rule of differentiation. It allows us to integrate a function which is written as a product. So we have the product rule of d, of u times v, dx, equals to the sum of u times dv/dx, and v times du/dx.

We then integrate both side with respect to x, and we have the equation as shown on the screen. By rearranging the equation from the previous slide, we obtain the following technique for integration which called integration by parts. We can use integration by parts to find an integral of u times dv/dx with respect to x provided that the integral of v times du/dx with respect to x exists. So we have derived the integration by parts formula, and that is the integration of u, times dv/dx with respect to x, equals to u, times v, take away the integration of v, times du/dx with respect to x. Let's have a look at an example. Find an integration of x squared, times cos dx. First of all, we let u equals to x squared. Differentiate this, we get the du/dx equals 2x. Next we let dv/dx equals cos , and we need to integrate this, and we get v equals to half times sin . Now we notice that the integration of x squared times cos dx is equivalent to the integration of u times dv/dx, dx which is the integration by parts formula. So it then equals to u times v, take away the integration of v times du/dx, dx. You notice on the left hand side, there is an arrow drawn from u to v, and then v going to du/dx.

That's just a little thing to help the students to remember how the formula started off. So the formula started off with u times v, and then the arrow drawn backwards, so that's take away the integration of v times du/dx. We are now substituting u equals x squared, v equals to half times sin , and du/dx equals 2x into the integration by parts formula. So we get x squared times half sin , take away the integration of 2x times half sin dx. Tidy up this equation, we get half times x squared, times sin , take away the integration of x times sin dx. From the previous screen, we need to find an integration of x times sin dx. So we are going to apply the integration by parts again. This time we'll let u equal to x, so du/dx equals to one, and we let dv/dx equals to sin . So we integrate this, and we get v equals to negative half cos .

We substitute all of these into the integration by parts formula. So the integration of x squared times cos dx is equal to half, times x square, times sin . Take away, all in the brackets, x times negative half cos which represents, u times v, take away the integration of one, times negative half cos dx which represents the integration of v, times du/dx, dx. Tidy up this, we get half times x squared, times sin , plus half x, times cos . Take away half times, the integration of cos dx, so we only have a tiny bit to do which is the integration of cos dx. So our final answer would be, half times x squared sin , plus half x, times cos .

Take away a quarter, times sin plus c. We are going to look at the second topic which is the area of revolution. First of all we are going to look at the rotation about the x-axis. So if the curve y equal to f of x from x equals to a, to x equals to b, it's rotated about the x-axis, then the area of the surface of revolution is given by A equals to 2pi, times the integration from a to b of y times the square root of one, plus dy/dx squared, dx. Teachers need to be aware that the formula will give only the lateral surface, not the ends. We also need to develop awareness that while the study design refers to the surface area of a solid of revolution, we can ask questions that will explore both cases. Here is the first example of finding the area of the surface of revolution when the curve is rotated about the x-axis.

Example, the curve given by y equals to square root of four, take away x squared where x is between negative one to one inclusive is rotated about the x-axis to form a solid of revolution. Find the total surface area of the closed volume of this solid. This question is required to find the total surface area of the closed volume of this solid. Hence, on top of using the area of the surface of revolution formula from the previous slide, to find the surface area, it also requires to find the area of the end of the solid which happens to be an area of two circles, each with a radius of square root of three. To find dy/dx, we need to re-write y as, y equals to four minus x squared, or to the power of half. Using the Chain Rule, we can find dy/dx equals to, negative x, over square root of four, minus x squared. Now let's recall the area of the surface of revolution formula from the the previous slide which is A equals to 2pi, integration from a to b, y times square root of one, plus dy/dx, all squared, dx.

We are now going to substitute a equals negative one, b equals to one, y equals to square root of four minus x squared, and dy/dx equals negative x, over square root of four minus x square into the area of the surface of revolution formula. So we get A equals to 2pi, integration from negative one to one, square root of four minus x squared, which represents y, square root of one plus x squared, over four minus x squared, dx. We tidy up the part underneath the square root of one plus x square over four minus x squared which is the same as a square root of four over, four minus x squared. So the area now becomes 2pi times the integration from negative one to one, square root of four minus x squared, times square root of four, over four minus x squared dx. This integration tidy up to be 2pi, integration from negative one to one, 2,dx.

So we integrate two, we get x, and we substitute the terminals into x, and so the surface area is now equals to 8pi. Now this is a curved surface area only, not the end bits yet. We need to now include the two ends of the solid which is we know they are the area of the two circles each with the radius of square root of three. So the total surface area of the closed volume of the solid is now equal to eight pi, plus two times, pi times square root of three squared, and that gives us a total of 14 pi square units. We are now going to look at the area of revolution which is rotating about the y-axis. So the formula is given as A equals to 2pi, integration from a to b, x times, square root of one, plus dx/dy all squared, dy. Please note that we use dx/dy here, not dy/dx when rotating around the y-axis, for convenience.

Please note that the form square root of one plus, dy/dx, or square, dx could also have been used. Maybe it is an exercise for teachers to prove equivalence at their leisure, or as a learning activity with students. So let's read through the example. The curve given by y equals to cube root of x is rotated about the y-axis to form a solid of revolution. Find the surface area of the part of this solid when x is from zero to eight inclusive. Note that the wording does not mention the total surface area, so only the curved surface of rotation would be required in this question.

So let's now have a look at the solution on the next slide. The integration formula is now with respect to y. The terminals are also the y value. So first of all we need to write x in terms of y, and we also need to have the terminals in terms of the y values. So y equals to cube of x becomes x equals to y cubed. Differentiate this, we get dx/dy equals to three y square. The interval of the x values from zero to eight, where then becomes the interval of y from zero to two, inclusive. We are now going to substitute all of these into the formula of A equals, 2pi, integration from a to b, x times, square root of one, plus dx/dy all squared dy, and that gives us 2pi, integration from zero to two, y cubed, times square root of one, plus in the brackets, we got three y square, close the bracket square, dy. Tidy up this and we get 2pi, integration from zero to two, y cubed, times square root of one, plus nine y to the power four, dy. Now we are going to use substitution method to integrate this. We let u equals to one plus nine, y to the power four. So du/dy gives us 36y cubed.

We also need to change the terminals from zero to two of y into the u values. So y equals to zero gives us u equals to one, and when y equals to two, it gives us the u value of 145. So now the integration becomes 2pi, integration from one to 145, y cubed, times u to the power of half, du divided by 36y cubed. Tidy up this part, we have, pi over 18, integration from one to 145, u to the power of half du. So we need to integrate u to the half. So this gives us pi over 27, u to the power of three on two, the terminals are from one to 145, substitute the terminals in, and we get the final answer of pi over 27, open the bracket, 145 times, root 145, minus one, close the bracket, square units. The last bit of the area of revolution is the surface area using parameters. The area of a surface of revolution formed by a parametric curve is found by using the given formula. It is assumed that f and g are differentiable on a to b inclusive, with f dash and g dash continuous.

For rotation about the x-axis, it is assumed that the curve is the graph of a function. The area of the surface formed by rotating a parametric curve about the y-axis can be found in a similar way by replacing g of t with f of t in this formula. An example of this case is as shown on the screen. Find the surface area of revolution formed when the curve defined by x equals to four thirds, times square root of t, plus one all cubed, and y equals to half, times t squared, and t is between zero to one inclusive is rotated about the x-axis. So please note that the wording will indicate that only the curved surface area would be required. We will go through the solutions in the next slide. From the formula, we can see that y equals to g of t. We also need dx/dt, and dy/dt. a and b are the t values from zero to one, which are given in the question. So we re-write x as four thirds, times t plus one, all to the power three on two.

Using the Chain Rule, we get dx/dt, equals to two, times t plus one to the power of half, and we have y equals half t squared. So dy/dt is equals to t. We substitute all this into the formula, and we get, A equals to 2pi, times the integration from zero to one, half t square times, square root of four times, t plus one, plus t squared dt. We tidy up this line and we get, pi times the integration from zero to one, t squared times t plus two, dt. We expand this and integrate this line, and we get, t to the power four over four, plus two t cubed divided by three, from zero to one. We substitute that one in, and our final answer comes up to be 11pi over 12 square units. And last but not least, the logistic equation. This is more likely to be a revised topic as most teachers will not have already incorporated into their teaching. However, it worth to keep in mind that if the rate is a negative quadratic, graphing the rate versus the y variable will provide an elegant means to identify the y value of the point of inflexion.

Let's have a look at the example here. The population P of t of bacteria in Petri dish satisfies the logistic differential equation given as dP/dt equals the 2P times, six minus, P over 8,000 where t is measured in hours, and the initial population is 4,000 bacteria. A. Find the maximum number of bacteria predicted by this model. b, find the number of bacteria when the population is growing the fastest, and c, solve the differential equation to find P as a function of t. First of all, we can see that this is a quadratic rate. An elegant solution may be found by utilising a sketch. So from the diagram we can see the maximum number of bacteria occurs when dP/dt equals to zero. Students might get this confused with finding P at the maximum rate instead. So now from the diagram, we know the maximum P occurs when dP/dt equals zero, which means 2P times six, minus P over 8,000, equals zero. We solve this equation, it gives us P equals zero, or six minus P over 8,000 equals zero, therefore P equals to 48,000 bacteria is the maximum.

Why the maximum rate occurs at 24,000 bacteria which is asked in the next question. Part b requires us to find the number of bacteria when the population is growing the fastest. The population is growing the fastest is actually when the derivative function is at maximum, and we can see it on the diagram. Since this is a positive quadratic function, hence maximum occurs at the turning point due to symmetry. It is the midpoint of the two P-intercepts, so therefore the population is growing fastest when P equals 24,000 bacteria. We were given dP/dt in the question. So let's flip this around and we got dt/dp equals to 4,000, over P, times 48,000 minus P. Using partial fractions, and u as an arbitrary variable with an initial population of 4,000, so now t equals to one over 12, times the integration from 4,000 to P of one on u, plus one over 48,000 minus u, du. This leads us to one on 12, in the brackets of log, absolute value of u, minus log of absolute value of 48,000, minus u, from 4,000 to P.

Substituting u equals to P, and 4,000. It leads us to an answer of one on 12, log of 11 P, over 48,000, minus P. We then rearrange this equation, so we get P equals to 48,000 over, one plus 11 times e to the power of negative 12 t, bacteria. And that brings us to the end of the 10 minute information bites on calculus. I hope you have found this presentation useful when planning the 2023 Specialist Maths Course. For further information, please contact, Michael MacNeill, VCAA Mathematics Curriculum Manager, and his details are as shown on this slide. Thank you so much for your interest and attention.

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